Inversion of the OIS Curve before the Great Recession and the Market Expectation of Monetary Policy

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Abstract

The short-term OIS curve is shown to be inverted before the Great Recession. The timely inversion begs the question whether the market anticipated the Federal Reserve’s interest rate cuts for the recession. Incorporating a database that precisely tracks the public information on the FOMC meeting schedule into the time-varying CIR model, we show that market expectations, instead of risk premiums, drive the dynamics of the OIS curve and explain the OIS curve inversion before the recession. Our empirical results indicate that the Federal Reserve was behind the curve before the recession but moved ahead the curve during the recession.

Keywords: OIS, FOMC, yield curve, term structure models

JEL Code: E4, E5

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1 Introduction

The overnight index swap (OIS) curve varied dramatically during 2002–2015, a period with more than one hundred Federal Open Market Committee (FOMC) meetings. Most strikingly, the OIS curve inverted before the Great Recession, which started in December 2007 and lasted through June 2009, according to the National Bureau of Economic Research (NBER). The timing of the OIS curve inversion is particularly thought-provoking. The spread between the three-month and one-month OIS rates became negative approximately a quarter before the recession (panel B of Figure 1). The spread between the six-month and one-month rates turned negative about half a year before the recession (panel C). The spread between the one-year and one-month rates started being negative in late 2006, preceding the recession by about a year (panel D). The timing of these inversions naturally begs the question whether the market anticipated the Federal Reserve’s interest rate cuts for the Great Recession.

Figure 1: The 1-month OIS rate and the term spreads of selected OIS rates from May 7, 2002 to December 31, 2015. The vertical gray area indicates the period of the Great Recession.

The question whether a yield curve reflects the expectation of market participants is important for monetary policies and for investment decisions as well. The answer to this question is the foundation for using the yield curve as a predictor of business cycles (see Harvey (1988) and Estrella and Hardouvelis (1991)). Traditionally, economists and practitioners watch the spread between the ten-year and two-year yields of Treasury bonds for signals of recessions. This yield spread was negative in fall 2006 and early 2007, during the year before the Great
However, the cause for the inversion of the long-term Treasury yield curve during 2006–2007 has spurred debate among economists. The issue in the debate is the cause of the inversion. Estrella and Trubin (2006) argue that the inversion results from the market’s anticipation that the Federal Reserve will cut interest rates to battle against the coming recession. By contrast, several studies attribute the inversion to other factors. Bernanke et al. (2011) argue that the Asian demand for long-term Treasury bonds caused long-term Treasury yields to drop below short-term yields. Kim and Wright (2005) argue that a drop in the inflation premium may have caused the Treasury yield curve to invert. Campbell et al. (2013) suggest that the negative beta of long-term Treasury bond returns led to the negative risk premiums in the bond yields.

The debate over the Treasury yield curve inversion is still alive and not limited to the academic circle. When the Treasury curve was flattening in summer 2018, the debate resurfaced in the industry (“Yield curve squeezed from both sides,” Wall Street Journal on July 6, 2018) and in the central bank (“Fed debates signal from yield curve”, WSJ on July 9, 2018). A well-known columnist recently argued that the Federal Reserve missed the recession signal of the 2006 yield curve inversion (Justin Lahart: “Fed shouldn’t ignore yield curve,” WSJ on July 23, 2018). An influential economist agrees that the Federal Reserve missed the signal in 2006 but maintains that other factors, such as the demand for Treasuries and the uncertainty of inflation, can cause the yield curve to invert (Burton Malkiel: “An inverted yield curve may not portend doom,” WSJ on July 31, 2018).

While a strong demand for long-term bonds, the premium for inflation, or the negative beta of long-term Treasury bond returns may drive the Treasury curve to invert, these are obviously unimportant for the short-term OIS. The short-term OIS, therefore, provides a unique setting to investigate the market expectation of the Fed’s monetary policy. In the literature, there has been no research explaining the dynamics of the OIS curve. Particularly, there has been no study showing and explaining the inversion of the OIS curve before the Great Recession. In this paper, we analyze how the market expectation of future interest rates drives the OIS curve dynamics, especially its inversion. The result, as well as the methodology, in our study of the OIS curve can be useful for understanding the yield curve dynamics in general.

An OIS contract exchanges a fixed interest rate (the OIS rate) for a compound overnight floating interest rate. For the OIS denominated in U.S. dollars, the overnight floating rate is the effective federal funds rate. OIS is an important tool for institutions to hedge interest rate risk and for investors to take strategic positions on the directions of debt or equity markets. In the modern financial market, practitioners and academics use OIS rates as measures of investors’ expectations of the average overnight rates (McAndrews et al., 2017). Major security
exchanges use OIS rates as benchmark default-free discount rates for security valuation and collateral settlement (Hull and White, 2012). The trading volume of OIS has become larger than the trading volume of other interest rate derivatives combined, according to reports by regulators and clearing institutions (CFTC, 2013).

Since the floating rate in the dollar-denominated OIS is the overnight federal funds rate, a critical element in the analysis of the OIS curve is the information on the timing of FOMC meetings, when the Federal Reserve sets the target for the overnight federal funds rate. The overnight federal funds rate fluctuates around the Federal Reserve’s target rate or within the target range.\footnote{The Federal Reserve’s target for the federal funds rate is a single point until December 16, 2008. Since then, the target has been a range.} If investors think the Federal Reserve may change the target at a FOMC meeting, they should price an OIS with a maturity date before the meeting differently from an OIS that matures after the meeting. The financial industry has been attentive to the precise information on the FOMC meeting schedule in pricing OIS.

We build a database that precisely tracks the public information on the FOMC meeting schedule each day. The source of the information is the Federal Reserve’s press release. In June of each year, the Federal Reserve releases the FOMC meeting schedule for the next calendar year. The Federal Reserve sometimes reschedules the dates of some meetings, and the rescheduled dates are also announced in the press release. Our database tracks both the original scheduling and the rescheduling of the FOMC meeting dates. We precisely incorporate the public information on the FOMC meeting schedule into OIS pricing.

We then decompose the OIS curve into the expectation component and the risk premium component. The results show that the OIS curve is mainly driven by the expectation component. When the OIS curve is inverted, the expectation component is negative while the risk premium component remains positive. The negative expectation component indicates that the OIS curve inversion before the recession reflects investors’ anticipation of the Federal Reserve’s rate cuts. Furthermore, we show that in the two years before summer 2008, the market expectation moved ahead of the Federal Reserve’s decisions on interest rates, but in the second half of 2008 the expectation trailed the Federal Reserve’s decisions. These results are evidence that the Federal Reserve was behind the curve before the recession but moved ahead the curve during the recession.\footnote{An interesting question beyond the scope of this paper is whether it is optimal for the Fed to be ahead of the curve. Policy makers appear to have concerns about the Fed falling behind the curve. On November 27, 2007, Larry Summers argued in Financial Times that “the Fed has to get ahead of the curve and recognize—as the market already has—that levels of the federal funds rate that were neutral when the financial system was working normally are quite contractionary today.” Bernanke (2015) recalls in his memoir that he tried in early 2008 to get the Fed ahead of the curve.}

We need a dynamic pricing model to extract the expectation component of the OIS term.
structure. Our approach to OIS pricing is based on the time-varying CIR model, in which the reversion center of the instantaneous rate varies over time. Cox et al. (1985) derives the model as an equilibrium in an economy with a well-specified production function and risk-averse consumer-investors. This model restricts the latent instantaneous rate to be positive, particularly suitable for analyzing the interest rates that were close to zero. We adapt the model so that its reversion center is consistent with the information about the FOMC meeting schedule. The result is a step function for the reversion center, which is constant during each intermeeting period but can be distinct in separate intermeeting periods. The CIR model with this special time-varying reversion center has a closed-form formula for term rates, very convenient for application to the OIS rates. This CIR model retains the tractability of the classic CIR model, which has a constant reversion center, but is far more flexible for fitting a large cross section of daily time series of interest rates. The estimated reversion center delivers a unique comparison between the market expectation in the OIS curve and the Federal Reserve’s target for the federal funds rate.

Our analysis of the OIS curve complements a large literature that explains the dynamics of the Treasury yield curve. Numerous studies link the yield curve to macroeconomic conditions and monetary policy. Those studies include Ang and Piazzesi (2003), Estrella and Hardouvelis (1991), Hördahl et al. (2006), Ang et al. (2008, 2011), Rudebusch and Wu (2008), and Joslin et al. (2013, 2014), among others. The link between the Treasury yield curve and FOMC meeting dates has been investigated by Piazzesi (2005), Heidari and Wu (2010), and Kim and Wright (2014). Ang and Bekaert (2002), Bansal and Zhou (2002), and Dai et al. (2007), among others, explain the dynamics of the Treasury yield curve by regime switching of business cycles. Hördahl et al. (2015) examine the Treasury yield curve’s response to the revisions in expectations at government economic announcements. A distinction of our analysis from the existing literature is that we incorporate the precise FOMC meeting schedule into the study of daily short-term interest rates.

Our results contribute to the understanding of the yield curve at the short end. Longstaff (2000) presents evidence on the expectations in the very short-term interest rates (maturities of three months or shorter). Corte et al. (2008) confirm that the very short-term interest rates are mostly determined by expectations. Our results about the importance of the expectation in short-term OIS rates dovetail the existing empirical evidence for the very short terms but do not ignore the risk premiums for medium terms. Academics have noted the challenge of explaining the short-term interest rates. For instance, Singleton (2006) conjectures that the short end of the yield curve may require a special factor. Our analysis suggests that the FOMC

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3The time-varying CIR model, which is rarely noted in the literature, is different from the single-factor or multi-factor CIR models typically seen in applications. We will discuss the differences in Section 3.
meeting schedule holds a key to the dynamics of short-term interest rates.

We organize the rest of the paper as follows. Section 2 reviews the OIS market and discuss the data on OIS rates. This section also describes the database of the public information on the FOMC meeting schedule. Section 3 develops our approach to OIS pricing that incorporates the precise public information on the FOMC meeting schedule. This section also outlines our econometric methodology. Section 4 presents the main empirical results, and Section 5 concludes the paper.

2 OIS and FOMC

2.1 The OIS Market

In an OIS contract, a party exchanges a fixed interest rate for the compound overnight floating rate with another party. The fixed rate is called the OIS rate. In the OIS denominated in U.S. dollars, the floating rate is the effective federal funds rate. There are also OIS contracts denominated in other currencies. For example, the OIS contracts denominated in euros use the Euro Overnight Index Average (EONIA) as the floating rate. The OIS contracts denominated in British pounds use the Sterling Overnight Index Average (SONIA) as the floating rate. We focus on the dollar-denominated OIS, but our analysis applies to the OIS contracts denominated in other currencies.

Like other interest rate swaps, an OIS is settled by cash at the end of each payment period. A payment period is also known as a “calculation period.” While OIS with maturities longer than one year follow other swaps to use either three months or one year as a payment period (Hull and White, 2012), a short-term OIS has a single payment period. This means that a short-term OIS has only one cash flow and the cash flow is on the maturity date. Therefore, a short-term OIS rate is equivalent to the yield on a discount bond with the same initiation date and maturity date as the OIS. We show this equivalence in Appendix A.1.

An OIS can be initiated on any business day, and the maturity can be customized to any length, from a few days to many years. The flexibility of the dates for initiation and maturity makes OIS a useful tool for institutions that need to hedge the risk of overnight funding. This flexibility also makes OIS an effective tool for money managers to bet on FOMC meeting decisions. Indeed, there are special OIS, called meeting-to-meeting OIS, which set maturity dates specifically for speculating on FOMC meeting outcomes.

An OIS may carry counterparty credit risk, but the credit risk premium is likely to be small because the two sides of the contract do not exchange the principal. Like other interest rate

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4Like other short-term money market instruments, short-term OIS use the actual/360 day-counting rule.
 swaps, the two sides of an OIS contract exchange only the difference in the accrued interests. This is called netting. There is no concern of losing the principal value in an OIS contract. Duffie and Huang (1996) show that netting substantially reduces the effect of counterparty credit risk on swap rates. Their calculations suggest that for a typical swap, a fixed-rate counterparty with a credit spread of one hundred basis points (bps) translates to a credit premium of one basis point (bp) in the swap rate.

Counterparty credit risk should be further mitigated by cash collateral, which has gradually become standard as the OIS market develops. The International Swaps and Derivatives Association (ISDA) standardizes the calculation and requirement of cash collateral for OIS. It eliminates the use of non-cash collateral for OIS to minimize credit risk. One of the ISDA’s objectives is to support the adoption of OIS rates as the benchmark risk-free rates for security valuation and settlement (see ISDA, 2013, *Standard Credit Support Annex*). If an OIS counterparty defaults, the cash collateral covers the loss accrued during the payment period. Johannes and Sundaresan (2007) have shown that collateral mitigates the counterparty credit risk in long-term interest-rate swaps. Hull and White (2012) analyze the effect of netting and collateral on OIS and support using OIS as default-free discount rates. Wang and Yang (2018) provide empirical evidence that credit risk premiums in the short-term OIS rates were small prior to 2007 and disappeared since then.

The industry practice appears to align with the ISDA’s goal in adopting OIS rates as benchmark risk-free rates. The three-month OIS rate has been subtracted from Libor of the same term to create the widely watched Libor-OIS spread. This spread is used as an index of the credit risk premiums in interbank loans. In December 2007, the Federal Reserve started to use OIS rates as risk-free rates that floor the bids for term loans in the Term Auction Facility. In June 2010, the LCH switched to OIS rates for discounting swaps. In July 2010, the International Derivatives Clearing Group moved to use OIS rates for discounting derivative securities. In August 2011, the Chicago Mercantile Exchange announced the use of OIS rates in settling collaterals.

OIS contracts are actively traded, and the size of a transaction is typically large because OIS traders are institutions. Based on the data provided by MarkitSERV to regulators, the Commodity Futures Trading Commission (2013) reports that there are 12,816 transactions of OIS during the three months from June 1 to August 31 in 2010. Those transactions amount to $16.878 trillion of notional value. This is 37% of the total notional value of interest rate swaps transacted during these three months. The average notional value of an OIS transaction is $1.293 billion during the same period. Based on the data compiled from the Depository Trust

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5However, several studies demonstrate that the Libor-OIS spread contains both credit premium and liquidity premium. For examples, see McAndrews et al. (2015) and Schwarz (2016).
& Clearing Corporation’s (DTCC) real time Swap Data Repository (SDR), the CFTC reports that 66% of the transactions of dollar-denominated OIS are block trades, which are large trades with minimum size requirements. For example, the minimum size for a block trade of three-month OIS is $4 billion.

### 2.2 OIS Rates

As we pointed out earlier, a short-term OIS rate is equivalent to the yield of a zero-coupon discount bound. This equivalence makes short-term OIS rates ideal for the application of dynamic term structure models. Since those models typically produce pricing formulas for the yields of discount bonds without coupons, researchers need to extract the yield curve of zero-coupon discount bonds if the data are yields of coupon bonds. For example, because interest rate swaps on Libor have multiple payment periods, researchers have to estimate the zero-coupon yield curve by bootstrapping from the swap curve. Bootstrapping can potentially be imprecise, and the method used in bootstrapping may affect econometric inference. Fama and Bliss (1987) bootstrap zero-coupon yield curves from the same set of coupon bond rates using two different procedures. Singleton (2006) shows that the yield curves produced by the two procedures have different properties and lead to different empirical results. By contrast, short-term OIS rates are already yields of discount bonds, and thus are free from the distortion of bootstrapping procedures.

The short-term OIS rates in our analysis are the daily OIS indices of monthly maturities up to one year. The data, obtained from Bloomberg, cover the period from May 7, 2002 to December 31, 2015. The FOMC held a meeting on May 7, 2002, and it did not change the target rate at the meeting. Since then, Bloomberg provides the daily OIS indices for maturities of every month up to a year. Table 1 presents the summary statistics of the short-term OIS rates in our data. The table shows that the slope of the short-term OIS curve is positive on average. The OIS rates of different terms have similar standard deviations, which are about 1.8%, and the rates vary in a wide range over time, from about 6 bps to over 500 bps.

Figure 1 shows the variations in the OIS rates. These variations occurred over a period roiled by a severe financial crisis in 2008. The economy went through the expansion in 2002–2004, the real estate boom in 2005–2006, the Great Recession in 2007–2009 (indicated by the shaded period in each panel of the figure), and the slow recovery in 2010–2015. The variations in the OIS rates coincide with a tumultuous history with more than one hundred FOMC meetings. During the same period, the Federal Reserve’s monetary policy changed from the gradual tightening in 2004–2005 to the extraordinary easing starting from the end of 2008. The changes in the Federal Reserve’s monetary policy caused phenomenal variations in the
<table>
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<th>Stdev</th>
<th>Min</th>
<th>Median</th>
<th>Max</th>
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<tr>
<td>One-month OIS rate</td>
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<td>0.22</td>
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<td>1.80</td>
<td>0.06</td>
<td>0.26</td>
<td>5.48</td>
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<td>Five-month OIS rate</td>
<td>1.49</td>
<td>1.80</td>
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<td>0.28</td>
<td>5.52</td>
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<td>Six-month OIS rate</td>
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<td>1.80</td>
<td>0.06</td>
<td>0.29</td>
<td>5.55</td>
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<td>1.81</td>
<td>0.06</td>
<td>0.31</td>
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<tr>
<td>Eight-month OIS rate</td>
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<td>1.81</td>
<td>0.07</td>
<td>0.33</td>
<td>5.60</td>
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<td>One-year OIS rate</td>
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<td>1.80</td>
<td>0.06</td>
<td>0.48</td>
<td>5.67</td>
</tr>
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</table>

Table 1: The summary statistics (in percentage points) of the daily observations of the OIS rates from May 7, 2002 to December 31, 2015.

overnight federal funds rate: it dropped to 1% in 2003, climbed above 5% in 2006, and then plunged below 25 bps in December 2008.

In Figure 1, the term spreads were positive before the Great Recession, turned negative as the economy was heading to the recession, and became positive again by the end of 2008. As we have highlighted at the beginning of this paper, the timing of the OIS curve inversion is particularly thought-provoking. Panel B shows that the spread between the three-month and one-month OIS rates became negative approximately a quarter before the recession. Panel C shows that the spread between the six-month and one-month rates turned significantly negative about half year before the recession. Panel D shows that the spread between the one-year and one-month rates started being negative in late 2006, preceding the recession by about a year. These inversions appear to predict the recession. To the best of our knowledge, the intriguing timing of the OIS curve inversion has never been noted or investigated in the literature prior to this paper.

It is useful to compare the OIS rates with Libor and Treasury bill (Tbill) rates because the latter two types of rates are often used as reference interest rates. It is widely recognized that Libor contains credit risk premiums and Tbill rates command liquidity premiums. In Figure 2, we plot the time series of the three-month and six-month OIS rates (panels A and B), and the spreads between the Libor and OIS rates of the same term (panels C and D).

The OIS rate typically lies below the Libor for the same term. Panel C of Figure 2 shows that the three-month OIS rate was lower than the three-month Libor by a few basis points before mid 2007. However, the three-month OIS was nearly 100 bps lower than the three-month Libor in late 2007 and in most days of 2008. The Libor-OIS spread shot up to hundreds of
Figure 2: The 3-month and 6-month OIS rates, Libor-OIS spreads, and Tbill-OIS spreads from May 7, 2002 to December 31, 2015. The vertical gray area indicates the period of the Great Recession.

basis points in late 2008. Panel D shows similar time-series variations in the Libor-OIS spread for the six-month maturity. The concern over banks’ credit risk was a major reason for the elevated Libor-OIS spread in 2007 and 2008 (Bernanke, 2013). McAndrews et al. (2017) show that liquidity of the interbank market also plays an important role in the spike of the Libor-OIS spread during the financial crisis.

The OIS rate typically lies above the Tbill rate for the same term. Panel C of Figure 2 shows that the three-month OIS rate was higher than the three-month Tbill rate by about 30 bps in 2006 and in the first half of 2007. In the second half of 2007 and in 2008, the three-month OIS rate was often 100 bps higher than the three-month Tbill rate. Panel D shows that the properties of the six-month rates are similar to the three-month rates. The Tbill-OIS spreads plotted in panels C and D of Figure 2 are consistent with the conventional view that the special treatment of Treasury bills in institutions, accounting, and taxation push down the Tbill rates (Longstaff, 1990).

2.3 FOMC Meeting Schedule

Since OIS rates are sensitive to the FOMC meeting schedule, we build a database to track the public information on the FOMC meeting schedule. The FOMC schedules eight meetings each year, with one meeting in roughly every six weeks. The scheduled meeting dates are made
public through the Federal Reserve’s press release. Since the FOMC issues a policy statement at the end of each meeting to announce its decision on the target for the federal funds rate, investors may revise their expectation of the future course of interest rates after a FOMC meeting. Even when the FOMC does not change the target, investor may still revise expectations because the FOMC provides forward guidance on the future path of its monetary policy in policy statements.

The database is based on the Federal Reserve’s press release. Typically, on a day in June, the Federal Reserve releases the tentative meeting dates for the next year. Right before the press release, investors know the meeting dates for only the next six months. Immediately after the press release, investors know the meeting dates for the next 18 months. As each day passes by, the time to the furthest scheduled meeting date shortens until the release of the meeting schedule for the next year. On the day right before the release of the following year’s meeting schedule, the time to the furthest scheduled meeting date is the shortest, which is slightly more than six months. Hence, on each day from June to the end of the current year, the meeting dates for at least a full year ahead are publicly known.

The Federal Reserve may reschedule some meeting dates, and it makes rescheduling public through its press release. Our database includes the information on rescheduling. Therefore, the database tracks precisely the publicly available information on the FOMC meeting dates and avoids any forward-looking bias. We do not assume that investors know all the actual dates of the future FOMC meetings.

The FOMC occasionally holds unscheduled meetings. At these meetings, the FOMC usually reviews economic and financial developments without changing monetary policy or issuing statements. However, the FOMC cut the target rate at two unscheduled meetings during our sample period, both in 2008: the FOMC cut the target rate from 4.25% to 3.5% at the meeting on January 22 and from 2% to 1.5% at the meeting on October 8. Although the Federal Reserve’s decisions at these two unscheduled meetings affected OIS rates, the market became aware of the meetings only after the Federal Reserve announced the meeting decisions. Thus, OIS rates incorporate the unscheduled meeting decisions only after the meeting announcements.

There have been many changes in the Federal Reserve’s open market operations since the global financial crisis. Traditionally, the Federal Reserve Bank of New York used repo (and reverse repo) transactions with the primary dealers, who have accounts at clearing banks, to reduce temporary deviations of the federal funds rate from the target. These transactions modify reserves in the banking system and affect the federal funds rate. The Federal Reserve suspended these operations in November 2008 when the FOMC directed the Federal Reserve Bank of New York to add reserves to the banking system through large-scale purchases of mortgage-
backed securities. The asset purchase program added so much liquidity to the banking system that it exceeded the Federal Reserve’s ability to offset by repo operations. The suspension of the repo operations lasted until December 16, 2015 when the FOMC raised the target to a range located 25 bps above zero.

About the same time when the Federal Reserve suspended its repo operations, it started to pay interest to depository institutions on excess reserves. On October 6, 2008, the FOMC announced that the interest on excess reserves (IOER) was effective on October 1, 2008. The FOMC set the initial IOER to be different from the target for the federal funds rate, but eventually set it at the upper bound of the 0~25 bps target range on December 17, 2008. The FOMC kept the IOER at the same level until December 16, 2015, when it raised the IOER to 0.50%. Theoretically, the IOER should serve as a floor of the federal funds rate: if the market rate of overnight funds is lower than the IOER, member banks of the Federal Reserve can arbitrage by borrowing funds from the market and holding money in the reserve accounts. However, the federal funds were traded regularly at rates well below the IOER. The federal funds rate was always within the target range during that period without the need for the Federal Reserve to conduct repo operations.

A consequence of the IOER is that the trading volume in the federal funds market fell substantially when banks held excess reserves in their accounts at the Federal Reserve. The daily volume of the overnight federal funds declined from more than $200 billion before the global financial crisis to about $60 billion by the end of 2012, based on the annual report of the Federal Reserve Bank of New York (2015). Since 2012, the daily volume has remained stable around $60 billion. The New York Fed reports that the overnight federal funds market has remained linked to other money market rates, despite the decline in the trading volume. Afonso et al. (2011) reach a similar conclusion on the federal funds market of 2008. The OIS rates are expected to be more closely linked to the federal funds rate than the other interest rates because the payoffs of OIS contracts are determined by the compound federal funds rate.

3 A Model of the OIS Term Structure

3.1 Pricing OIS with the FOMC Meeting Schedule

To incorporate the information on the FOMC meeting schedule, we build up a model on the original time-varying CIR model introduced by Cox et al. (1985). The original model is a dynamic term structure model with a sound economic foundation. The model is the result of

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6The Federal Reserve Bank of New York (2014) attributes the failure of arbitrage to large lenders such as Fannie Mae and Freddie Mac, which do not have reserve accounts to earn interest from the Federal Reserve, and to the changes in regulations that eliminate banks' economic benefit of arbitrage.
a general equilibrium framework with a well-specified production technology and risk-averse agents. In this model, the instantaneous rate \( r_t \) summarizes the economic condition and follows a mean-reverting, square-root process:

\[
d r_t = \kappa (\theta_t - r_t) \, dt + \sigma \sqrt{r_t} \, dw_t ,
\]

where \( \kappa \) and \( \sigma \) are positive constants, \( \theta_t \) is a positive, deterministic function of time, and \( w_t \) is a Wiener process. The diffusion coefficient \( \sigma \sqrt{r_t} \) is the local volatility. The instantaneous rate \( r_t \) is always positive in this model and tends to revert to \( \theta_t \). Thus, \( \theta_t \) is called the reversion center of the instantaneous rate. Since the reversion is faster if \( \kappa \) is larger, \( \kappa \) measures the reversion speed.

In the original time-varying CIR model, the term rate, i.e., the yield of a discount bond initiated at time \( t \) and to mature at time \( t + s \) is

\[
y_t(s) = s^{-1} [ b(s) r_t - c_t(s) ] , \tag{2}
\]

\[
b(s) = \frac{2(e^{\gamma s} - 1)}{\delta(e^{\gamma s} - 1) + 2\gamma} , \tag{3}
\]

\[
c_t(s) = -\kappa \int_0^s \theta_{t+u} b(u) du , \tag{4}
\]

where \( \gamma^2 = (\kappa + \lambda)^2 + 2\sigma^2 \) and \( \delta = \gamma + \kappa + \lambda \). Cox et al. (1985) derive the time-varying CIR model and the formula of the yield. In the formula, \( \lambda \) is a parameter for the market price of risk. More specifically, a negative (positive) \( \lambda \) implies a positive (negative) risk premium. If \( \lambda \) is zero, the term rate contains no risk premium and reflects purely the expectation of the path of the future instantaneous rate.

There is no analytical solution of \( c_t(s) \) for a general \( \theta_t \), and thus no closed-form formula for the term rate in the original time-varying CIR model. Moreover, the term rate in equation (2) depends on \( \theta_{t+u} \) for all \( u \in (0, s) \), instead of a limited number of parameters. Unless we impose additional assumptions on \( \theta_t \), econometric estimation of the original time-varying CIR model is difficult.

Cox et al. (1985) consider a simple assumption about the reversion center. They assume that \( \theta_t \) is a constant \( \theta \). Then, stochastic process (1) becomes \( dr_t = \kappa (\theta - r_t) \, dt + \sigma \sqrt{r_t} \, dw_t \), and \( c_t(s) \) in equation (4) becomes independent of \( t \). Under that assumption, the term rate admits a simple closed-form formula: \( y_t(s) = s^{-1} [ b(s) r_t - a(s) \theta ] \), where \( b(s) \) is the same as in equation (3) and \( a(s) \) is

\[
a(s) = \frac{2\kappa}{\sigma^2} \left[ \frac{\delta}{2} s + \log \frac{2\gamma}{\delta(e^{\gamma s} - 1) + 2\gamma} \right] . \tag{5}
\]
This is the classic CIR model, in which the term rate is an affine function of the instantaneous rate. The closed-form formula is instrumental for econometric estimation, as shown by Brown and Dybvig (1986), Gibbons and Ramaswamy (1993), Pearson and Sun (1994), Lamoureux and Witte (2002), and many others.

Neither the original time-varying CIR nor the classic CIR with a constant reversion center accounts for the information on the FOMC meeting schedule. Recent studies observe that the FOMC meeting decisions, as well as other economic events, may cause market conditions to change discontinuously. Piazzesi (2005) specifies the target rate as a process that jumps on FOMC meeting dates. Heidari and Wu (2010) include factors that jump only on FOMC meeting dates. Kim and Wright (2014) highlight the jumps resulted from scheduled government announcements. Some other studies introduce regime switches to account for the discontinuity of the interest rate process. Examples of such studies are Bansal and Zhou (2002), Ang and Bekaert (2002), and Dai, Singleton, and Yang (2007), among others.

Motivated by these developments in the literature, we modify the time-varying CIR model to make the reversion center adapted to the FOMC meeting schedule. Specifically, the reversion center $\theta_t$ is specified to be a step function of time and is discontinuous on the FOMC meeting dates. Thus, $\theta_t$ is constant during each intermeeting period but may change on a FOMC meeting date. The step function is compatible with the economic environment in which the Federal Reserve adjusts the target for the federal funds rate at FOMC meetings and keeps the target unchanged in each intermeeting period. Not only is the step function flexible enough to account for the changes of monetary policy, it also produces a closed-form formula for the term rates, which is convenient for econometric identification.

Suppose at time $t$ investors know $k-1$ scheduled FOMC meeting dates up to time $t+s$. These FOMC meeting dates divide the interval $(t, t+s]$ into $k$ subintervals. The reversion center is constant in each subinterval. Let the values of $\theta_t$ during $(t, t+s)$ be $\theta_t(s) = (\theta_1, \cdots, \theta_k)'$ and let $\tau_i$ for $i = 1, \cdots, k-1$ be the meeting dates during $(t, t+s)$. For convenience of notation, let $\tau_0 = t$ and $\tau_k = t+s$. The step function of $\theta_t$ during $(t, t+s]$ is

$$\theta_t = \sum_{i=1}^{k} \theta_i I(\tau_{i-1} < t \leq \tau_i), \quad (6)$$

where $I(\cdot)$ is the indicator function that equals 1 or 0 depending on whether the logical condition in its argument is true or not. A meeting time $\tau_i$ is a discontinuity point of $\theta_t$ if $\theta_{i-1} \neq \theta_i$, and a continuity point if $\theta_{i-1} = \theta_i$. Since the FOMC announces its decision after the meeting, the step function is left continuous. We refer to equation (6), along with equation (1), as the CIR-FOMC model. Figure 3 illustrates the FOMC meeting dates and the steps of the reversion center involved in the time-$t$ price of a discount bond that matures at time $t+s$. 
Figure 3: The steps of the reversion center, $\theta_t = \theta_i$ for $t \in (\tau_{i-1}, \tau_i]$, involved in pricing a discount bond at time $t$ with the time to maturity $s$ in the CIR-FOMC model. The FOMC meeting dates are $\tau_i$ for $i = 1, \ldots, k - 1$, with $\tau_0 = t$ and $\tau_k = t + s$.

It is important to point out that the assumption of constant $\theta_t$ in an intermeeting period does not imply a constant expectation during the period. The expectation of the future interest rate depends on both $\theta_t$ and $r_t$. Although $\theta_t$ is constant in an intermeeting period, $r_t$ still changes when new information about the economic condition arrives each day, as in the equilibrium framework laid out by Cox et al. (1985). The CIR-FOMC model adapts $\theta_t$ to the FOMC meeting schedule to separate the effects of the FOMC meeting decisions from the effects of economic events and information on other days. That is, $\theta_t$ captures the effects of the FOMC decisions on the meeting days, while $r_t$ captures the effects of the economic conditions that can change at any time. We should stress that $\theta_t$ is not the Fed's target for the federal funds rate because $\theta_t$ is the reversion center of the instantaneous rate whereas the Fed's target is set for the overnight rate. Moreover, we cannot assume that $\theta_t$ is the Fed’s target when the target is a range instead of a number.

In the CIR-FOMC model, we have a closed-form formula for the term rate:

$$y_t(s) = s^{-1} \left[ b(s) r_t - a_t(s)' \theta_t(s) \right],$$

where $b(s)$ is the same as in equation (3) and $a_t(s)$ is an $n \times 1$ vector with the $i$th element defined by

$$a_{ti}(s) = \frac{2 \kappa}{\sigma^2} \left[ \frac{\delta}{2} (\tau_i - \tau_{i-1}) + \log \frac{\delta(e^{\gamma(t+s-\tau_i)} - 1) + 2\gamma}{\delta(e^{\gamma(t+s-\tau_{i-1})} - 1) + 2\gamma} \right].$$

The pricing formula can be derived from equations (2)–(4) by showing $c_t(s) = a_t(s)' \theta_t(s)$. This, in turn, is obtained by substituting the step function (6) into equation (4) and then integrating over each subinterval individually.\(^7\) The formula shows that the term rate impounds the effects

\(^7\)Wang (1996) originally derives the pricing formula in an attempt to match the step function with the swap maturity dates to better calibrate the CIR model for swaps. A new perspective of our pricing formula for the OIS is to link the step function of the reversion center to the FOMC meeting schedule.
of both the current and future reversion centers. More specifically, the term rate depends on both the reversion center in the current intermeeting period and the reversion centers in the future intermeeting periods up to time $t + s$. The effect of reversion center $\theta_i$ is determined by $a_{t_i}(s)$, the $i^{th}$ element in vector $a_t(s)$. The relation of $a_{t_i}(s)$ to the meeting dates is given by equation (8).

The CIR-FOMC model is an extension of the classic CIR model. If there are no scheduled FOMC meeting dates until the maturity date, only the reversion center in the current intermeeting period is relevant. In this case, the reversion center is a constant until the maturity date. Then, equations (7) and (8) reduce to the term rate formula for the case of a single constant reversion center.

The CIR-FOMC model occupies an advantageous middle ground between the original time-varying CIR model and the CIR model with a constant reversion center. The CIR model with a general time-varying reversion center does not have a tractable solution of term rates, which is necessary for econometric estimation. The CIR model with a constant reversion center is too restrictive to fit the observed variations in the term structure of interest rates. The CIR-FOMC model offers a closed-form formula for term rates and has a time-varying reversion center. The special, and most important, feature of the CIR-FOMC model is that the time variation of the reversion center is adapted to the FOMC meeting schedule.

The CIR-FOMC model is not an affine term structure model, although it may appear so if one imagines that $\theta_t$ is another latent factor. Duffie and Kan (1996) stress that an important property of affine models is time-homogeneity: the form of the affine function should depend only on the time to maturity ($s$) but not on the current time ($t$). By contrast, $a_t(s)$ in the CIR-FOMC model depends on $t$.

The CIR-FOMC model resembles regime-switching models but is different from them. The regime-switching term structure models such as Bansal and Zhou (2002), Ang and Bekaert (2002), and Dai, Singleton, and Yang (2007) specify stochastic processes with different parameters across a small number of latent regimes. In those models, regimes are usually unobservable, and the timing of regime changes is stochastic. Although the intermeeting periods in the CIR-FOMC model are analogous to regimes, they are observable and deterministic. Thus, changes in the reversion center in the CIR-FOMC model are pre-scheduled events, different from the stochastic switches in regime-switching models.

The CIR-FOMC model is different from the models that specify the reversion center as a continuous stochastic process (e.g., Balduzzi et al. (1996, 1998) and Chen (1996), among others). Dai and Singleton (2000) show that those models are special cases of multi-factor dynamic term structure models. In Appendix A.3, we discuss the poor empirical performance of multi-factor models in explaining the dynamics of the OIS curve.
The CIR-FOMC model abstracts from the potential risk premiums associated with the jumps on FOMC meeting dates. To investigate the potential effects of jump risk premiums, we estimate a jump Gaussian model, which introduces a factor that jumps on FOMC meeting dates. This model builds on Piazzesi (2009), Heidari and Wu (2010), and Kim and Wright (2014). In Appendix A.4, we show that the jump Gaussian model and the CIR-FOMC model give similar results for the period before the global financial crisis. For the period since the crisis, when the federal funds rate was mostly near zero, the jump Gaussian model produces counter-intuitive empirical results. Gaussian models do not restrict the instantaneous rate to be positive. The challenge for Gaussian models in fitting near-zero interest rates has been recognized in the literature.\(^8\)

### 3.2 Econometric Method

Our empirical study follows Lamoureux and Witte (2002), who conduct a Bayesian analysis of the multi-factor CIR models. The estimation of our model uses only the OIS rates with maturities within the time covered by the published meeting schedules, avoiding the unrealistic assumption that investors know all the future FOMC meeting dates. More specifically, on each day the estimation uses the maturities up to but not beyond the furthest scheduled meeting date. Because the time to the furthest scheduled meeting date varies over time, the maturities of the OIS rates used in the estimation change day by day. For each day, the estimation uses the OIS rates with maturities up to at least six months or more but excludes the OIS rates of maturities longer than a year or beyond the end of the published meeting schedule. Therefore, the data in our estimation are daily observations of twelve or fewer OIS rates.

Let \(y_{tj} = y_t(s_j)\) be the \(j\)th OIS rate with term \(s_j\) observed on date \(t\). Suppose \(m_t\) is the total number of the OIS rates on date \(t\) used in the estimation. Since the longest maturity on each day used in the estimation can be six months or longer (up to one year), \(m_t\) ranges from six to twelve. Let vector \(\theta_t(s_j)\) be the reversion centers involved in pricing \(y_t(s_j)\) in the CIR-FOMC model, and let \(\Theta\) be the vector of all the reversion centers during the sample period. The remaining parameters are \(\phi = (\kappa, \sigma, \lambda)'\). Furthermore, we introduce the following vectors:

\[
A_t = \left( a_t(s_1)' \theta_t(s_1) / s_1 , \cdots , a_t(s_{m_t})' \theta_t(s_{m_t}) / s_{m_t} \right)',
\]

\[
B_t = \left( b(s_1) / s_1 , \cdots , b(s_{m_t}) / s_{m_t} \right)'.
\]

Both \(A_t\) and \(B_t\) are functions of \(\phi\), and \(A_t\) depends on \(\Theta\). It follows from equation (7) that the model-implied interest rates are \(B_t r_t - A_t\).

---

\(^8\)See Kim and Singleton (2012), Priebsch (2013), Christensen and Rudebusch (2015), and Bauer and Rudebusch (2016), among others.
Following Lamoureux and Witte (2002), we assume that all observed interest rates contain pricing errors. This allows the inclusion of a large cross section of OIS rates in our estimation. The pricing errors have different variances for different maturities and are assumed to follow independent normal distributions. Thus, the $j^{th}$ observed interest rate on date $t$ is
\[ y_{tj} = B_{tj}r_t - A_{tj} + \epsilon_{tj}, \quad \epsilon_{tj} \sim N(0, \omega_j^2), \quad \text{for } j = 1, \cdots, m_t, \]
where $\omega_j^2$ is the variance of the pricing error. Since the estimation uses at most twelve terms of OIS rates, we need to estimate $\omega_j$ for twelve maturities. We denote the variance matrix of the pricing errors by $\Omega = \text{diag}(\omega_1^2, \cdots, \omega_{m_t}^2)$. The sub-matrix of $\Omega$ relevant to the rates on date $t$ used in the estimation is $\Omega_t = \text{diag}(\omega_1^2, \cdots, \omega_{m_t}^2)$.

By $Y$ we denote the collection of the OIS rates on all $T$ days in the estimation. We can obtain the likelihood function of the observed interest rates conditioning on the realization of the instantaneous rate, although the instantaneous rate $r_t$ is unobservable. Let $R$ be the instantaneous rates on all $T$ days. The conditional likelihood function of $Y$ has the following expression:
\[ L(Y | R, \phi, \Theta, \Omega) = \prod_{t=1}^{T} L_t(Y_t | r_t, \phi, \Theta, \Omega), \]
\[ L_t(Y_t | r_t, \phi, \Theta, \Omega) \propto |\Omega_t|^{-1/2} e^{-\frac{1}{2} (Y_t + A_t - B_t r_t) \Omega_t^{-1} (Y_t + A_t - B_t r_t)'}, \]
where $|\Omega_t|^{-1/2} = (\omega_1 \cdots \omega_{m_t})^{-1}$. To obtain the marginal likelihood function of the observed OIS rates, we need to calculate the integral over the unobservable instantaneous rates on $T$ days:
\[ L(Y | \phi, \Theta, \Omega) = \int_R L(Y | R, \phi, \Theta, \Omega) p(R | \phi, \Theta) dR, \]
where $p(R | \phi, \Theta)$ is the probability density of $R$ implied by the CIR-FOMC model.

The maximum likelihood method is not a feasible method for estimating the CIR-FOMC model because the marginal likelihood function in equation (14) is intractable. First, the optimization of the likelihood function does not have a closed-form solution when there is no closed-form formula for the integration. Second, the numerical procedures for both integration and optimization suffer from the curse of dimensionality. The integration has a very large dimension, which is $T = 3563$, because we need to integrate over $r_t$ on 3,563 trading days in our sample. Not only is the computation intensive, but the numerical error also grows quickly with the dimension. In addition, the likelihood function $L(Y | \phi, \Theta, \Omega)$ needs to be optimized

\[ 9 \text{An alternative approach is to impute the latent factors from some observed interest rates by assuming these rates are precisely priced by the model while the other rates are priced with errors. This approach is often used in the maximum likelihood method. A disadvantage of this approach is that the empirical results depend on the choice of the interest rates for imputation.} \]
over $\Theta$, which is also a vector of a sizable dimension.

To estimate the CIR-FOMC model, we combine Bayesian inference with the Markov chain Monte Carlo (MCMC) sampling technique. In this approach, we view the model parameters as random variables characterized by the posterior distribution. The MCMC is an iterative scheme to generate samples from the posterior distribution. To deal with the unobservable instantaneous rate, we apply the data augmentation method of Tanner and Wong (1987). The posterior distribution of the unknown parameters $\phi$, $\Theta$, and $\Omega$ and the instantaneous rates $R$ is

$$p(\phi, \Theta, \Omega, R | Y) \propto L(Y | R, \phi, \Theta, \Omega) \cdot p(R | \phi, \Theta) \cdot p(\phi, \Theta, \Omega),$$

(15)

where $L(Y | R, \phi, \Theta, \Omega)$ is the likelihood function, $p(R | \phi, \Theta)$ is the distribution of the instantaneous rates, and $p(\phi, \Theta, \Omega)$ denotes the prior distribution of the parameters. We provide the details of our implementation of the MCMC technique in Appendix A.2.

The Bayesian-MCMC approach is a powerful alternative to the maximum likelihood method. For the classic treatise on the advantages of Bayesian inference, see Zellner (1971) and Zellner (1985). A comprehensive treatise on the advantages of combining Bayesian inference and MCMC sampling is available in Geweke (2005). Lamoureux and Witte (2002) explain the advantages of the Bayesian-MCMC approach in estimating the multi-factor CIR models. Johannes and Polson (2010) survey the applicability of this approach to a class of continuous-time models in finance. Smith and Gelfand (1992) and Casella and George (1992) explain why MCMC sampling is an effective way to overcome the curse of dimensionality in obtaining statistical inference for nonlinear models. The combination of Bayesian inference and MCMC sampling is particularly useful for the CIR-FOMC model.

To measure the model’s goodness of fit, we focus on the model’s performance in explaining the term spreads. A widely used measure in conventional linear regressions is the $R$-squared, which is the ratio of the variation explained by a model to the total variation in the data. This idea can be extended to Bayesian inference. Let $z_{tj} = y_{tj} - r_t$ be the term spread between the $j^{th}$ term rate and the instantaneous rate. The total variation in all the term spreads is $\sum_{t=1}^{T} \sum_{j=1}^{m} (z_{tj} - \bar{z})^2$, where $\bar{z}$ is the sample average of all $z_{tj}$. The term spread implied by the model is $\hat{z}_{tj} = \hat{y}_{tj} - r_t$. Thus, the sum of the unexplained variation in the term spreads is $\sum_{t=1}^{T} \sum_{j=1}^{m} (z_{tj} - \hat{z}_{tj})^2$. The proportion of the total variation explained by the model is

$$R^2 = 1 - \frac{\sum_{t=1}^{T} \sum_{j=1}^{m} (z_{tj} - \hat{z}_{tj})^2}{\sum_{t=1}^{T} \sum_{j=1}^{m} (z_{tj} - \bar{z})^2}.$$  

(16)

We obtain the marginal posterior distribution of $R^2$ from the MCMC samples and refer to the posterior mean of $R^2$ as the Bayesian $R$-squared and denote it by $\mathbb{R}^2$. This goodness-of-fit
measure follows the idea of Gelman and Pardoe (2006), who introduce a similar Bayesian version of the $R$-squared for hierarchical models in statistics.

## 4 Empirical Results

### 4.1 Expectation Component in the OIS Curve

We use the daily data of the short-term OIS rates to estimate the CIR-FOMC model and report the summary statistics of the posterior distributions of volatility ($\sigma$), reversion speed ($\kappa$), and price of risk ($\lambda$) in Table 2. The reversion centers, which are the unique feature of the CIR-FOMC model, will be discussed subsequently. Also reported in Table 2, the Bayesian $R$-squared of the CIR-FOMC model is 91.34%. This level of goodness of fit is rather high for a cross section of twelve interest rates over more than 3,000 days. The fit is remarkable in view of the substantial variation in the term structure over the sample period. The model’s goodness of fit is visible in panel A of Figure 4, which plots the time series of the model-implied term spread between the six-month and one-month OIS rates. The plot also shows the observed term spread between the six-month and one-month OIS rates. Clearly, the model-implied term spread closely tracks the observed term spread.

<table>
<thead>
<tr>
<th>$R^2$ = 91.34%</th>
<th>Mean</th>
<th>Stdev</th>
<th>5th %</th>
<th>Median</th>
<th>95th %</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>0.0404</td>
<td>0.0006</td>
<td>0.0395</td>
<td>0.0404</td>
<td>0.0412</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.6255</td>
<td>0.0918</td>
<td>0.4787</td>
<td>0.6176</td>
<td>0.8095</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>-0.2987</td>
<td>0.0923</td>
<td>-0.4866</td>
<td>-0.2906</td>
<td>-0.1521</td>
</tr>
<tr>
<td>$\theta_t$</td>
<td>0.0128</td>
<td>0.0019</td>
<td>0.0096</td>
<td>0.0127</td>
<td>0.0165</td>
</tr>
</tbody>
</table>

Table 2: The statistics of the marginal posterior distribution of the parameters in the CIR-FOMC model and the Bayesian $R$-squared of the model. The parameters include the volatility ($\sigma$) of the instantaneous interest rate, the reversion speed ($\kappa$), and the market price of risk ($\lambda$). The last row presents the time-series averages of the statistics of the posterior distribution of the reversion center ($\theta_t$). (The estimated reversion center at each time is presented in Figure 5.)

Table 2 shows that $\sigma$, $\kappa$, and $\lambda$ in the CIR-FOMC model are estimated precisely. The estimation of $\sigma$ is particularly precise. The posterior mean of $\sigma$ is 0.0404, while the posterior standard deviation is only 0.0006. The central ninety-percentile interval of the posterior distribution of $\sigma$ is a narrow range of 0.0395~0.0412. The reversion speed $\kappa$ falls into a range between 0.4787 and 0.8095 with a central 90% posterior probability. The estimation of $\lambda$ has a similar precision; its central ninety-percentile interval is (-0.4866, -0.1521). The estimated reversion center at each time is plotted in Figure 5. We will discuss the plot in detail in the next section. In Table 2, we present the time-series averages of the statistics of the posterior
distribution of the reversion centers. The average posterior standard deviation is only a small fraction of the average posterior mean, suggesting that the data are very informative about the reversion center.

Using the CIR-FOMC model, we extract the expectation component of an OIS rate by setting the market price of risk to zero. Specifically, we obtain the expectation component, \( \bar{y}_t(s) \), by setting \( \lambda \) to zero in equations (8) and (3) while keeping the other parameters unchanged. The expectation component of a term spread is the difference between the expectation components of two OIS rates. Let \( s_1 \) and \( s_2 \) be the terms of two OIS rates and assume \( s_1 < s_2 \). The expectation component of the term spread is \( \bar{y}_t(s_2) - \bar{y}_t(s_1) \). A positive (negative) expectation component of the term spread shows the market anticipation of the rising (falling) instantaneous rate in interval \( (s_1, s_2) \).

![A: Model-Implied Spread](image1)

![B: Expectation Component](image2)

![C: Risk Premium Component](image3)

![D: Risk Premium Proportion](image4)

Figure 4: The spread between the 6-month and 1-month rates implied by the CIR-FOMC model, its expectation and risk premium components, and the risk premium proportion in the 6-month rate. The gray line is the observed term spread, and the vertical gray area indicates the period of the Great Recession.

In panel B of Figure 4, we plot the posterior mean of the expectation component of the term spread between the six-month and one-month OIS rates. To compare with the data, we also plot the observed term spread. The plots show that the expectation component varied drastically over the sample period. The component was positive in 2004 and 2005 but negative in 2006 and 2007. It suggests that the negative expectation component is the reason for the inversion of the OIS curve during 2006–2007. Moreover, panel B of Figure 4 displays an asynchronous relation between the market expectation and the term spread. The expectation
component started descending more than a half year earlier than the observed term spread. It became negative in early 2006, more than a half year before the term spread turned negative. This asynchronous relation is corroborated by the estimated reversion center, which will be discussed in the next subsection.

The risk premium component is the difference between the model-implied term rate and its expectation component. That is, the risk premium component is \( \tilde{y}_t(s) = \hat{y}_t(s) - \bar{y}_t(s) \). The risk premium component of a term spread is the difference between the risk premium components of two term rates. Let \( s_1 \) and \( s_2 \) be the terms of two OIS rates and assume \( s_1 < s_2 \). The risk premium component of the term spread is \( \tilde{y}_t(s_2) - \tilde{y}_t(s_1) \). It measures the difference in the risk compensation for different terms.

Panel C of Figure 4 shows that the risk premium component of the term spread moved upward as the term spread became negative in the years before the global financial crisis. Particularly, it was positive in the second half of 2007, when the term spread was very negative. This result suggests that when the yield curve inverted in 2007, the market expected the coming drop in interest rates, and investors demanded a positive premium for the uncertainty of the future interest rates. In fact, the risk premium component is positive for the entire sample period. Note that neither the model nor the econometric method restricts the risk premium to be positive. A positive risk premium is consistent with the presumption that investors are risk averse and interest rate risk is a concern of the OIS investors. Most importantly, the empirical result shows that the inversion of the OIS curve before the Great Recession was not driven by negative risk premiums.

To assess the importance of risk premiums in OIS rates, we measure the importance of the risk premium in the OIS rate by \( \tilde{y}_t(s)/\hat{y}_t(s) \), which is the risk premium proportion of the OIS rate \( \tilde{y}_t(s) \). In panel D of Figure 4 we plot the risk premium proportion of the six-month OIS rate. We see an increasing trend in the risk premium proportion in the three years preceding the Great Recession. The risk premium proportion peaked in early 2008. Since then, it was on a decreasing trend and reached a low point at the end of 2008. During the same period, the Federal Reserve was on the path of easing monetary policy by moving the federal funds target to a narrow range near zero. The uncertainty of the monetary policy should have been low at the end of 2008 because the FOMC stated clearly in its forward guidance: “The Committee anticipates that weak economic conditions are likely to warrant exceptionally low levels of the federal funds rate for some time.” Overall, the risk premium proportion estimated from the CIR-FOMC model is small, varying between 5% and 7%. This confirms that risk premiums play a much smaller role than expectations in the dynamics of the OIS curve.

Similarly, we can measure the importance of the expectation component by \( \bar{y}_t(s)/\hat{y}_t(s) \), but we do not need to report it because it is simply one minus the risk premium proportion.
4.2 Reversion Center of the Instantaneous Rate

A unique advantage of the CIR-FOMC model is that it reveals the dynamics of the reversion center of the instantaneous rate. In Figure 5, we display the time series of the posterior mean of the reversion center. For comparison, we display the Federal Reserve’s target for the federal funds rate in the same figure. The target was a time-varying rate until December 16, 2008. Since then, it was a range of 0~25 bps until December 16, 2015 and then a range of 25~50 bps for the last half month of our sample period.

Figure 5: The posterior mean of the reversion center in the CIR-FOMC model (solid line). The gray line is the Federal Reserve’s target rate/range for the federal funds rate. The vertical gray area indicates the period of the Great Recession.

Figure 5 reveals an interesting asynchronous relation between the reversion center and the Federal Reserve’s monetary policy. The target rate was 1% at the beginning of the sample period. The Federal Reserve started tightening monetary policy on June 30, 2004, increasing the target rate by 25 bps at each FOMC meeting until August 8, 2006, when the FOMC decided to maintain the target rate at 5.25%. The posterior mean of the reversion center was in an upward trend in 2004 and in the first half of 2005, ahead of the increase in the target rate. The elevation of the reversion center ahead of the rise of the target rate suggests that market participants anticipated the Federal Reserve’s tightening. In late 2005, when the target was
still rising, the reversion center was declining. The reversion center was in a downward trend in the period from fall 2006 through fall 2007, when the FOMC kept the target rate unchanged at 5.25%. The FOMC did not begin lowering the target until September 18, 2007. The decline in the reversion center well ahead of the cuts in the target rate shows the market’s anticipation of the Federal Reserve’s easing of its monetary policy.

From fall 2006 to fall 2007, when the Federal Reserve kept the target rate at 5.25%, the OIS term spreads began to turn negative (panels B, C, and D of Figure 1). It is well known that the yield curve of long-term Treasury bonds also inverted from fall 2006 to fall 2007, and it is controversial whether the inversion reflected investors’ expectation of the Federal Reserve’s easing of its monetary policy. Estrella and Trubin (2006) take the view that the inverted yield curve of Treasury bonds in fall 2006 predicted that a recession would happen within a year and that the Federal Reserve would cut the target rate. However, Bernanke et al. (2011) hold a different view and argue that the inversion did not reflect the expected future cuts in the target rate. In their view, the inversion was the result of “global savings glut” — the investments of some Asian countries’ large surplus in the U.S. Treasury bonds pushed down the long-term yields and caused the risk premiums to be negative.

It is difficult to discriminate between these two views without explicitly analyzing how the flow of global savings into the U.S. Treasury bonds affect the yield curve. However, the downward trend of the reversion center from fall 2006 to fall 2007 in Figure 5 appears more consistent with the view that the market expectation of the Federal Reserve’s easing caused the inversion of the OIS curve. As we have noted earlier, the reversion center was in a steep downward trend in this period, while the target rate was constant.

An excess demand for the long-term Treasury bonds is unlikely to affect the OIS rates with maturities up to only one year. Based on a report by the U.S. Treasury and the Federal Reserve,\(^\text{11}\) the foreign holding of the long-term U.S. Treasury debt indeed increased from $1,429 billion in June 2004 to $1,965 billion in June 2007. This is consistent with Bernanke et al. (2011). However, the foreign holding of the short-term (maturities of one year or less) U.S. Treasury debt reduced from $317 billion in June 2004 to $229 billion in June 2007. The increase in the foreign holdings of long-term Treasury debt may cause the inversion of the yield curve at the long end, but the reduction in the foreign holding of the short-term Treasury debt cannot be an explanation for the inversion of the short-term OIS curve.

The Great Recession indeed came in fall 2007, as predicted by Estrella and Trubin (2006). On September 18, 2007, the Federal Reserve cut the target rate from 5.25% to 4.75%. In the subsequent seven meetings, the FOMC cut the target rate down to 2% by the end of April

\(^{11}\)See the “Report on Foreign Portfolio Holdings of U.S. Securities as of June 30, 2008” jointly issued by the U.S. Treasury and the Federal Reserve in April 2015.
2008. In the same period, the posterior mean of the reversion center dropped from about 2.5% to 0.13%. The drop in the reversion center was concurrent to the negative spreads shown in Figure 1. The drop was then quickly reversed in mid 2008 when the FOMC surprised the market by its decisions to keep the target rate at 2% at the June and August meetings, and again at the September meeting, which was immediately after the collapse of Lehman Brothers. In the hindsight, Bernanke (2015) points out that keeping the interest rate at 2% at the September meeting was “certainly a mistake.” The reversion center moved back to 2% on the day of the September meeting. Then, the FOMC cut the target rate by 50 bps at the October 8 meeting and by another 50 bps at the October 29 meeting. The FOMC finally set the target to the range of 0~25 bps at the meeting on December 16, 2008. Over a year since the December 2008 meeting, the reversion center fell gradually from 2% to the middle of the 0~25 bps range.

A comparison of the changes in monetary policy with the reversion center shed light on the question whether the Federal Reserve’s decision spearheaded or followed the market expectation around the global financial crisis. The phrase of this question on Wall Street is whether the Federal Reserve was “ahead of (or behind) the curve.” Some academic scholars (e.g., Fama, 2012) have even raised the question whether the Federal Reserve controls interest rates. Figure 5 suggests that the Federal Reserve was behind the curve in all the days up to early 2008 because the changes in the target rate followed the changes in the reversion center in this period. It appears that the Federal Reserve moved ahead of the curve in the second half of 2008. After the reversion center dropped to nearly zero in early 2008, it moved back to 2%, the same as the target rate. The reversion center was then moving down slowly in the last quarter of 2008 when the FOMC cut the target to the near-zero range.

The reversion center in Figure 5 stayed mostly inside the 0~25 bps range of the target during 2011–2014, except a brief rise above the range in summer 2011. The nearly-constant reversion center inside the target range suggests that the Federal Reserve convinced the market that it would keep the interest rate near zero for an extended period. Indeed, the FOMC statement of each meeting since June 2010 included a clear forward guidance that the economic conditions warranted exceptionally low levels of the federal funds rate for an extended period. The brief rise of the reversion center in summer 2011 was a short-lived episode of investors’ expectation that the Federal Reserve might soon begin to normalize monetary policy. The FOMC statement on June 22, 2011 noted that inflation had picked up. On the same day, as part of its forward guidance, the FOMC released its economic projection of upward trends in both GDP growth and inflation for the coming year. Meanwhile, the Federal Reserve was completing the program (QE2) of purchasing $600 billion long-term Treasury bonds by June 2011, but the FOMC statement gave no hint on any additional large-scale asset purchase program. The FOMC did not announce QE3 until September 13, 2012. The meeting detail released on July
12, 2011 showed that the FOMC considered and adopted plans and principles for monetary policy normalization at its June meeting. However, the subsequent August and September meeting statements, which noted that inflation had moderated, quickly changed the market expectation. The reversion center fell back into the near-zero range in late 2011, consistent with the Federal Reserve’s later actions.

The reversion center moved up steadily in 2015, clearly showing that the market anticipated the Federal Reserve’s normalization of monetary policy in 2015. After the economic data of the last quarter of 2014 presented a solid economic growth and a strong job gain but slightly declined inflation, the FOMC said in its statement on January 28, 2015 that it “expects inflation to rise gradually toward 2 percent over the medium term as the labor market improves further.” The committee held the target unchanged at its first meeting of 2015 and told the public that the timing of the target increase would depend on employment and inflation. Since then, the OIS term spreads gradually moved higher (panels C and D of Figure 1), and the reversion center embarked on a steep, upward trend (Figure 5).

The labor market improvement was robust during the period spanning through the next six meetings in 2015, but the economic growth was moderate. The annualized inflation rate released in each month during January–October, 2015 was only 0.2% or lower, although in each of the six meeting statements the FOMC expressed its confidence that inflation would “move back to its 2 percent objective over the medium term.” Finally, despite the continued weak inflation data for November, the FOMC raised the target range to 25~50 bps at its last meeting of 2015, as expected by the market.

5 Conclusion

This paper brings attention to the intriguing timing of the OIS curve inversion before the Great Recession. Our empirical results demonstrate that market expectation is the main driver of the movements of the short-term OIS curve. The results suggest that the OIS curve inversion is a timely signal of the 2007–2009 recession. The results also indicate that the Federal Reserve’s monetary policy was behind the market expectation in the years prior to the global financial crisis and moved ahead of the expectation in the middle of the crisis. Our analysis of the OIS curve obviate the confounding factors that complicate the interpretation of the Treasury yield curve and its inversion before the recession.

We propose a term structure model of OIS for analyzing the tailspin of the short-term OIS curve. The model highlights the importance of the FOMC meeting schedule in understanding the dynamics of the OIS term structure. We build a database that precisely tracks the public

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12 The inflation rates quoted here are the Consumer Price Index (CPI) published by the Bureau of Labor Statistics.
information on the FOMC meeting schedules. Incorporating the database into the term structure model of OIS, we deliver intuitive explanations for the dynamics of the expectation in the OIS curve. The reversion center of the instantaneous rate in the model offers a unique insight into the dynamics of the OIS term structure throughout the pre-crisis years, the crisis period, and the post-crisis era.

References


Appendix

A.1 Equivalence between OIS Rates and Term Rates

The cash settlement of an OIS reflects the profit or loss in the daily interest payments accrued during the payment period. Let $s = n/360$ be the length of the payment period from time $t$, where $n$ is the number of calendar days of the OIS term. Let $b = 1, \ldots, n_b$ track the business
days in chronological order during the period from \( t \) to \( t + s \), \( F_t(h_i) \) be the overnight interest rate for the \( i \)-th business day, and \( h_i \) be the length of “overnight.” The length of “overnight” (from one business day to the next business day) varies. For example, \( h_i = 1/360 \) if both date \( i \) and the next calendar day are business days, whereas \( h_i = 3/360 \) if date \( i \) is a Friday and the next Monday is a business day. If there is a bank holiday between date \( i \) and date \( i + 1 \), then the length of \( h_i \) adjusts accordingly.\(^\text{13}\) It follows that \( \sum_{i=1}^{n_b} h_i = n/360 \). The accrued floating interest is \( L \left[ \prod_{i=1}^{n_b} (1 + F_t(h_i)h_i) - 1 \right] \), where \( L \) is the notional amount of the loan in the OIS. Let \( R_{t}^{\text{OIS}}(s) \) be the fixed rate in the OIS for term \( s \) at time \( t \). The accrued fixed interest is simply \( LsR_{t}^{\text{OIS}}(s) \). The two parties of the OIS use cash to settle the difference between the accrued floating interest and the fixed interest at the end of the payment period.

Consider the side of the OIS contract that pays the floating overnight rate on \( L \) dollars and the other side that receives the fixed OIS rate on \( L \) dollars. The value of the loan with the floating interest rate is simply \( V_{t}^{\text{float}} = L \). Let \( R_{t}^{0}(s) \) be the annualized zero-coupon discount rate at time \( t \) for a dollar paid at time \( t + s \). The \( L \)-dollar loan that pays the fixed OIS rate and matures at \( t + s \) should be valued at \( V_{t}^{\text{fixed}} = L \left[ 1 + sR_{t}^{\text{OIS}}(s) \right] / \left[ 1 + sR_{t}^{0}(s) \right] \). The loans on the two sides of a fair OIS contract should have the same value: \( V_{t}^{\text{float}} = V_{t}^{\text{fixed}} \), which implies \( R_{t}^{\text{OIS}}(s) = R_{t}^{0}(s) \). That is, a short-term OIS rate is the zero-coupon discount bond yield for the same term. Here, \( R_{t}^{0}(s) \) (or \( R_{t}^{\text{OIS}}(s) \)) is a simple interest rate without compounding. In the estimation of the term structure models, we use the continuously compounded interest rate \( y_{t}(s) = s^{-1} \ln (1 + sR_{t}^{\text{OIS}}(s)) \).

### A.2 Details of MCMC Sampling

The first step of Bayesian estimation is to specify the prior distributions of the unknown parameters. To extract the maximum information from the data, we use non-informative priors. Since \( \kappa, \sigma^2 \), and \( \omega_j \) are positive by definition, the prior distributions of \( \log \kappa \), \( \log \sigma^2 \), and \( \log \omega_j \) are flat, following the standard approach in Bayesian statistics. Zelner (1971) provides the rationale for this type of priors. Since \( \lambda \) may be either positive or negative, the prior distribution of \( \lambda \) is flat.

The reversion center, \( \theta_i \), should be positive, and the potential change from \( \theta_i \) to \( \theta_{i+1} \) should be distributed around zero. Thus, \( \log \theta_i \) has a flat distribution, and \( \log \theta_{i+1} - \log \theta_i \) has a normal distribution with mean \(-0.5\) and variance \(1.0\). The normally distributed change of \( \log \theta \) ensures that \( \theta_i \) is always positive and that the potential change is smaller if the current \( \theta \) is lower. The mean of \(-0.5\) implies \( E[\theta_{i+1} | \theta_i] = \theta_i \) in the prior distribution. This prior is

\(^\text{13}\)If a holiday separates the \( i \)-th business day from the next business day, then \( h_i = 2/360 \). If Monday is a holiday, then the “overnight” from Friday to Tuesday is actually four-day long: \( h_i = 4/360 \).
diffuse because two standard deviations of $\theta_{i+1}$ cover the range of $(0.08\theta_i, 4.48\theta_i)$. We find that the estimation results do not change if we use a larger variance in the prior.

We also impose the Feller condition, $2\kappa\theta_i \geq \sigma^2$, in the prior distribution. This condition, as noted in Cox et al. (1985), guarantees that the instantaneous rate is always positive. This constraint is easy to implement in MCMC sampling because we can simply discard the random draws of the parameters that violate the condition. The ease of imposing nonlinear constraints is another advantage of the MCMC Bayesian method. By contrast, a nonlinear constraint such as the Feller condition of the CIR model can be difficult to incorporate in the maximum likelihood method.

To deal with the latent state variable, we apply the data augmentation method of Tanner and Wong (1987). Besides the unknown parameters, we also obtain random draws of the unobserved instantaneous rate $r_t$ in MCMC sampling. In the CIR model, the transition probability distribution from $r_{t-1}$ to $r_t$ is:

\begin{align}
  cr_t &\sim \chi^2(u,v), \quad c = 4\kappa / \{\sigma^2[1 - \exp(-h\kappa)]\}, \\
  u & = 4\kappa\theta_i / \sigma^2, \quad v = cr_{t-1}\exp(-h\kappa),
\end{align}

where $\chi^2(u,v)$ is the non-central $\chi^2$ distribution with $u$ degrees of freedom and the non-centrality parameter $v$, and $h$ denotes the overnight term. Let $f(r_t | r_{t-1})$ denote the transition probability density. The distribution of the instantaneous rates on all the dates in our sample is

\begin{align}
  p(R | \phi, \Theta) = \prod_{t=1}^T f(r_t | r_{t-1}).
\end{align}

The above distribution depends on $r_0$, which is also treated as an unknown parameter. We expand $\phi$ to include $r_0$, so that $\phi = (\kappa, \sigma, \lambda, r_0)$. We set the prior distribution of $\log(r_0)$ to be flat.

The joint posterior distribution of the unknown parameters $\phi$, $\Theta$, and $\Omega$ and the latent variable $R$ is

\begin{align}
  p(\phi, \Theta, \Omega, R | Y) \propto L(Y | R, \phi, \Theta, \Omega) \cdot p(R | \phi, \Theta) \cdot p(\phi, \Theta, \Omega),
\end{align}

where $L(Y | R, \phi, \Theta, \Omega)$ is the likelihood function, and $p(\phi, \Theta, \Omega)$ denotes the joint prior distribution of the parameters described earlier.

The posterior distribution in the above equation is not a known type. We apply the MCMC method—we draw random samples of each unknown parameter and the state variable (or a subset of them) while conditioning on the others, and iterate among them. The basic idea of the MCMC is that the distribution of the iterative samples from the conditional posterior distributions converges to the joint posterior distribution.
The conditional posterior distributions of some parameters and the state variable are not standard distributions for which sampling tools are readily available. To draw samples from these distributions, we apply the Metropolis-Hastings algorithm, which proposes a draw and then decides whether to accept or reject. Chib and Greenberg (1995) provides an introduction to the Metropolis-Hastings method. Following the recommendation from the literature, we use the random walk proposal distribution with a target average acceptance of about 30%. The only exception is the conditional distribution of the pricing error variance $\omega_j^2$, which can be derived from equation (11):

$$p(\omega_j^2 | r, \phi, \Theta, \Omega_{-j}, Y) \propto \frac{1}{\omega_j} \exp \left[ -\frac{1}{2} \sum_{t=1}^r I(j \leq m_t) (y_{tj} + A_{tj} - B_{tj} r_t)^2 / \omega_j^2 \right],$$

where $\Omega_{-j}$ represents the subset of $\Omega$ obtained by excluding $\omega_j$, and $n_j = \sum_{t=1}^r I(j \leq m_t)$ is the total number of days when the $j^{th}$ maturity is included in the estimation. The summation in the above equation is over the dates with $j \leq m_t$ because the estimation uses only the interest rates with maturities $s_j \leq s_m$, on each date $t$. The above density function is an inverse gamma distribution, from which we draw samples of $\omega_j^2$.

The MCMC Bayesian inference of the multi-factor CIR models in Appendix A.3 follows the same idea. Lamoureux and Witte (2002) describe the MCMC Bayesian inference of the multi-factor CIR models. This estimation approach can be applied to the jump Gaussian model as well. The application of the jump Gaussian model to OIS are examined in Appendix A.4.

### A.3 Conventional Factor Models

In this section, we investigate the application of multi-factor models to the OIS term structure.

A multi-factor CIR model assumes that the instantaneous rate is the sum of $k$ independent factors: $r_t = x_{1t} + \cdots + x_{kt}$. Each factor is a mean-reverting square-root process:

$$dx_{it} = \kappa_i(\theta_i - x_{it}) dt + \sigma_i \sqrt{x_{it}} dw_{it}, \quad i = 1, \cdots, k,$$

where $\{w_{it}\}_{i=1}^k$ are independent Wiener processes. A discount bond yield in the multi-factor CIR model is an affine function of the factors and has an analytical pricing formula:

$$y_t(s) = s^{-1} \sum_{i=1}^k \left[ b_i(s) x_{it} - a_i(s) \theta_i \right],$$

$$b_i(s) = \frac{2(\gamma_i - 1)}{\delta_i (\gamma_i - 1) + 2 \gamma_i},$$

$$a_i(s) = \frac{2 \kappa_i}{\sigma_i^2} \left[ \frac{\delta_i}{2} + \log \frac{2 \gamma_i}{\delta_i (\gamma_i - 1) + 2 \gamma_i} \right],$$

where $\gamma_i$ is the drift rate, $\kappa_i$ is the mean reversion rate, $\sigma_i$ is the volatility, $\delta_i$ is the negative of the inverse of the diffusion coefficient.
where $\gamma_i^2 = (\kappa_i + \lambda_i)^2 + 2\sigma_i^2$, $\delta_i = \gamma_i + \kappa_i + \lambda_i$, and $\lambda_i$ is the parameter that determines the market price of risk for the $i^{th}$ factor. Cox et al. (1985) derive the closed-form formula for multi-factor CIR models. Pearson and Sun (1994) implement the maximum likelihood estimation of these models. Lamoureux and Witte (2002) introduce Bayesian inference to the models.

![Figure 6: The spread between the 6-month and 1-month rates implied by the two-factor CIR model, its expectation and risk premium components, and the risk premium proportion in the 6-month rate. The gray line is the observed term spread, and the vertical gray area indicates the period of the Great Recession.](image)

We have experimented with various numbers of factors and settled down to the two- and three-factor models. We skip the one-factor CIR model as it fits the OIS curve very poorly, with an $R$-squared less than 20%. Our choices are supported by the evidence of principal component analysis. We find that the first principal component explains 94.90 percent of the variation in the short-term OIS rates, the first two principal components explain 98.58 percent, and the first three principal components explain 99.38 percent. Two factors appear sufficient for fitting the short-term OIS rates. Since long-term bond yields typically need three factors to explain (Litterman and Scheinkman, 1991), we also apply the three-factor CIR model to OIS. We find that the $R$-squared of the two-factor model is 94.59%. This high goodness of fit is reflected in panel A of Figure 6, in which the model-implied term spread tracks the observed term spread very closely. The fit is slightly better with the three-factor CIR model, which is shown in panel A of Figure 7. The $R$-squared of the three-factor model is 97.53%.

The summary statistics of the posterior distributions of the parameters in the two- and
Figure 7: The spread between the 6-month and 1-month rates implied by the three-factor CIR model, its expectation and risk premium components, and the risk premium proportion in the 6-month rate. The gray line is the observed term spread, and the vertical gray area indicates the period of the Great Recession.

three-factor CIR models are reported in Tables 3 and 4. The estimation of the parameters in the multi-factor CIR models is very precise, similar to that in the CIR-FOMC model.

We decompose the term spread between the six-month and one-month OIS rates into the expectation and risk premium components using the multi-factor CIR models. The expectation component obtained from the two-factor CIR model (panel B of Figure 6) appears unrelated to the Federal Reserve’s monetary policy. The expectation component stayed negative in the period from early 2004 to late 2008. The negative expectation in 2004 and 2005 was particularly inconsistent with the FOMC’s announced plan to raise the target rate in those years. During the second half of 2004 and most of 2005, the Federal Reserve carried out a series of target rate hikes, and the OIS curve was upward sloping. The negative expectation, however, suggests that in 2004 and 2005 investors expected the interest rates to fall. The expectation component was only slightly negative at the end of 2007 when the term spread was very negative and when the Federal Reserve started cutting the target rate.

The expectation component obtained from the three-factor CIR model is small, similar to the two-factor CIR model. The expectation component stays negative for the period from early 2004 to late 2008 (panel B of Figure 7). The plot shows that the three-factor CIR model attributes only a small part of the variations in the OIS curve to the changes in the market
Table 3: The statistics of the marginal posterior distribution of the parameters in the two-factor CIR model and the Bayesian $R^2$-squared of the model. The parameters include the volatilities ($\sigma_1$ and $\sigma_2$), the reversion speeds ($\kappa_1$ and $\kappa_2$), and the market prices of risks ($\lambda_1$ and $\lambda_2$), and the reversion centers ($\theta_1$ and $\theta_2$) of the two factors.

Table 4: The statistics of the marginal posterior distribution of the parameters in the three-factor CIR model and the Bayesian $R^2$-squared of the model. The parameters include the volatilities ($\sigma_1$, $\sigma_2$, and $\sigma_3$), the reversion speeds ($\kappa_1$, $\kappa_2$, and $\kappa_3$), and the market prices of risks ($\lambda_1$, $\lambda_2$, and $\lambda_3$), and the reversion centers ($\theta_1$, $\theta_2$, and $\theta_3$) of the three factors.

The multi-factor CIR models attribute the variations in the OIS curve mostly to the variations in the risk premiums. Panel C of Figure 6 and panel C of Figure 7 show that the risk premium components estimated from the multi-factor models nearly coincide with the observed term spread. In particular, the risk premium is very negative in the beginning of the Great Recession.

The risk premium proportions implied by the multi-factor CIR models are presented in panel D of Figure 6 and panel D of Figure 7. In both models, the risk premium proportion was decreasing over the three years preceding the Great Recession and sharply turned negative in...
late 2007. The multi-factor CIR models seem to suggest that the risk premium became smaller
during this period and turned negative when a sweeping financial crisis was getting underway,
when the economy was entering one of the worst recessions in history, and when investors
were anxious about what the Federal Reserve would do.

In addition, the risk premium proportions estimated from the multi-factor CIR models vary
so much that they are sometimes unreasonably large. The proportion fluctuates between -10% and
20% in the two-factor CIR model and between -50% and 30% in the three-factor CIR
model. These results are inconsistent with the intuition that the risk premiums in short-term
OIS rates should be small.

We have also estimated two- and three-factor Gaussian term structure models and obtained
similar results. These models attribute the variations in the OIS rates to the risk premium
components, and the risk premium proportions demonstrate wide oscillations.

In summary, the multi-factor CIR and Gaussian models, which ignore the FOMC meeting
schedule, explain the OIS curve almost entirely by the risk premium component. The variations
in the risk premiums estimated from the multi-factor models appear to be difficult to recon-
cile with the changes in the economy and the financial market during the sample period. By
comparison, the empirical results obtained from the CIR-FOMC model are intuitive and easy
to interpret.

A.4 Jump Gaussian Model

In this section, we investigate the application of the jump Gaussian model to the OIS term
structure.

The instantaneous rate in the jump Gaussian model is an affine function of two factors:
\( r_t = \delta + x_{1t} + x_{2t} \). The first factor is a stochastic process that jumps only on the scheduled
FOMC meeting dates:
\[
dx_{1t} = J_t \, dC_t ,
\]
where \( C_t \) is a counting process that increases by 1 on each scheduled FOMC date, and \( J_t \) is the
size of the jump. The jump size follows a normal distribution: \( J_t \sim N(\mu_t, \sigma_1^2) \), where \( \mu_t \) and
\( \sigma_1^2 \) are the mean and variance of the distribution. The variance \( \sigma_1^2 \) is a positive constant. The
mean varies over time and depends on both factors:
\[
\mu_t = -\kappa_{11} x_{1t}^- - \kappa_{12} x_{2t} ,
\]
where \( \kappa_{11} \) and \( \kappa_{12} \) are constants, and \( x_{1t}^- = \lim_{s \downarrow t} x_{1s} \). If \( \kappa_{11} \) is positive, the jump process is
mean reverting. The second factor is a Gaussian diffusion process:

\[ dx_{2t} = -\kappa_{21} x_{1t} dt - \kappa_{22} x_{2t} dt + \sigma_2 dw_t , \]  

where \( w_t \) is a Wiener process.

For the jump factor, the market price of risk is determined by \( \rho_1 + \lambda_{11} x_{1t} + \lambda_{12} x_{2t} \). For the diffusion factor, the market price of risk is determined by \( \rho_2 + \lambda_{21} x_{1t} + \lambda_{22} x_{2t} \), where \( \rho_i \) and \( \lambda_{ij} \) for \( i, j = 1, 2 \) are constants. In this model, the yield at time \( t \) on a discount bond maturing at \( t+s \) is

\[ y_t(s) = s^{-1} \left[ b_{1t}(s)x_{1t} + b_{2t}(s)x_{2t} - a_t(s) \right] , \]

where \( a_t(s) \), \( b_{1t}(s) \), and \( b_{2t}(s) \) are determined by three ordinary differential equations, which can be solved numerically. The numerical procedure for solving the equations is similar to those in Piazzesi (2009), Heidari and Wu (2010), and Kim and Wright (2014).

| \( R^2 = 90.62\% \) Mean Stdev 5% Median 95% |
|---|---|---|---|---|
| \( \sigma_1 \) | 0.0030 | 0.0002 | 0.0028 | 0.0030 | 0.0034 |
| \( \kappa_{11} \) | 0.0228 | 0.0123 | 0.0033 | 0.0282 | 0.0404 |
| \( \kappa_{12} \) | 0.0988 | 0.0475 | 0.0242 | 0.0984 | 0.1825 |
| \( \sigma_2 \) | 0.0132 | 0.0002 | 0.0128 | 0.0131 | 0.0135 |
| \( \kappa_{21} \) | -0.3431 | 0.1920 | -0.6463 | -0.3454 | -0.0202 |
| \( \kappa_{22} \) | 1.2196 | 0.4430 | 0.5230 | 1.2283 | 1.9899 |
| \( 10 \times \rho_1 \) | -0.0031 | 0.0002 | -0.0034 | -0.0031 | -0.0027 |
| \( \lambda_{11} \) | 0.0058 | 0.0123 | -0.0122 | 0.0058 | 0.0252 |
| \( \lambda_{12} \) | -0.0322 | 0.0478 | -0.1182 | -0.0313 | 0.0439 |
| \( 10 \times \rho_2 \) | 0.0114 | 0.0020 | 0.0080 | 0.0114 | 0.0149 |
| \( \lambda_{21} \) | -0.0572 | 0.1913 | -0.3846 | -0.0526 | 0.2472 |
| \( \lambda_{22} \) | -0.8559 | 0.4431 | -1.6333 | -0.8576 | -0.1641 |

Table 5: The statistics of the marginal posterior distribution of the parameters in the jump Gaussian model and the Bayesian \( R \)-squared of the model. The parameters of the jump factor are \( \sigma_1 \), \( \kappa_{11} \), and \( \kappa_{12} \). The parameters of the diffusion factor are \( \sigma_2 \), \( \kappa_{21} \), and \( \kappa_{22} \). For the jump factor, the parameters of the market price of risks are \( \rho_1 \), \( \lambda_{11} \), and \( \lambda_{12} \). For the diffusion factor, the parameters of the market price of risks are \( \rho_2 \), \( \lambda_{21} \), and \( \lambda_{22} \).

The jump Gaussian model has advantages and disadvantages. Similar to the reversion center in the CIR-FOMC model, the jump factor captures the changes in interest rates concentrated on FOMC meeting dates. The model allows us to solve numerically for term rates, although there is no closed-form formula. The model also features potential risk premiums for the jumps on FOMC meeting dates. A major disadvantage of the jump Gaussian model is that interest rates in the model are not restricted to be positive, inconsistent with the U.S. monetary policy that keeps the interest rate positive.
We estimate the jump Gaussian model using the Bayesian MCMC method. Table 5 reports the summary statistics of the posterior distribution of each parameter in the jump Gaussian model. The $R$-squared is 90.62%, suggesting that the model fits the observed OIS curve nearly as well as the CIR-FOMC model. The jump Gaussian model's goodness of fit is also evident in panel A of Figure 8.

Figure 8: The spread between the 6-month and 1-month rates implied by the jump Gaussian model, its expectation and risk premium components, and the risk premium proportion in the 6-month rate. The gray line is the observed term spread, and the vertical gray area indicates the period of the Great Recession.

Like the CIR-FOMC model, the jump Gaussian model mainly relies on the expectation component to fit the observed term spread. Panel B of Figure 8 shows that the expectation component estimated from the jump Gaussian model moves up and down substantially throughout the sample period. The model attributes most of the movements of the term spread to the changes in the market expectation. The expectation component tracks the observed spread closely, except for a few diversions. One deviation is in the period from mid-2006 to the end of 2007. The expectation component in this period is negative and substantially lower than the observed term spread. The negative expectation component of the term spread corroborates the empirical results obtained from the CIR-FOMC model.

Until late 2008, the risk premium component estimated from the jump Gaussian model in panel C of Figure 8 is very similar to the risk premium component estimated from the CIR-FOMC model. It confirms the increasing trend of the risk premium component prior to the
Great Recession. However, the risk premium proportion estimated from the jump Gaussian model varies widely since late 2008. The proportion fluctuates between -40% and 50% (panel D of Figure 8). These unreasonably large variations in the risk premium may result from a disadvantage of the jump Gaussian model discussed earlier: the model does not restrict the instantaneous rate to be positive. When the federal funds rate fluctuates in a narrow range near zero, the model falls short of producing reasonable estimates.