Executive Compensation and Short-Termism

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Abstract

The stock market is widely believed to pressure executives to deliver short-term earnings at the expense of long-term value. This paper develops a model of the interaction between executive compensation and stock market prices, and analyzes its implications for corporate short-termism. I show that inefficient short-termism can arise in equilibrium as a self-fulfilling prophecy, due to strategic complementarities between the firm’s investment horizon and investors’ decision to acquire information about short-term performance or long-term value. However, the severity of the underlying agency problem between the manager and shareholders fully determines whether short-termism is an equilibrium outcome. This implies both that the stock-market cannot be identified as the cause of corporate short-termism and that it actually has the potential to alleviate the problem. The model helps us assess evidence presented in the “myopia” debate and yields novel implications regarding ownership structure, executive compensation, and managerial horizon.
1 Introduction

Managerial short-termism is a hotly debated issue in corporate, policy-making, and academic circles. Within the debate, two general and opposing views have taken shape. The most widely-held view, argued since the late 1970s (Lipton 1979), is that short-termism is a significant obstacle for firms in sustaining long-term value and the stock market is the primary culprit. The stock market pressures executives to deliver short-term earnings at the expense of long-term value; this encourages executives to hold back long-term investments and harms the firm and the economy. In support of this view, empirical evidence is often cited confirming the existence of short-termism (Graham et al. 2005, Budish et al. 2015, Edmans et al. 2017a,b) and its detrimental effects. Recently, however, some have questioned the widespread concern about corporate short-termism and have cast doubt on its popular diagnosis. Instead, they claim that firms choose their investment horizon optimally; the stock market simply reflects these choices and does not drive inefficient short-termism. Some of these dissenters point to a lack of long-term evidence that is consistent with the predictions of the short-term critics (Kaplan 2017). Others have gone so far to say that corporate short-termism is an imaginary problem (Roe 2018).

The high stakes in this debate have naturally led to a substantial academic literature. Yet, previous work on short-termism either takes as exogenous the dependence of the manager’s contract on the stock price (e.g., Stein 1989, Bebchuk and Stole 1993, Edmans 2009) or ignores the stock market and focuses on agency conflicts within the firm that make short-termism a second best (e.g., Narayanan 1985, Von Thadden 1995, Thakor 2018). This is surprising, given that short-termism is about both markets and compensation: stock prices can pressure managers to deliver short-term earnings at the expense of long-term value, but whether managers care about this pressure depends on the structure of their compensation. The objective of this paper is to explore the causes and consequences of corporate short-termism within a formal model in which both the optimal design of executive compensation and the stock market price are endogenously determined.

In the model, shareholders provide a manager with incentives to take a risky project. They can chooses either a short-term project that boosts current earnings or a long-term project that pays out in the future. The long-term project has higher returns but is costlier to incentivize, because the manager is risk-averse and the realization of the firm’s long-term value is more volatile. The stock price can be used in the contract, but its informativeness

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1 Graham finds that 78% of surveyed executives would destroy economic value to boost earnings. This year, prominent business leaders have spoken out about the excessive focus on short-term performance, warning about its potential effects on the overall economy (Dimon and Bufeet, 2018).

2 I survey the literature on executive compensation in the next subsection.
is endogenous (as in Kyle 1985): a speculator acquires information to profit off of liquidity traders, and a market-maker clears the market. An important and novel feature of the model is that the speculator chooses whether to acquire information about the firm’s short-term performance and/or the value of its long-term projects.

I show that the strategic interaction between executive compensation and the informativeness of the stock price is characterized by a two-way feedback. One way goes from the firm to the stock market: when the firm is expected to invest in the long-term project, the speculator acquires information about it and this information is partly incorporated into the price through her trading. However, if the firm is expected to undertake the short-term project, the speculator only acquires information about the firm’s short-term performance: there is no gain in learning about the long-term project, since the firm is not expected to invest in it. The second direction of the feedback runs from the stock market to the firm: if the speculator acquires information about the firm’s long-term project, the stock price can be used to incentivize the manager to undertake a long-term project, enabling the shareholders to design a more efficient contract. In turn, implementing a long-term project becomes more attractive for the firm.

This two-way feedback generates a strategic complementarity in the choice of horizons between the shareholders and the speculator. This strategic complementarity can lead to multiple equilibria, where inefficient short-termism can arise in equilibrium as a self-fulfilling prophecy, due to coordination failure between the speculator and the firm. When both long-termism - i.e., the firm investing in the long-term project and the speculator acquiring information about it - and short-termism - i.e., the firm investing in the short-term project and the speculator only acquiring information about short-term performance - are equilibria of the game, firm value is strictly larger under long-termism. The speculator, however, might be better off under short-termism, when the cost of acquiring information about the long-term project is higher or when she is looking for a quick profit from trades. In this case, the shareholders and the speculator’s preferences over equilibria are not aligned, and coordination failure is a serious issue.

The modeled interaction and the resulting strategic complementarity uncovers a new mechanism by which the stock market can feed corporate short-termism through an excessive focus on short-term performance. This is the first main result of the paper. Whereas previous

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3This is different from other papers on market monitoring, like Holmstrom and Tirole (1993) and Edmans (2009), where the informed-trader can only acquire information about the firm’s long-term value.

4It is worth emphasizing that an improvement in contracting is achieved even though I let the manager’s contract be contingent on the firm’s present and future return streams. The reason is that the speculator has information about the executive’s choice that is not in the return realization. Therefore, the price contains unique information about managerial performance.
work on short-termism takes as exogenous the dependence of the manager’s contract on the stock price, here shareholders are free to choose the structure of executive compensation. Yet, despite this freedom, inefficient short-termism can still arise in equilibrium as a self-fulfilling prophecy. But does this support the claim that the stock market is the primary culprit of corporate short-termism?

To address this question, I analyze a benchmark where the stock price cannot be part of the manager’s contract; this is the case, for example, when the firm shares are not publicly traded. Comparing the equilibrium outcomes in the benchmark with those in the model with stock prices yields two important implications. First, firms that were long-termist in the benchmark model continue to be long-termist when the stock price can be included in the contract. Therefore, the stock market does not increase the mass of short-termist firms in the economy. Second, a mass of firms that were short-termist in the benchmark can sustain efficient long-termism when the stock price can be included in the contract and is informative about the firm’s long-term value.

Together, these two observations suggest that the role of the stock market in relation to corporate short-termism may be fundamentally misunderstood. The real cause of corporate short-termism is the underlying agency problem between the shareholders and the manager, which makes it more costly for shareholders to incentivize long-term projects. Far from being the primary culprit of corporate short-termism, the stock market can be a (potentially) alleviating force: when the stock price is informative about long-term value, it enables a more efficient contract design that sustains efficient long-termism. However, an excessive focus on short-term performance in the stock market fails to alleviate the existing agency problem and leads to inefficient short-termism.

The analysis discussed so far naturally raises a question: which factors make coordination failure less likely to occur? If the speculator has a preexisting stake in the firm (i.e., if the speculator is a blockholder), her preferences over equilibria are closer to the one of the shareholders. Having a preexisting stake in the firm does not affect the speculator’s trading strategies and, hence, does not affect her profits from trading. However, it creates a link between the speculator’s expected payoff and the firm ex-ante value, aligning shareholders’ and speculator’s preferences over different equilibria: if the stake is sufficiently large, the equilibrium with long-termism Pareto dominates the one with short-termism. Therefore, coordination failure is less likely. This result uncovers a new strategic complementarity between inside and outside (the speculator in the model) shareholders. Compared to other informed-traders in the market, outside shareholders have a stronger incentive to trade on information about the long-term prospect of a firm, as this enables inside shareholders to design more efficient managerial contracts and increase firm value. These findings contribute
to a new literature that tries to explain the predominance in the U.S. of small transient blockholders, “who typically lack control rights and instead follow the ‘Wall Street Rule’ of ‘voting with their feet’ - selling their stock if dissatisfied” (Edmans 2009).

The strategic complementarity between the firm’s and informed-traders’ investment horizons has important empirical implications. For example, the model predicts that, in firms with high growth opportunities, (i) executive pay should be more linked to stock prices and (ii) stock prices should be more informative about long-term value. The reason is that investing in long-term projects is always a dominant strategy for these firms. The speculator anticipates this and acquires information about the firm’s long-term projects. As a result, the stock price is informative about long-term value and can be used to incentivize the manager. This is consistent with the evidence that stock-options are more prevalent in high-tech, “new economy” firms and more generally in growth industries, such as computer, software, and pharmaceutical firms (Murphy 1999, Core and Guay 2001, Ittner et al. 2003). At the same time, Price/Earnings ratios are higher in these industries, which implies that the stock market is taking into account the potential for future profits (Kaplan, 2017).

A second set of empirical implications relates to the importance blockholders have in the equilibrium selection. While the role of blockholders in encouraging long-term investments (Cronqvist and Fahlenbrach (2009)) and deterring myopia (Dechow et al. (1996), Farber (2005), Burns et al. (2008)) is well documented, there is less evidence about the specific channel that leads to this effect - see Edmans and Holderness (2017) for a review of the literature on blockholders. Blockholders can intervene directly into a firm’s operations (voice) or simply trade on information about the firm (exit); if this information is impounded into the stock-price, this also disciplines management. The second channel works through the stock price and, thus, relies on prices being used in the manager’s compensation. Because both compensation and price informativeness are endogenous in my model, my results offer new insights into how to empirically distinguish the two channels. The model predicts that, if the channel is exit, the increase in long-term investments associated with the presence of outside blockholders will be accompanied with (i) executive pay being linked more to stock prices and (ii) prices being more informative about the firm's long-term value. More broadly, while previous work has focused on the role of inside or outside shareholders taken alone, the model suggests that the study of their interaction may motivate new interesting avenues for empirical research.

Next, I offer a summary of the related theoretical literature.
Theoretical Literature

This paper contributes to the literature on managerial short-termism. The earlier work on the topic focused on the distortions that result from exogenous short-term concerns of the managers (e.g., Narayanan 1985, Stein 1989, Bebchuk and Stole 1993). More recent work instead analyzes short-termism in an optimal contracting setting. Bolton et al. (2006) show that optimal compensation contracts may emphasize short-term stock performance at the expense of long-run value, when current shareholders can sell the stock in the future to potentially overoptimistic investors. Peng and Roell (2014) analyze the trade-off between short-term incentives, that expose shareholders to the risk of manipulation by the manager, and long-term, which expose the manager to the risk of future contingencies. In both papers, the investors’ incentives to acquire information are not examined and the informativeness of the stock price is exogenous. My paper contributes to this literature by examining a model in which both the structure of compensation and the informativeness of the stock price are endogenous.

The interaction between executive compensation and the informativeness of the stock price is a type of feedback effect. There is a substantial literature on feedback effects of market prices, which examines how markets affect real decisions and the resulting feedback loop between the two - see Bond, Edmans, and Goldstein (2012) for a review. Within this literature, the paper closest to mine is Holmstrom and Tirole (1993), which examines the value of the stock market as a monitor of managerial performance. The speculator in their model can only acquire information about the firm’s long-term value, while she chooses which type of information (about short-term and/or long-term value) to acquire in my model. Moreover, in my paper the structure of compensation affects the speculator’s incentives to acquire information. Therefore, the interaction between compensation and the stock market is two-way. This feature is absent in Holmstrom and Tirole, as the manager there only chooses effort, which does not affect the ex-ante uncertainty about firm value. Edmans (2009) also connects feedback effects and short-termism, focusing on the role of blockholders as a solution to managerial myopia: by gathering information about a firm’s fundamental value and impounding it into prices, blockholders prevent managers from discarding efficient long-term investments that reduce short-term profits. The dependence of the manager’s contract on current stock prices and the fact that the blockholders trade on long-term information is taken as given in his model, while both are endogenous choices in mine; this allows me to examine the interaction between inside and outside shareholders and its implications for executive compensation and the firm’s optimal investment horizon.

The rest of the paper is organized as follows. Section 2 describes the basic model. In
Section 3 I describe the equilibrium trading strategies and information acquisition choice in the stock market, and how these are affected by the firm’s investment horizon and the managerial contract. Section 4 describes the optimal contracts and how these depend on the informativeness of the stock market. This allows me to solve for the equilibrium and describe its properties in Section 5. In Section 6 I discuss the empirical implications of the model and related evidence. Finally, Section 7 concludes. Detailed proofs are presented in the Appendix.

2 The Model

I consider a publicly traded firm, run by a risk-averse manager and owned by different categories of risk-neutral investors. These categories are (i) inside owners, who hold a constant fraction of shares in each period; (ii) liquidity traders, who buy shares for investment purposes but may have to sell shares when unexpected events occur; and (iii) speculators (a single one in the model), who can collect information about the future value of the firm and make money by trading on that information.

The model has two periods, indexed \( t = 1, 2 \). At time \( t = 1 \), the insiders hire a manager to run the firm and a market for the shares of the firm takes place. The firm’s short-term earnings then realize at the end of the period. Finally, at time \( t = 2 \) the firm is liquidated to shareholders. All agents in the model are rational. For simplicity, I assume that there is no discounting and, therefore, the timing of payments is immaterial.

A. The Firm

At time \( t = 1 \), the shareholders (through the board of directors) choose the firm’s investment horizon. They can choose either a short-term project that boosts the firm’s earnings in the first period or a long-term project that pays out only in the second period. The expected return of a project increases with managerial effort. The interpretation is that, for a given investment horizon, the manager screens among different investment opportunities with the same horizon: the more effort he exerts in screening projects, the higher the expected return of the project that ends up being implemented.

At time \( t = 1 \), the firm produces earnings (gross of payments to the manager) in the amount

\[
\pi_1 = \omega_1 + \eta_1. \tag{1}
\]

The random variable \( \omega_1 \) represents the return on the short-term project. If the manager undertakes the short-term project, \( \omega_1 \) is normally distributed with mean \( e \) and variance \( \sigma^2_\omega \),
where $e$ is the manager’s effort. If the manager does not undertake the short-term project, $\omega_1 = 0$. The random variable $\eta_1$ is a noise term, representing other factors outside the manager’s control that affect the firm’s short-term performance, and is normally distributed with mean 0 and variance $\sigma_1^2$; without loss of generality, I normalize $\sigma_1^2$ to 1.

At time $t = 2$, the firm is liquidated to shareholders. The resulting liquidation proceeds (gross of payments to the manager) are

$$\pi_2 = \omega_2 + \eta_2. \quad (2)$$

The random variable $\omega_2$ represents the return on the long-term project. If the manager undertakes the long-term project, $\omega_2$ is normally distributed with mean $\mu e$ and variance $\sigma_\omega^2$, where $e$ is the manager’s effort. If the manager does not undertake the long-term project, $\omega_2 = 0$. The random variable $\eta_2$ is a noise term, representing other factors outside the manager’s control that occur during the second period and affect the firm’s liquidation value. I assume that $\eta_2$ is normally distributed with mean 0 and variance $\sigma_2^2$, and is independent of $\eta_1$.

For simplicity, I assume that the manager’s effort can only take two values, i.e., $e \in \{0, 1\}$. I let $C(e)$ denote the manager’s private cost of effort, where $C(1) = c$ and $C(0) = 0$. The shareholders cannot observe the manager’s choice of effort. For a given investment horizon (short-term or long-term), they will have to write a compensation contract that incentivizes the manager to choose the desired level of effort. Notice that when $e = 0$, neither project creates value in expectation for shareholders. Therefore, a project is worth being implemented only if $e = 1$, regardless of its horizon. I make the following assumption regarding the cost of the manager’s effort.

**Assumption 1:** $c + \frac{e^2}{2} (\sigma_\omega^2 + 1) < 1$.

Assumption 1 ensures that incentivizing the manager to exert $e = 1$ creates value for shareholders (as $c$ is not too large), at least for the short-term project. Therefore, shareholders will always want to hire the manager in equilibrium.

Of course, providing incentives on a short-term or a long-term project requires different contracts and, therefore, implies different agency costs. Shareholders take into account both the expected return on the project and the relative agency costs when they choose which

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5The left-hand side of the inequality in Assumption 1 describes the total cost of incentivizing a short-term project, under an optimal contract. The first term ($c$) is the manager’s cost of effort; the second term is the manager’s risk-premium, since the contract will link his pay to $\pi_1$, which has volatility $\sigma_\omega^2 + 1$ when the short-term project is implemented. This total cost has to be lower than 1, which is the expected return on a short-term project.
type of projects to pursue. The following assumption characterizes the key trade-off in their choice of the firm’s investment horizon:

**Assumption 2**: $\mu > 1; \sigma_2 > 1$.

In a scenario where the stock price cannot be part of the contract (or is not informative about the manager’s actions), shareholders have two options. They can link the manager’s pay to $\pi_1 = \omega_1 + \eta_1$ and have the manager exert effort on a short-term project. Otherwise, they can link the manager’s pay to $\pi_2 = \omega_2 + \eta_2$ and incentivize the long-term project. On the one hand, the long-term project has higher returns ($\mu > 1$). On the other hand, since the manager is risk-averse and $\eta_2$ is more volatile than $\eta_1$ ($\sigma_2 > 1$), incentivizing effort on the long-term project is more costly for shareholders. The rationale behind this assumption is the idea that, since the liquidation value $\pi_2$ realizes much later in the game, many factors that are outside the manager’s control can affect its realization and contribute to the volatility of the future contingencies $\eta_2$. Therefore, shareholders face a trade-off between higher-returns and lower risk-premium to the manager when they choose the firm’s investment horizon. The analysis in this paper explores the effect that the information contained in the stock price has on this trade-off.\footnote{The assumption that $\sigma_2 > 1$ makes the analysis interesting, otherwise shareholders would always choose the long-term project regardless of the information contained in the stock price. Similarly, if $\mu \leq 1$ and $\sigma_2 > 1$, they would always choose the short-term project.}

**B. The Stock Market**

At time $t = 1$, after the manager undertakes the investment project, a market for the shares of the firm takes place. Trading occurs among liquidity/noise traders, one speculator and a competitive market maker, and the share price $p$ is determined in a model à la Kyle (1985). In this model, market participants first submit their demands, and then prices are set such that expected trading profits are zero conditional on aggregate demand.

Let $u$ denote the aggregate demand of the liquidity traders. This variable is assumed normally distributed with mean zero and variance $\sigma_u^2$, and is independent of $\eta_1$ and $\eta_2$. As usual, liquidity traders serve the purpose of disguising the trades of the informed; otherwise, prices would fully reveal the speculator’s information and there would be no returns to collecting information for the speculator.

Before submitting her demand, the speculator can gather information about the firm’s value. She can learn the firm’s short-term earnings $\pi_1$, at a cost $g_1$; she can also learn the return/quality of the firm’s long-term project $\omega_2$, at a cost $g_2$.\footnote{The assumption that the speculator perfectly learns $\pi_1$ and $\omega_2$ simplifies the analysis but does not affect any of the results. I could have that the speculator observes imperfect signals $s_1 = \pi_1 + \epsilon_1$ and $s_2 = \omega_2 + \epsilon_2$, where the error terms $\epsilon_1$ and $\epsilon_2$ are both normally distributed.} Let $s = (s_1, s_2)$ denote the sig-
nals observed by the speculator; $s$ takes three possible values, i.e., $s \in \{(\pi_1, \emptyset), (\emptyset, \omega_2), (\pi_1, \omega_2)\}$, where $\emptyset$ signifies that a signal was not acquired, since the speculator can decide to learn $\pi_1$ only, $\omega_2$ only, or both $\pi_1$ and $\omega_2$. After observing her signals, the speculator submits a demand $x$. The other agents in the model cannot observe the speculator’s signals $s$ or her demand $x$.

Notice that the speculator cannot learn the firm’s liquidation value $\pi_2$, but just the component of this that depends on the firm’s long-term project. The rationale for this assumption is that the firm’s liquidation value at time $t = 2$, i.e., $\pi_2$, depends on future contingencies ($\eta_2$) that cannot be predicted (or are too costly to predict) by the speculator at time $t = 1$. This is similar to the specification of the speculator’s signal in Holmstrom and Tirole (1993).

Stock-market participants do not observe the type of project the firm implements or the manager’s choice of effort, but form conjectures about them. The market’s conjecture about the return on the short-term project is denoted by $\overline{w}_1$. Similarly, $\overline{w}_2$ denotes the conjecture about the return on the long-term project. The speculator and the market maker know that a project adds value to the firm only if $e = 1$ and, therefore, their conjecture about effort is always $\overline{e} = 1$. Given that $\overline{e} = 1$ and that the firm either invests in the short-term project or in the long-term project, the pair $(\overline{w}_1, \overline{w}_2)$ takes only two values: if the speculator and the market-maker expect the manager to undertake the short-term project, then $(\overline{w}_1, \overline{w}_2) = (\overline{w}_1 \sim N(1, \sigma_{\omega}^2), 0)$; if they expect the manager to undertake the long-term project, then $(\overline{w}_1, \overline{w}_2) = (0, \overline{w}_2 \sim N(\mu, \sigma_{\omega}^2))$.

For simplicity, I assume that the cost of acquiring information ($g_1$ and $g_2$) is small compared to the volatility of liquidity trading:

**Assumption 3:** $g_1 + g_2 < \frac{1}{2} \sigma_u (1 + \sigma_{\omega}^2)^2$.

This assumption makes sure that the speculator acquires a signal whenever this grants her an informational advantage over the market maker. This simplifies the exposition and allows us to focus on the type of information the speculator chooses to acquire rather than whether she acquires information or not.

**C. Managerial Contract**

There are three sources of performance information in the model: the share price $p$, the firm’s short-term performance $\pi_1$, and the firm’s liquidation value $\pi_2$. As is standard in

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8In Section 5.4, I show that the key results of the paper are robust to a symmetric specification where $s \in \{\omega_1, \emptyset, (\emptyset, \omega_2), (\omega_1, \omega_2)\}$. 

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the theoretical literature on executive compensation, I will only consider contracts that are linear in these three performance measures, i.e., of the form:

\[ w = \alpha + \beta p + \gamma_1 \pi_1 + \gamma_2 \pi_2. \] (3)

The manager’s preference over income \( w \) is represented by an exponential utility function. The manager’s private cost of exerting effort \( c(e) \) is independent of his wealth. This implies that the manager’s evaluation of the normally distributed (in equilibrium) income lottery \( w \) can be represented by the certain equivalent measure

\[ U(w, e) = E(w) - \frac{r}{2} Var(w) - C(e), \] (4)

where \( r \) denotes the manager’s coefficient of absolute risk aversion.

For simplicity, I set the manager’s reservation utility to zero. The initial shareholders’ problem is then to choose (through the board of directors) the firm’s investment horizon and the contract \( (\alpha, \beta, \gamma_1, \gamma_2) \) in order to maximize the firm’s expected value at the beginning of time \( t = 1 \), i.e., the expected value of \( \pi_1 + \pi_2 - w \). The contract must satisfy both the manager’s participation and incentive constraints. Stock-market participants do not observe the managerial contract; I let \( (\overline{\alpha}, \overline{\beta}, \overline{\gamma}_1, \overline{\gamma}_2) \) denote their conjectures about \( (\alpha, \beta, \gamma_1, \gamma_2) \), respectively.

\[ D. \text{ Sequence of Events} \]

The timing of the model is summarized in what follows.

**Time \( t = 1 \):**

(i) Shareholders privately choose the firm’s investment horizon and the managerial contract \( (\alpha, \beta, \gamma_1, \gamma_2) \).

(ii) The manager chooses whether to accept the contract or not. If the contract is accepted, the manager privately chooses the effort level \( e \).

(iii) The speculator privately chooses her signal \( s \). Having observed \( s \), she privately chooses demand \( x \).

(iv) Liquidity traders, the speculator, and the market maker trade shares at the market-clearing price \( p \).

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9I show in Proposition 1 that the stock price \( (p) \) follows a normal distribution in equilibrium. As a consequence, the wage \( w \) is normally distributed as well in equilibrium.
(v) The firm’s short-term earnings $\pi_1$ publicly realize.

**Time** $t = 2$:

(vi) The firm’s liquidation value $\pi_2$ publicly realize. The firm’s total value $\pi_1 + \pi_2$ is divided among shareholders after deducting the manager’s pay $w$.

I use *Perfect Bayesian Equilibrium* as the solution concept.

## 3 The Stock Market

Proceeding by backward induction, I first characterize the equilibrium in the stock market for given conjectures about the managerial contract and the firm’s investment horizon, i.e., for given $(\pi, \beta, \gamma_1, \gamma_2)$ and $(\overline{\omega}, \overline{\omega})$.

For given conjectures about the managerial contract and the firm’s investment horizon, the equilibrium price will depend on the speculator’s trading and information acquisition strategies. Conversely, the speculator’s optimal strategies will depend on how his trading affects the price. I am looking for a rational expectations equilibrium in which, for given conjectures $(\pi, \beta, \gamma_1, \gamma_2)$ and $(\overline{\omega}, \overline{\omega})$, the market-maker’s beliefs about the speculator’s behavior coincide with his actual behavior.

### 3.1 Preliminaries

Let $\overline{x}(s)$ denote the market-maker’s conjecture about the speculator’s demand as a function of her private signals $s$. I posit that $\overline{x}(s)$ takes the following linear form:

$$\overline{x}(s) = \phi_1^s s_1 + \phi_2^s s_2 + k^s.$$  \hfill (5)

The coefficients $(\phi_1^s, \phi_2^s)$ determine how aggressively the speculator trades on each signal, depending on which signals she decided to acquire. For a given choice of signals $s$, $k^s$ represents a constant term in $\overline{x}(s)$. I emphasize that the linear specification implies that the coefficients $(\phi_1^s, \phi_2^s, k^s)$ are free to change depending on which information the speculator decided to acquire, i.e., whether she observed $\pi_1$ only, $\omega_2$ only, or both $\pi_1$ and $\omega_2$, but do not depend on the exact realization of $\pi_1$ and $\omega_2$. The market maker does not observe the speculator’s choice of the signals; let $\overline{s}$ denote his conjecture about $s$.

The market maker observes total demand $q = x + u$ and sets a price

$$p = E[\pi_1 + \pi_2 - \overline{\omega} \mid \overline{x}(s) + u = q],$$  \hfill (6)

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where the expectation in (6) is taken with respect to $u, \pi_1, \text{ and } \pi_2$, conditional on the observed $q$ and the conjectured speculator’s strategies $(\pi, \bar{s})$.

The speculator takes as given the market-maker’s conjectures $\pi$ and $\bar{s}$. She then chooses her demand $x$, knowing the true signals $s$ but unaware of $u$ and, thus, of $q$. Therefore, her optimal demand $x$ solves:

$$\pi(s) = \arg\max_x x \{ E[\pi_1 + \pi_2 - w \mid x, \bar{s}] - E[p \mid x]\}, \quad (7)$$

where the expectation in (7) takes into account that the price $p$ is a function of both $x$ and $u$, as described by equation (6).

### 3.2 Stock Market Equilibrium

The market equilibrium is determined by the linearity restriction (5), the pricing rule (6), and the rationality condition on the speculator’s trading strategy (7). The following proposition characterizes the equilibrium.

**Proposition 1** Fix the market-maker’s and speculator’s conjectures about the manager’s contract and the firm’s investment horizon, i.e., ($\alpha, \beta, \gamma_1, \gamma_2$) and ($\bar{\omega}_1, \bar{\omega}_2$). There exists a unique equilibrium satisfying conditions (5) to (7). In this equilibrium, we have:

1. If the market-maker and speculator expect the manager to undertake the short-term project (i.e., $\bar{\omega}_1 \sim N(1, \sigma^2_\omega), \bar{\omega}_2 = 0$), the speculator acquires information only about short-term earnings $\pi_1$.
   
   (a) The speculator’s demand strategy is:
   $$x^* = \frac{1 - \gamma_1}{2\lambda^*} (\pi_1 - 1), \quad (8)$$
   
   where $\lambda^* = \frac{1}{2\sigma_u} \left[(1-\gamma_1)^2 (\sigma^2_\omega + 1)\right]^{\frac{1}{2}}$;
   
   (b) The equilibrium price $p^*$ is
   $$p^* = \frac{1}{1 + \beta} \left[1 - \gamma_1 - \alpha + \frac{1 - \gamma_1}{2} \pi_1 + \lambda^* u\right]. \quad (9)$$

2. If the market-maker and speculator expect the manager to undertake the long-term project (i.e., $\bar{\omega}_1 = 0, \bar{\omega}_2 \sim N(\mu, \sigma^2_\omega)$), the speculator acquires information about both short-term earnings $\pi_1$ and the long-term project $\omega_2$. 

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(a) The speculator’s demand strategy is
\[
x^{**} = \frac{1 - \overline{\gamma}_1}{2\lambda^{**}} \pi_1 + \frac{1 - \overline{\gamma}_2}{2\lambda^{**}} (\omega_2 - \mu),
\] (10)

where \(\lambda^{**} = \frac{1}{2\sigma_w} \left[ (1 - \overline{\gamma}_1)^2 \cdot 1 + (1 - \overline{\gamma}_2)^2 \sigma_w^2 \right]^{\frac{1}{2}}\).

(b) The equilibrium price \(p^{**}\) is
\[
p^{**} = \frac{1}{1 + \beta} \left[ (1 - \overline{\gamma}_2) \mu - \overline{\alpha} + \frac{1 - \overline{\gamma}_1}{2} \pi_1 + \frac{1 - \overline{\gamma}_2}{2} (\omega_2 - \mu) + \lambda^{**} u \right].
\] (11)

The characterization in Proposition 1 calls for several comments. First, the firm’s investment choice affects the uncertainty about the firm’s value and, thus, the speculator’s incentives to acquire information about it. The speculator’s information is partly incorporated into the stock price via her trading activity. Therefore, through the effect on the speculator’s equilibrium strategies, the firm’s investment horizon affects the informativeness of the stock price: this is the first direction of the two-way feedback in the model. The intuition behind this result is the following. When the manager is expected to undertake the short-term project, there is no value for the speculator in acquiring information about the firm’s long-term project, as this is not expected to be implemented (\(\overline{\omega}_2 = 0\)). Therefore, the speculator will only acquire information about the firm’s short-term earnings in this case. As a consequence, the stock price will only incorporate information about the firm’s short-term performance.

On the contrary, when the manager is expected to undertake the long-term project, the speculator can acquire information about it and profit off from the uninformed (liquidity) traders in the market. Therefore, the speculator will acquire information about both \(\pi_1\) and \(\omega_2\). Notice that, when the manager undertakes the long-term project (i.e., \(\overline{\omega}_1 = 0\) and \(\omega_2 \sim N(\mu, \sigma_w^2)\)), the firm’s short-term performance is fully determined by the first-period contingencies \(\eta_1\), since \(\pi_1 = \eta_1\) in this case. This implies that there is still uncertainty about the firm’s short-term performance and, thus, incentives for the speculator to acquire information about it. As a result, the stock price will incorporate information about both the firm’s short-term performance and its long-term value in this case.

Second, given the conjecture of a linear trading strategy for the speculator, the expected firm value conditional on aggregate demand \(q\) depends linearly on \(q\). The coefficients \(\lambda^*\) and \(\lambda^{**}\) in Proposition 1 measure the sensitivity of expectations to the order flow; that is, \(\lambda^*\) and \(\lambda^{**}\) measure how informative aggregate demand is. Of course, this depends on the type of information the speculator is trading on. Therefore, the sensitivity of the stock price to the order flow changes depending on the conjecture about the firm’s investment horizon which,
in turn, determines the speculator’s information acquisition decision.

Finally, how aggressively the speculator trades on each signal depends on the conjectures about the parameters $\tau_1$ and $\tau_2$ of the manager’s contract. The intuition for this result is as follows. Suppose $\gamma_1$ increases: a smaller share of the firm’s short-term earnings $\pi_1$ goes to shareholders (a larger fraction goes to the manager) and, thus, affect the value of the shares. This has a negative externality on the speculator, who knows the realization of $\pi_1$ and profits from trading on it. The speculator reacts by trading less aggressively on $\pi_1$, so that a smaller fraction of $\pi_1$ is revealed to the market-maker. As a consequence, less of her signal is incorporated into the price. Therefore, the sensitivity of the stock price to the realization of $\pi_1$ ($\omega_2$) decreases with $\tau_1$ ($\tau_2$).

Summing up, the informativeness of the stock price depends both directly and indirectly on managerial incentives. Directly, through the effect that the contract has on how aggressively the speculator trades on each signal. Indirectly, through the effect that the firm’s choice of projects has on the speculator’s incentives to acquire information. Since the stock price can be part of the contract, this has important implications for the structuring of managerial incentives themselves.

4 Optimal Contracting

Having characterized the equilibrium trading and information acquisition strategies in the stock market, I can now characterize the optimal contracts for a given project type and given conjectures about the informativeness of the stock price.

Both the manager and shareholders take the market’s conjectures about the contract $(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}_1, \tilde{\gamma}_2)$ and the firm’s investment horizon $(\bar{w}_1, \bar{w}_2)$ as given. Of course, they understand the structure of the equilibrium in the stock market and, therefore, how these conjectures affect the informativeness of the stock price. I begin by describing the manager’s choice of effort and how this depend on the contract and on the informativeness of the stock price.

4.1 Managerial Incentives

Given the contract $(\alpha, \beta, \gamma_1, \gamma_2)$ and the type of the project (short-term or long-term), the manager chooses the effort level ($e \in \{0, 1\}$) that maximize the certainty equivalent measure of his utility (equation (4)).

Lemma 1 The manager’s choice of effort is characterized as follows:
1. When the market-maker and speculator expect the manager to undertake the short-term project (i.e., \( \omega_1 \sim N (1, \sigma^2_{\omega}) \), \( \omega_2 = 0 \)):

(a) If the manager were to undertake the short-term project, he would choose \( e = 1 \) if \( \beta \frac{1 - \gamma_1}{2(1 + \beta)} + \gamma_1 \geq c \);

(b) If the manager were to undertake the long-term project, he would choose \( e = 1 \) if \( \gamma_2 \mu \geq c \).

2. When the market-maker and speculator expect the manager to undertake the long-term project (i.e., \( \omega_1 = 0 \), \( \omega_2 \sim N (\mu, \sigma^2_{\omega}) \)):

(a) If the manager were to undertake the short-term project, he would choose \( e = 1 \) if \( \beta \frac{1 - \gamma_1}{2(1 + \beta)} + \gamma_1 \geq c \);

(b) If the manager were to undertake the long-term project, he would choose \( e = 1 \) if \( \beta \frac{1 - \gamma_2}{2(1 + \beta)} \mu + \gamma_2 \mu \geq c \).

Lemma 1 characterizes the manager’s optimal effort choice for a given contract \((\alpha, \beta, \gamma_1, \gamma_2)\) and given market’s conjectures about both the contract and the choice of the project. First, consider the case when the stock-market expects the firm to implement the short-term project. Suppose also this conjecture is consistent: shareholders have asked the manager to invest in the short-term project, and the manager is contemplating whether to exert effort in screening the project or not. Effort increases the expected return on the project \((\omega_1 \sim N (\epsilon, \sigma^2_{\omega}))\). When the manager shirks and chooses \( e = 0 \), the expected short-term earnings go down, since \( \pi_1 = \omega_1 + \eta_1 \). This reduces the manager’s expected pay in two ways. First, via its short-term incentives \( \gamma_1 \): the manager loses \( \gamma_1 \cdot 1 \). Second, via the expected stock price, which goes down by a factor \( \frac{1 - \gamma_1}{2(1 + \beta)} \). This is because the speculator observes the true realization of \( \pi_1 \), which is distributed as \( \pi_1 \sim N (\omega_1, \sigma^2_{\omega} + 1) \) when \( \omega_1 \sim N (0, \sigma^2_{\omega}) \). Therefore, in expectation, the speculator finds out that the firm is overvalued (the market maker expects \( e = 1 \) and, thus, \( \pi_1 \sim N (1, \sigma^2_{\omega} + 1) \)) and sells shares, driving down the expected stock price. The speculator acts as a monitor for the manager and contributes to incentivizing high effort. If the losses from shirking are greater than the cost of effort, the manager is better off by choosing \( e = 1 \). This describes the inequality in Part 1.a of the Lemma.

Even if the stock market expects the short-term project, shareholders might prefer to incentivize the manager to take a long-term project instead. In this case, since the speculator has not acquired information about \( \omega_2 \) (the conjecture is \( \omega_1 \sim N (1, \sigma^2_{\omega}), \omega_2 = 0 \)), the stock price will not reflect that the manager did not exert effort in screening the project, i.e., that
Therefore, if the stock market expects the short-term project will be implemented, the stock price is of no use in incentivizing effort on the long-term project. This describes the inequality in Part 1.b of the Lemma.

A similar logic yields the incentive constraints in Part 2 of the Lemma. Notice that the constraint in Part 2.a is the same as the one in Part 1.a. This is because regardless of the conjectures about project choice, the speculator always acquires information about \( \pi_1 \). Therefore, when the market expects the long-term project and the manager invests instead in a short-term project, he can boost the expected stock price by exerting effort on the project: \( \pi_1 \) would be then distributed as \( \pi_1 \sim N(1, \sigma^2 + 1) \), while the market-maker and the speculator expect \( \pi_1 \sim N(0, \sigma^2 + 1) \). \(^{10}\)

### 4.2 Optimal Contracts

Like the manager, shareholders take the market’s conjectures about the contract \((\pi, \beta, \gamma_1, \gamma_2)\) and the firm’s investment horizon \((\bar{\omega}_1, \bar{\omega}_2)\) as given when choosing the project type and the contract that maximize the firm’s value. Following Grossman and Hart (1983), it is useful to think about this choice as a two-stage process. First, for a given project (short-term or long-term project), the shareholders find the optimal contract to incentivize the manager to exert high effort \( e = 1 \). This contract determines the firm’s optimal value for a given project choice. Second, they compare the firm’s value in the two scenarios and choose the investment horizon (and the associated contract) that leads to higher value. The analysis of contracting in this section describes the first step of this two-stage process. The properties of the strategic interaction between managerial incentives and the informativeness of the stock price will then be used in the next section to characterize the firm’s optimal investment horizon and the equilibrium of the game.

The manager’s participation constraint is always binding under an optimal contract, so that the following inequality is always satisfied with equality:

\[
\alpha + E (\beta p + \gamma_1 \pi_1 + \gamma_2 \pi_2) - \frac{T}{2} Var (\beta p + \gamma_1 \pi_1 + \gamma_2 \pi_2) - c \geq 0 \tag{12}
\]

Equation (12) describes the manager’s participation constraint. The distribution of \( \pi_1 \) and \( \pi_2 \) in the equation is conditional on \( e = 1 \) and on the choice of the project, which affects the distribution of returns \((\omega_1, \omega_2)\). The distribution of \( p \) depends on the true distribution of \((\omega_1, \omega_2)\) and the market’s conjectures (about \((\omega_1, \omega_2)\) and the contract), via the effect on \(^{10}\)The sensitivity of the stock-price to short-term performance is also the same in both cases, i.e., \( \frac{1 - \gamma_1}{2(1 + \beta)} \) (see Proposition 1).
the stock market equilibrium.

Given that the participation constraint is always binding under an optimal contract, the optimal contract for a given investment horizon simply minimizes the manager’s risk-premium under the respective incentive constraints. Therefore, for a given investment horizon, the optimal contract solves:

$$\min_{(\alpha, \beta, \gamma_1, \gamma_2)} \frac{r}{2} Var (\beta \pi_1 + \gamma_1 \pi_1 + \gamma_2 \pi_2)$$

subject to the respective incentive constraint in Lemma 1.

As before, the distribution of \(\pi_1\) and \(\pi_2\) in program (13) is conditional on \(e = 1\) and on the choice of the project, which affects the distribution of returns \((\omega_1, \omega_2)\). The distribution of \(p\) depends on both the true distribution of \((\omega_1, \omega_2)\) and the market’s conjectures (about \((\omega_1, \omega_2)\) and the contract), via the effect on the stock market equilibrium.\(^{11}\)

The following proposition characterizes the optimal contracts for the short-term and the long-term projects.

**Proposition 2** Fix the market-maker’s and speculator’s conjectures about the manager’s contract and the firm’s investment horizon, i.e., \((\bar{\alpha}, \bar{\beta}, \bar{\gamma}_1, \bar{\gamma}_2)\) and \((\bar{\omega}_1, \bar{\omega}_2)\). We have:

1. The optimal contract that incentivizes the manager to exert effort on a short-term project features: \(\gamma_1^* = c, \beta^* = \gamma_2^* = 0\);

2. The optimal contract that incentivizes the manager to exert effort on a long-term project features:

   (a) If the market-maker and the speculator expect the manager to undertake the short-term project (and, thus, \(p\) is not informative about the long-term project \(\omega_2\)), the contract features \(\gamma_1^* = \beta^* = 0, \gamma_2^* = \frac{c}{\mu}\);

   (b) If the market-maker and the speculator expect the manager to undertake the long-term project (and, thus, \(p\) is informative about \(\omega_2\), the contract features \(\gamma_1^* = 0, \beta^* > 0\) and both \(\gamma_2^* > 0\). The exact value of \(\beta^*\) and \(\gamma_2^*\) is characterized by

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\(^{11}\)The exact expression for the variance in (13), for a given investment horizon and conjectures \((\bar{\alpha}, \bar{\beta}, \bar{\gamma}_1, \bar{\gamma}_2)\) and \((\bar{\omega}_1, \bar{\omega}_2)\), is described in Appendix B.2.
the following system of equations:

\[ \gamma_2^{\up} = \frac{c - 2 \left( \frac{1 - \pi_1}{1 - \pi_2} \right)^2 + \sigma_2^2}{\mu 2 \left( \frac{1 - \pi_1}{1 - \pi_2} \right)^2 + \sigma_2^2 + \sigma_\omega^2}; \]

\[ \gamma_2^{\up} + \beta_2^{\up} \frac{1 - \pi_2}{2 \left( 1 + \beta \right)} = \frac{c}{\mu}. \] (15)

For all three cases, the fixed component of pay \( \alpha \) is chosen so that the manager’s participation constraint in equation (12) is binding.

Notice that \( \gamma_2^{\up} \) is always lower than \( \frac{c}{\mu} \). Therefore, the optimal contract for the long-term project always links the manager’s pay to both \( p \) and \( \pi_2 \). Moreover, as the volatility of future contingencies (\( \sigma_2^2 \)) increases, the contract puts less weight on \( \pi_2 \) and more on \( p \), i.e., \( \gamma_2^{\up} \) decreases and, thus, \( \beta_2^{\up} \) increases with \( \sigma_2^2 \).\(^{12}\)

Regardless of the market’s conjecture about the firm’s investment horizon, the optimal contract for the short-term project links the manager’s pay to the realization of short-term earnings only. The intuition for this result is the following. Depending on the market’s conjectures, the stock price \( p \) can be informative about (i) \( \pi_1 \) only or about (ii) both \( \pi_1 \) and \( \omega_2 \). In case (i), \( p \) and \( \pi_1 \) contain the same type of information.\(^{13}\) However, the speculator does not fully reveal his information about \( \pi_1 \) to the market. Therefore, only a fraction of the realization of \( \pi_1 \) is incorporated into \( p \). This means that the incentive power of \( p \) is lower and, thus, a larger \( \beta \) is required to provide incentives via \( p \). This translates into a larger risk-premium, since a larger part of the manager’s wealth is subject to risk. As a consequence, it is optimal to use only \( \pi_1 \) in the contract. In case (ii), \( p \) contains information about \( \omega_2 \) as well, which is not useful to incentivize the short-term project. Of course, using only \( \pi_1 \) in the contract is optimal also in this case.

However, when it comes to incentivizing the manager to take the long-term project, the speculator’s information acquisition becomes crucial. If the speculator acquires information about both \( \pi_1 \) and \( \omega_2 \), \( p \) provides additional information about the manager’s effort. This is because \( p \) does not include future contingencies (\( \eta_2 \)) that will affect the firm’s liquidation value \( \pi_2 \) but do not depend on the manager’s effort. On the other hand, \( p \) includes current

\(^{12}\) The firm’s liquidation value is \( \pi_2 = \omega_2 + \eta_2 \). As the volatility of \( \eta_2 \) (i.e., \( \sigma_2^2 \)) increases, linking the manager’s pay to \( \pi_2 \) becomes more expensive to shareholders, since the manager will require a higher risk-premium. In the limit as \( \sigma_2 \) tends to infinity, \( \gamma_2^{\up} \) vanishes and \( \beta_2^{\up} = \frac{c}{\mu} \).

\(^{13}\) It is worth noticing that, while the exact realization of \( p \) also depends on the realization of liquidity trading \( u \), the volatility of \( u \) has no direct effect on the equilibrium distribution of \( p \). This is because the speculator’s optimal trading strategy is such that \( x(s) \) will adjust in response to changes in \( \sigma_u \) precisely so that the distribution of price is independent of the level of liquidity trading. Therefore, liquidity trading has only an indirect effect on the distribution of \( p \), by incentivizing information collection by the speculator.
contingencies \( \eta_1 \) that affect the firm’s short-term performance \( \pi_1 \) but do not depend on the manager’s effort in screening among long-term projects. Therefore, shareholders will want to use a mix of both \( p \) and \( \pi_2 \) in the contract. However, when the speculator acquires information about \( \pi_1 \) only, \( p \) is of no use in incentivizing the manager. Therefore, shareholders can use only \( \pi_2 \) in the contract.

5 Equilibrium

I have found that the strategic interaction between executive compensation and the informativeness of the stock price is characterized by a two-way feedback. One way goes from the firm to the stock market: when the firm is expected to invest in the long-term project, the speculator acquires information about it and this information is partly incorporated into the price through her trading. The other direction of the feedback goes from the stock market to the firm: if the speculator acquires information about the firm’s long-term project, the stock price can be used to incentivize effort on a long-term project; therefore, implementing a long-term project becomes more attractive for the firm.

This two-way feedback generates a strategic complementarity in the choice of horizons between the shareholders and the speculator. As is typical in games with strategic complementarities, this can lead to multiple equilibria.

Definition 1 Depending on the parameters of the model, there exist two types of equilibria:

1. An equilibrium with short-termism, where: the manager undertakes the short-term project; the speculator acquires information only about short-term earnings \( (\pi_1) \); shareholders set a contract \( \gamma_1^* = c, \beta^* = \gamma_2^* = 0 \);

2. An equilibrium with long-termism, where: the manager undertakes the long-term project; the speculator acquires information about both the long-term project \( (\omega_1) \) and \( \pi_1 \); shareholders set a contract \( \gamma_1^{**} = 0, \beta^{**} > 0, \gamma_2^{**} > 0 \). The exact value of \( \beta^{**} \) and \( \gamma_2^{**} \) is characterized by the following system of equations:

\[
\gamma_2^{**} = \frac{c}{\mu} \left( 2 \left( 1 - \frac{\gamma_2^{**}}{1 - \gamma_2^{**}} \right) \sigma^2 + \frac{\sigma_\omega^2}{2 (1 + \beta^{**})} \right); \quad (16)
\]

\[
\gamma_2^{**} + \beta^{**} \left( \frac{1 - \gamma_2^{**}}{2 (1 + \beta^{**})} \right) = \frac{c}{\mu}. \quad (17)
\]

In both equilibria, the manager chooses \( e = 1 \) and the fixed component of pay \( \alpha \) is chosen so that the manager’s participation constraint in equation [12] is binding.
When the market conjectures about the contract are consistent with the actual contract, i.e., when $\beta = \beta^\ast$, $\tau_1 = 0$, $\tau_2 = \gamma_2^\ast$, the system of equations that characterizes the optimal contract for the long-term project (i.e., equations (16) and (17)) is the same as the one in Proposition 2. The exact value of $\gamma_2^\ast$ is a fixed point of equation (16). In Appendix B.2, I show that this equation has a unique fixed point in $[0, \frac{c}{\mu}]$ and, thus, that $\gamma_2^\ast$ is unique. Given $\gamma_2^\ast$, the value of $\beta^\ast$ is uniquely pinned down by the incentive constraint in equation (17).

The analysis of the equilibrium becomes easier if we first analyze a benchmark where the stock price is not contractible. Therefore, I will first characterize the equilibrium in such a benchmark. This will help us characterize the equilibrium of the game in Proposition 3.

5.1 Benchmark without Stock Market

Consider a benchmark where the stock price $p$ is not contractible. This is the case, for example, when the firm’s shares are not publicly traded.

Lemma 2 When the stock price cannot be part of the contract, we have:

1. The optimal contracts for the short-term and long-term projects are as described in Part 1 and Part 2.a of Proposition 2, respectively: the optimal contract for the short-term project is $\gamma_1^\dagger = c$, $\beta^\dagger = \gamma_2^\dagger = 0$; the optimal contract for the long-term project is $\gamma_1^\dagger = \beta^\dagger = 0$, $\gamma_2^\dagger = \frac{c}{\mu}$;

2. There exists a (unique) threshold value $\overline{\mu}$, with $\overline{\mu} > 1$, such that:

   (a) when $\mu \leq \overline{\mu}$, shareholders choose the short-term project;
   
   (b) when $\mu > \overline{\mu}$, shareholders choose the long-term project.

Lemma 2 is quite intuitive. Even when the stock price can be part of the contract, the optimal contract for the short-term project only uses the realization of short-term earnings $\pi_1$. Therefore, this contract does not change in the benchmark where the stock price is not contractible. When $p$ is not informative about $\omega_2$, the optimal contract for the long-term project only uses the realization of the firm’s liquidation value $\pi_2$, as the stock price is not useful in incentivizing the manager. Since the stock price is not used in this case, this contract is the same as in the benchmark. This equivalence plays an important role in the characterization of the equilibrium.

Shareholders compare the firm’s value under the two investment horizons and choose the one that leads to higher value. They choose to have the long-term project implemented, if
the following condition is satisfied:

$$\mu - c - \frac{r}{2} \left( \frac{c}{\mu} \right)^2 \left( \sigma_2^2 + \sigma_1^2 \right) \geq 1 - \frac{r}{2}c^2 \left( \sigma_2^2 + 1 \right).$$

(18)

The inequality above determines the (unique) threshold value $\bar{\mu}$. The right-hand side of the inequality is the firm’s expected value when the short-term project is implemented; Assumption 1 ensures that this is non-negative. The left-hand side is instead the firm’s expected value when the long-term project is implemented. This expected value increases with $\mu$ for two reasons. First, the long-term project becomes more profitable when $\mu$ goes up. Second, it becomes easier to incentivize the manager (the incentive constraint in Part 2.a of Lemma 1 becomes slacker), as now the manager loses more from shirking. When $\mu = \bar{\mu}$, these two effects perfectly compensate for the fact that $\eta_2$ is more volatile than $\eta_1$ ($\sigma_2 > 1$). Therefore, the long-term project is, all else equal, more costly to incentivize: shareholders are indifferent between the two investment horizons. As $\mu > \bar{\mu}$, they are strictly better off when the long-term project is implemented.

5.2 The Equilibrium

Having characterized the optimal investment horizon in a benchmark model where the stock price was not contractible, I can now describe the equilibrium of the game in the full model. The following proposition characterizes the equilibria as a function of the expected return of the long-term project ($\mu$).

**Proposition 3** There exist two thresholds $\bar{\mu} > 1$ and $\underline{\mu} \in [1, \bar{\mu})$, where $\bar{\mu}$ is the same as in Lemma 2, such that:

1. If $\mu \leq \underline{\mu}$, short-termism is the unique equilibrium of the game;

2. If $\underline{\mu} < \mu < \bar{\mu}$, both short-termism and long-termism are equilibria of the game;

3. If $\mu > \bar{\mu}$, long-termism is the unique equilibrium of the game.
Figure 1 describes the results in Proposition 3. First, consider the case when the market’s conjecture is that the manager undertakes the short-term project. In this case, the speculator does not acquire information about the long-term project and, thus, $p$ cannot be used to incentivize the long-term project. Therefore, the optimal contracts are as described in Part 1 and Part 2.a of Proposition 2. As discussed earlier, these optimal contracts are the same as in the benchmark without the stock market and, thus, the firm’s optimal value for a given investment horizon is the same as in the benchmark. Shareholders then choose the firm’s investment horizon according to the inequality (18). If $\mu \leq \overline{\mu}$, they choose the short-term project; if $\mu > \overline{\mu}$, they choose the long-term project. This has two important implications. First, it implies that the initial conjecture we started with, i.e., that the manager undertakes the short-term project, can be consistent only when $\mu \leq \overline{\mu}$. Therefore, short-termism can be an equilibrium of the game if and only if $\mu \leq \overline{\mu}$. Second, it means that, when $\mu > \overline{\mu}$, the unique possible equilibrium is long-termism.

Now, consider the opposite market conjecture, i.e., that the manager undertakes the long-term project. In this case, the speculator acquires information about the long-term project and so $p$ can be used to incentivize effort on the long-term project. What happens to the shareholders’ optimal investment horizon in this case? Shareholders can now use a (strictly) better contract: the marginal firm $\overline{\mu}$, that was indifferent under the previous conjecture (and in the benchmark model), is strictly better off with the long-term project now. Therefore, the indifference condition obtains for a strictly lower threshold $\underline{\mu}$.

The fact that $\mu < \overline{\mu}$ induces the multiplicity of equilibria, since when $\mu < \mu < \overline{\mu}$, both short-termism and long-termism are equilibria. If the market conjectures that the manager implements the short-term project, shareholders are strictly better off incentivizing the short-term project ($\mu < \overline{\mu}$), since the stock price cannot be used in the contract: short-termism is an equilibrium. If the market conjectures that the manager implements the long-term project, shareholders are strictly better off choosing the long-term project ($\mu > \underline{\mu}$): long-termism is an equilibrium as well.

The interval $(\underline{\mu}, \overline{\mu})$ becomes arbitrarily large when the volatility of future contingencies $\eta_2$ is large. In the limit as $\sigma_2$ tends to infinity, we have $\lim_{\sigma_2 \to \infty} \underline{\mu} < \infty$ and $\lim_{\sigma_2 \to \infty} \overline{\mu} = \infty$. This is because, as $\sigma_2$ becomes large, the optimal contract in the equilibrium with long-termism only depends on the price $p$ ($\lim_{\sigma_2 \to \infty} \gamma_2^{**} = 0$ and $\lim_{\sigma_2 \to \infty} \beta^{**} = \frac{c}{\mu}$). Therefore, the contract does not expose the manager to the volatility of $\sigma_2$ and, thus, the threshold $\underline{\mu}$ is finite even when $\sigma_2 \to \infty$. However, if the price does not incorporate information about $\omega_2$, the contract for the long-term project needs to depend on $\pi_2$: $\overline{\mu}$ goes to infinity in the limit as $\sigma_2$ tends to infinity.
5.3 Equilibrium Selection

When both long-termism and short-termism are equilibria of the game, shareholders prefer long-termism, as firm value is strictly larger under long-termism. But what about the speculator? The following inequality characterizes her preference over the two equilibria.

\[
\frac{\sigma_u}{2} \left[ (1 - \gamma_1^{**})^2 \left( \sigma_u^2 + 1 \right) \right]^{\frac{1}{2}} - g_1 > \frac{\sigma_u}{2} \left[ 1 + (1 - \gamma_2^{**})^2 \sigma_u^2 \right]^{\frac{1}{2}} - (g_1 + g_2). 
\]  
(19)

The left-hand side of the inequality in (19) describes the speculator’s expected payoff under short-termism; the right-hand side instead describes her payoff under long-termism. When the inequality is satisfied, the speculator prefers short-termism.

Notice that the net profit from trading is always larger under long-termism (as we have \( \gamma_1^{**} > \gamma_2^{**} \) and \( \gamma_1^{**} < 1 \)). However, information acquisition costs are also larger under long-termism, since the speculator learns about both short-term performance and long-term value. Therefore, if \( g_2 \) is sufficiently large, the speculator prefers short-termism. In this case, the shareholders’ and the speculator’s preferences over equilibria are not aligned, and coordination failure is a serious issue.\(^{14}\)

The misalignment of preferences is (partly) due to the speculator having no direct stake in the firm at the information acquisition stage, which implies that she does not care if firm value is lower under short-termism. Recent empirical work documents the predominance in U.S. public companies of small transient blockholders, who typically lack control rights to directly intervene into a firm’s operation but can sell their shares if the firm underperforms (or possibly consolidate their position if the firm overperforms).\(^{15}\) Motivated by this stylized fact, the rest of this section explores the implications of having a pre-existing stake in the firm for the speculator’s incentives.

Let \( \Delta \) denote the speculator’s endowment of shares at the information acquisition stage. The following Lemma describes the effect of \( \Delta \) on the equilibrium.

\(^{14}\)The notion of Risk-dominance (Harsanyi and Selten (1988)) does not apply to games with incomplete information. Therefore, I focus only on Payoff-dominance as an equilibrium selection criterion. It is often argued that players will coordinate on the Pareto-dominant equilibrium (provided one exists) if they are able to talk to one another before the game is played. The intuition for this is that, even though the players cannot commit themselves to play the way they claim they will, the preplay communication lets the players reassure one another about the low risk of playing the strategy of the Pareto-dominant equilibrium (Fudenberg and Tirole (1991)).

\(^{15}\)Holderness (2009) documents that, when blockholders are defined as 5% shareholders, 96% of U.S. firms contain a blockholder. However, when the minimum ownership is defined as 20%, La Porta et al. (1999) find that only 20% (10%) of large (medium) U.S. firms contain a blockholder. They also estimate that a 20% stake gives effective control if the shareholder is an insider, while the threshold is likely to be higher for outside shareholders.
Lemma 3 A speculator’s preexisting stake in the firm (Δ) has no direct effect on the equilibrium strategies. However, if Δ is sufficiently large, i.e., Δ ≥ Δ, the equilibrium with long-termism Pareto dominates the one with short-termism; otherwise, the two equilibria cannot be Pareto-ranked.

The result in Lemma 3 is quite intuitive. Having a pre-existing stake in the firm does not affect the speculator’s trading strategies and, as a consequence, does not affect her profits from trading. This is because the speculator is allowed to short-sell, so Δ does not affect how much she can trade. Moreover, her profits from trading only depend on the difference between firm value and the expected stock price, not on their absolute value: the speculator sells if, given her signals, she expects that the firm will be overvalued; she buys otherwise. However, Δ creates a link between the speculator’s expected payoff and the firm’s ex-ante value. In expectation, the speculator’s demand is null (the price is correct in expectation). Therefore, she expects to keep the same stake Δ in the firm after trading. This means that, if firm value increases, the value of her expected stake in the firm increases as well. As a consequence, Δ aligns shareholders’ and speculator’s preferences over different equilibria: if the stake is sufficiently large, the equilibrium with long-termism Pareto dominates. Therefore, coordination failure is less likely.¹⁶

This result uncovers a new strategic complementarity between inside and outside shareholders (the speculator in the model). Compared to other informed-traders in the market, outside shareholders have a stronger incentive to trade on information about the long-term prospect of a firm, because this enables inside shareholders to design more efficient managerial contracts and increase firm value.

5.4 Alternative Specification for the Speculator’s Signal

Here, I show that the key results of the paper are robust to a setting where the speculator can only acquire signals about the firm’s projects ω₁ and ω₂, but not about the contingencies that affect its current or future profits, i.e., η₁ or η₂. This implies that the speculator’s signal s takes values in {ω₁, η₂, ω₁, η₁} as opposed to {(ω₁, η₁), (η₁, ω₂), (ω₁, ω₂)}, which is the case in the baseline specification of the model.

In this modified setting, the speculator will only acquire information about either ω₁ or ω₂, i.e., in equilibrium we have s ∈{ω₁, η₂, ω₁, η₁}. The intuition is quite straightforward. The firm either implements the short-term project, in which case (ω₁, η₂) =

¹⁶The strategic interaction in the model boils down to two players, the principal (shareholders) and the speculator. Especially if the speculator has a stake in the firm and so has a way to communicate with the controlling shareholders, it is natural for players to coordinate on the payoff-dominant equilibrium.
(\omega_1 \sim N(1, \sigma_1^2), 0), \text{ or the long-term project, i.e., } (\omega_1, \omega_2) = (0, \omega_2 \sim N(\mu, \sigma_2^2)). \text{ Therefore, the speculator can never profit from acquiring information about both. This has an important implication on the informativeness of the stock price: when the speculator acquires information about } \omega_1, \text{ the price is useful to incentivize effort on the short-term project.}\footnote{The logic is the same as for the long-term project in the baseline model. When the speculator observes } \omega_1, \text{ the price does not incorporate the realization of contingencies } \eta_1, \text{ which does not depend on the manager’s effort but affects the realization of the firm’s profits } \pi_1. \text{ Therefore, the price incorporates unique information about the manager’s effort on the short-term project. This partly compensates for the fact that the speculator does not reveal all of his information about } \omega_1 \text{ and, thus, shareholders use both } p \text{ and } \pi_1 \text{ in the contract.}

This result strengthens the strategic complementarity between the firm’s investment horizon and the speculator information acquisition choices.

**Definition 2** Depending on the parameters of the model, there exist two types of equilibria:

1. An equilibrium with short-termism, where: the manager undertakes the short-term project; the speculator acquires information only about \( \omega_1 \); shareholders set a contract \( \gamma'_1 > 0, \beta'_1 > 0, \gamma'_2 = 0 \);

2. An equilibrium with long-termism, where: the manager undertakes the long-term project; the speculator acquires information only about \( \omega_2 \); shareholders set a contract \( \gamma''_1 = 0, \beta'' > 0, \gamma''_2 > 0 \).

In both equilibria, the manager chooses \( e = 1 \) and the fixed component of pay \( \alpha \) is chosen so that the manager’s participation constraint in equation (12) is binding.

Like in the baseline model, I characterize the equilibria as a function of the expected return of the long-term project (\( \mu \)).

**Proposition 4** There exist two thresholds \( \mu'' > 1 \) and \( \mu' \in [1, \mu''] \), and \( \bar{\mu} \in (\mu', \mu'') \), such that:

1. If \( \mu \leq \mu' \), short-termism is the unique equilibrium of the game;
2. If \( \mu' < \mu < \mu'' \), both short-termism and long-termism are equilibria of the game;
3. If \( \mu > \mu'' \), long-termism is the unique equilibrium of the game.

The key difference with respect to the baseline model is that now the interval with multiple equilibria extends to firms that were long-termist in the model where the stock price cannot be included in the contract (i.e., \( \mu'' > \bar{\mu} \)). To understand this result, consider the case when the market’s conjecture is that the manager undertakes the short-term project.
In the new setting, the information in \( p \) makes also short-termism more attractive for the firm: if the price incorporates information about \( \omega_1 \), it enables a more efficient contract to incentivize effort on the short-term project. This moves the firm’s indifference (between the two projects type) condition to some \( \mu'' \) strictly larger then \( \bar{\mu} \).

Intriguingly, although the possibility to include the stock price in the design of compensation enlarges the mass of firms that could be short-termist in equilibrium (depending on which equilibrium is played), it never makes the firms worse off. Therefore, in line with the results in the baseline model, the stock market may fail to alleviate the problem of corporate short-termism, but never makes it worse.

### 6 Implications

This paper examines how the structure of managerial incentives and the informativeness of the stock price affect each other and the resulting consequences for corporate short-termism. This section discusses some implications of the model and the related empirical evidence.

The broad objective of this paper is to show that the role of the stock market in relation to corporate short-termism may be fundamentally misunderstood. Although regarded as the primary culprit of short-termism, the stock market should instead be seen as a potentially alleviating force. This has important implications for the myopia debate. For example, Kaplan (2018) points to a lack of long-term evidence consistent with the predictions of the short-term critics. One of his main arguments relates to the role of Venture Capital (VC) or Private Equity (PE) investments over time. VC and PE firms can make long-term investments in innovative companies with time horizons of five to ten years or more. Their long-term commitment shelters management from short-term market pressures. Therefore, if (i) short-termism has increased over time, the scope for VC and PE funds has increased over time, and (ii) if short-termism is a problem, then VC and PE funds should be highly profitable. Kaplan shows that neither hypothesis is consistent with the evidence. My results suggest that a key factor might be missing in this argument: the underlying agency problem between management and shareholders. Even if VC or PE funds take a firm private, this problem remains. Therefore, the evidence in Kaplan (2017) is not necessarily indicative that corporate short-termism is not a problem.

Both Kaplan and Roe (2018) also point to high Price/Earnings (P/E) ratio as evidence against the short-term critics. A high P/E ratio suggests that the stock market is valuing a firm much more than can be justified by its current earnings, because it is taking into account the potential for future profits. The current P/E ratio of the S&P 500 is 25, versus a historical median of 15. The high valuations of unicorns, despite them making little or
even negative earnings, indicates that the stock market must be valuing something other than current profits. In the model, firms with high growth opportunities \((\mu > \overline{\mu})\) benefit the most from being public. This is because investors anticipate that they will always invest in long-term projects and, thus, acquire information about them. This enables the firms to design a more efficient long-term contract for their managers. However, in firms with intermediate growth opportunities \((\mu < \mu < \overline{\mu})\), the investment horizon depends on whether the stock market is focused on short-term performance or not. Therefore, the high valuations of unicorns cannot be considered as evidence that stock-market-driven short-termism is not a problem at all.

A second set of implications relates to the strategic complementarity between the firm’s and informed-traders’ investment horizons. A cross-sectional interpretation of the results in Proposition 3 is that, in firms with high growth opportunities \((\mu > \overline{\mu})\), (i) executive pay should be more linked to stock prices and (ii) stock prices should be more informative about long-term value. This is consistent with the evidence that stock-options are more prevalent in high-tech, “new economy” firms and more generally in growth industries, such as computer, software, and pharmaceutical firms (Murphy 1999, Core and Guay 2001, Ittner et al. 2003). At the same time, P/E ratios are higher in these industries, which implies that the stock market is taking into account the potential for future profits (Kaplan, 2017). The results in Proposition 3 also imply that, in firms with intermediate growth opportunities \((\mu < \mu < \overline{\mu})\), the investment horizon depends crucially on the horizon of informed-trading. This is supported by the findings in Hansen and Hill (1991), Bushee (1998), and Wahal and McConnell (2000), which show a positive association between R&D and institutional ownership; the latter is typically highly correlated with blockholdings. Bushee also finds that myopia is driven by momentum investors who trade on current earnings and have small holdings.

Finally, the results in Lemma 3 relate to the importance blockholders have in the equilibrium selection. While the role of blockholders in encouraging long-term investments (Cronqvist and Fahlenbrach (2009)) and deterring myopia (Dechow, Sloan, and Sweeney (1996), Farber (2005), Burns, Kedia, and Lipson (2008)) is well documented, there is less evidence about the specific channel that leads to this effect. Blockholders can intervene directly into a firm’s operations \((voice)\) or simply trade on information about the firm \((exit)\); if this information is impounded into the stock-price, this also disciplines management. The second channel works through the stock price and, thus, relies on prices being used in the manager’s compensation. Because both compensation and price informativeness are endogenous in my model, the results in this paper offer new insights into how to empirically distinguish the two channels. Lemma 3 shows that, if the block is sufficiently large, the equilibrium with
long-termism Pareto dominates and, thus, coordination failure is less likely. Therefore, the model predicts that, if the channel is exit, the increase in long-term investments associated with the presence of outside blockholders will be accompanied with (i) executive pay being linked more to stock prices and (ii) and prices being more informative about the firm’s long-term value.

7 Conclusion

This paper studies the interaction between executive compensation and stock market prices, analyzing its implications for corporate short-termism. I show that inefficient short-termism can arise in equilibrium as a self-fulfilling prophecy, due to strategic complementarities between the firm’s investment horizon and investors’ decision to acquire information about short-term performance or long-term value. This uncovers a new mechanism by which the stock market can feed corporate short-termism through an excessive focus on short-term performance. At the same time, the analysis suggests that the real cause of corporate short-termism is the underlying agency problem between shareholders and management. Moreover, far from being the primary culprit, the stock market can be an alleviating force.

References


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A Appendix A

A.1 Proof of Proposition 1

Equilibrium with short-termism

Trading Strategies. Consider the case when the market-maker and speculator expect the manager to undertake the short-term project (i.e., $\varpi_1 \sim N(\mu, \sigma_\omega^2)$, $\varpi_2 = 0$; Part 1 of Proposition 1). As a consequence, the speculator acquires information only about short-term earnings $\pi_1$. The market-maker posits the following demand strategy for the speculator:

$$\bar{x}^* = \phi_1^* \pi_1 + k^*.$$ (20)

Let $\hat{\pi}_1$, $\hat{\pi}_2$, and $\hat{\omega}_1$ represent the expected values of $\pi_1$, $\pi_2$, and $\omega_1$, respectively, according to the conjectures about project choice. This implies

$$p = \hat{\pi}_1 + \hat{\pi}_2 - (\alpha + \beta p + \gamma_1 \hat{\pi}_1 + \gamma_2 \hat{\pi}_2) + \frac{\text{cov}(V, q)}{\text{var}(q)} q$$

$$\Rightarrow (1 + \beta) p = (1 - \gamma_1) \hat{\pi}_1 + (1 - \gamma_2) \hat{\pi}_2 - \alpha + \frac{\text{cov}((1 - \gamma_1) \pi_1 + (1 - \gamma_2) \pi_2, q)}{\text{var}(q)} q$$

$$\Rightarrow p = \frac{1}{1 + \beta} [(1 - \gamma_1) \hat{\pi}_1 + (1 - \gamma_2) \hat{\pi}_2 - \alpha + \lambda (x + u)].$$

Equation (20) implies that

$$\lambda = \frac{\phi_1^* (1 - \gamma_1) (\sigma_\omega^2 + 1)}{(\phi_1^*)^2 (\sigma_\omega^2 + 1) + \sigma_u^2}$$

The speculator’s optimal trading strategy $x^*$ solves

$$\max_x x E \{ V - p \mid \pi_1 \}.$$ (21)
We have:

\[
V - p = \pi_1 + \pi_2 - \left( \alpha + \frac{\beta}{\beta_2} p + \pi_1 \pi_1 + \pi_2 \pi_2 \right) - p \tag{22}
\]

\[
= \frac{1 + \beta}{1 + \beta} \left[ (1 - \beta) \pi_1 + (1 - \beta_2) \pi_2 - \alpha \right] + (1 + \beta) p
\]

\[
= (1 - \beta_1) (\pi_1 - \hat{\pi}_1) + (1 - \beta_2) (\pi_2 - \hat{\pi}_2) - \lambda (x + u).
\]

This implies

\[
E \{ V - p \mid \pi_1 \} = (1 - \beta_1) (\pi_1 - \hat{\pi}_1) - \lambda x,
\]

since \( E (\pi_2) = \hat{\pi}_2 \) if the speculator doesn’t have information about \( \omega_2 \).

Therefore, the speculator’s problem is

\[
\max_x \left[ (1 - \beta_1) (\pi_1 - \hat{\pi}_1) - \lambda x \right]
\]

The first-order condition gives us

\[
(1 - \beta_1) (\pi_1 - \hat{\pi}_1) - \lambda x - \lambda x = 0.
\]

Thus, we have

\[
x^* = \frac{1 - \beta_1}{2 \lambda \phi_1^*} (\pi_1 - 1),
\]

since \( \hat{\pi}_1 = 1 \) when \( \omega_1 \sim N (1, \sigma_1^2) \).

The equilibrium values \( \{ \phi_1^*, k^*, \lambda^* \} \) solve the following system of equations:

\[
\begin{align*}
\lambda^* &= \frac{\phi_1^*(1 - \pi_1)(\sigma_2^2 + 1)}{(\phi_1^*)^2(\sigma_2^2 + 1) + \sigma_2^4}, \\
\phi_1^* &= \frac{1 - \pi_1}{2 \lambda^*}, \quad k^* = \frac{1 - \pi_1}{2 \lambda^*},
\end{align*}
\]
We have:

\[
\lambda = \frac{\frac{1}{2\lambda} (1 - \bar{\gamma}_1)^2 (\sigma^2 + 1)}{(\frac{1}{2\lambda})^2 (1 - \bar{\gamma}_1)^2 (\sigma^2 + 1) + \sigma_u^2}
\]

\[\iff \lambda = \frac{4\lambda^2 \frac{1}{2\lambda} \chi^*}{4\lambda^2 (\frac{1}{2\lambda})^2 \chi^* + \sigma_u^2} \iff \lambda = \frac{2\lambda \chi^*}{\chi^* + 4\lambda^2 \sigma_u^2}
\]

\[\iff \chi^* + 4\lambda^2 \sigma_u^2 = 2\chi^*.
\]

This implies that

\[
\lambda^* = \frac{1}{2\sigma_u} [(1 - \bar{\gamma}_1)^2 (\sigma^2 + 1)]^{\frac{1}{2}};
\]

\[
x^* = \frac{1 - \bar{\gamma}_1}{2\lambda^*} (\pi_1 - 1).
\]

**Price distribution.** The equilibrium trading strategies described in equations (23) and (24) imply that, for given realizations of \(\pi_1\) and \(u\), the price is

\[
p^* = \frac{1}{1 + \beta} \left[ (1 - \bar{\gamma}_1) \widehat{\pi}_1 + (1 - \bar{\gamma}_2) \widehat{\pi}_2 - \bar{\alpha} + \lambda (x + u) \right]
\]

\[
= \frac{1}{1 + \beta} \left[ 1 - \bar{\gamma}_1 - \bar{\alpha} + \frac{1 - \bar{\gamma}_1}{2} (\pi_1 - 1) + \lambda^* u \right].
\]

The expected price is then

\[
E(p^*) = \frac{1}{1 + \beta} \left[ 1 - \bar{\gamma}_1 - \bar{\alpha} + \frac{1 - \bar{\gamma}_1}{2} [E(\pi_1) - 1] + \lambda^* u \right].
\]

If the market conjectures that the firm implements the short-term project is consistent, we have \(E(\pi_1) = 1\) and, thus, the speculator expected demand is zero, i.e., \(E(x^*) = 0\). This implies

\[
E(p^*) = \frac{1}{1 + \beta} [1 - \bar{\gamma}_1 - \bar{\alpha}].
\]

Price volatility is instead
$$Var(p^*) = \frac{1}{(1+\beta)^2} \left[ \left( \frac{1-\tau_1}{2} \right)^2 Var(\pi_1) + (\lambda \sigma_u)^2 \right]$$
$$= \frac{1}{(1+\beta)^2} \left[ \left( \frac{1-\tau_1}{2} \right)^2 Var(\pi_1) + \left( \frac{1-\tau_1}{2} \right)^2 (\sigma_\omega^2 + 1) \right].$$

If the market conjectures that the firm implements the short-term project is consistent, we have $Var(\pi_1) = Var(\omega_1 + \eta_1) = \sigma_\omega^2 + 1$. This implies

$$Var(p^*) = \frac{1}{(1+\beta)^2} \left( \frac{1-\tau_1}{2} \right)^2 (\sigma_\omega^2 + 1);$$

$$Cov(\pi_1, p^*) = \frac{1-\tau_1}{2(1+\beta)} (\sigma_\omega^2 + 1).$$

**Speculator’s expected profit.** Let $ER^*$ denote the speculator’s ex-ante profit from trading in the equilibrium with short-termism. We have

$$ER^* = E \{ x^*(V - p^*) \} - g_1.$$

Equations (22) and (24) imply that

$$V - p^* = (1 - \tau_1)(\pi_1 - 1) - \lambda^* (x^* + u).$$
$$= (1 - \tau_1)(\pi_1 - 1) - \frac{1-\tau_1}{2} (\pi_1 - 1) - \lambda^* u$$
$$= \frac{1-\tau_1}{2} (\pi_1 - 1) - \lambda^* u.$$

Therefore, we can write:

$$E \{ x^*(V - p^*) \} = E \{ x^*(x^* \lambda^* - \lambda^* u) \}$$
$$= \lambda^* Var(x^*),$$

since both $E(x^*)$ and $E(u)$ are equal to zero.
This implies

\[
ER^* = \lambda^* \left( \frac{1 - \gamma_1}{2\lambda^*} \right)^2 \left( \sigma_\omega^2 + 1 \right) - g_1
\]
\[
= \frac{1}{4} \left( 1 - \gamma_1 \right)^2 \left( \sigma_\omega^2 + 1 \right) - g_1
\]
\[
= \frac{\sigma_u}{2} \left( 1 - \gamma_1 \right)^2 \left( \sigma_\omega^2 + 1 \right)^{1/2} - g_1.
\]

As we will see, in equilibrium we have \( \gamma_1 = c \). Therefore, we need

\[
\frac{\sigma_u}{2} \left( 1 - c \right)^2 \left( \sigma_\omega^2 + 1 \right)^{1/2} > g_1 + g_2
\]
\[
\sigma_u^2 > 4 \frac{g_1^2 (1 - c)^2 \sigma_\omega^2}{(1 - c)^2 \sigma_\omega^2 + 1}.
\]

Assumption 3 ensures that the above inequality is satisfied.

**Equilibrium with long-termism**

**Trading Strategies.** Consider now the case when the market-maker and speculator expect the manager to undertake the long-term project (i.e., \( \bar{\omega}_1 = 0, \bar{\omega}_2 \sim N(\mu, \sigma_\omega^2) \); Part 2 of Proposition 1). As a consequence, the speculator acquires information about both short-term earnings \( \pi_1 \) and the long-term project \( \omega_2 \). The market-maker posits the following demand strategy for the speculator:

\[
\pi^{**} = \phi^{**}_1 \pi_1 + \phi^{**}_2 \omega_2 + k^{**}.
\]  

(25)

Let \( \hat{\pi}_1, \hat{\pi}_2, \) and \( \hat{\omega}_2 \) represent the expected values of \( \pi_1, \pi_2, \) and \( \omega_2, \) respectively, according to the conjectures about project choice. This implies

\[
p = \hat{\pi}_1 + \hat{\pi}_2 - \left( \alpha + \beta p + \frac{\gamma_1}{\var{\hat{\pi}_1 \hat{\pi}_2}} + \frac{\text{cov} \left( V, q \right)}{\var{q}} \right) w
\]
\[
\Leftrightarrow (1 + \beta) p = (1 - \gamma_1) \hat{\pi}_1 + (1 - \gamma_2) \hat{\pi}_2 - \alpha + \frac{\text{cov} \left( (1 - \gamma_1) \pi_1 + (1 - \gamma_2) \pi_2, q \right)}{\var{q}}
\]
\[
\Leftrightarrow p = \frac{1}{1 + \beta} \left[ (1 - \gamma_1) \hat{\pi}_1 + (1 - \gamma_2) \hat{\pi}_2 - \alpha + \lambda (x + u) \right].
\]

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Equation (25) implies that
\[
\lambda = \frac{\phi_1 (1 - \bar{\gamma}_1) + \phi_2 (1 - \bar{\gamma}_2) \sigma_w^2}{\phi_1^2 + \phi_2^2 \sigma_w^2 + \sigma_u^2}
\]

The speculator solves \( \max_x x E \{ \mathcal{V} - p \mid \pi_1, \omega_2 \} \); we have
\[
\mathcal{V} - p = \pi_1 + \pi_2 - \left( \alpha + \bar{\beta} p + \bar{\gamma}_1 \pi_1 + \bar{\gamma}_2 \pi_2 \right) - p \\
= \left( 1 - \bar{\gamma}_1 \right) \pi_1 + \left( 1 - \bar{\gamma}_2 \right) \pi_2 - \lambda (x + u)
\]
\[
\mathcal{V} = \left( 1 - \bar{\gamma}_1 \right) (\pi_1 - \hat{\pi}_1) + \left( 1 - \bar{\gamma}_2 \right) (\pi_2 - \hat{\pi}_2) - \lambda (x + u)
\]
This implies
\[
E \{ \mathcal{V} - p \mid \pi_1, \omega_2 \} = \left( 1 - \bar{\gamma}_1 \right) (\pi_1 - \hat{\pi}_1) + \left( 1 - \bar{\gamma}_2 \right) (\omega_2 - \hat{\pi}_2) - \lambda x.
\]
Therefore, the speculator’s problem is
\[
\max_x x \left[ (1 - \bar{\gamma}_1) (\pi_1 - \hat{\pi}_1) + (1 - \bar{\gamma}_2) (\omega_2 - \hat{\pi}_2) - \lambda x \right]
\]
The first-order condition gives us
\[
(1 - \bar{\gamma}_1) (\pi_1 - \hat{\pi}_1) + (1 - \bar{\gamma}_2) (\omega_2 - \hat{\pi}_2) - \lambda x - \lambda x = 0.
\]
Thus, we have
\[
x^{**} = \frac{1 - \bar{\gamma}_1}{2 \lambda} \pi_1 + \frac{1 - \bar{\gamma}_2}{2 \lambda} (\omega_2 - \mu),
\]
since \( \hat{\pi}_1 = 0 \) and \( \hat{\pi}_2 = \mu \) when \( \overline{\omega}_1 = 0 \) and \( \overline{\omega}_2 \sim N (\mu, \sigma_w^2) \).

The equilibrium values \( \{ \phi_1^{**}, \phi_2^{**}, k^{**}, \lambda^{**} \} \) solve:
\[
\lambda^{**} = \frac{\phi_1 (1 - \bar{\gamma}_1) + \phi_2 (1 - \bar{\gamma}_2) \sigma_w^2}{[\phi_1 (1 - \bar{\gamma}_1)]^2 + [\phi_2 (1 - \bar{\gamma}_2)]^2 \sigma_w^2 + \sigma_u^2}; \\
\phi_1^{**} = \frac{1 - \bar{\gamma}_1}{2 \lambda^{**}}; \quad \phi_2^{**} = \frac{1 - \bar{\gamma}_2}{2 \lambda^{**}}; \quad k^{**} = -\frac{1 - \bar{\gamma}_2}{2 \lambda^{**}} \mu.
\]
We have:

\[ \lambda = \frac{\frac{1}{2\lambda} \left[ (1 - \bar{\gamma}_1)^2 + (1 - \bar{\gamma}_2)^2 \sigma^2 \right]}{(\frac{1}{2\lambda})^2 \left[ (1 - \bar{\gamma}_1)^2 + (1 - \bar{\gamma}_2)^2 \sigma^2 \right] + \sigma_u^2} \]

\[ \Leftrightarrow \lambda = \frac{4\lambda^2}{4\lambda^2 (\frac{1}{2\lambda})^2 \chi^{**}} \Leftrightarrow \lambda = \frac{2\lambda \chi^{**}}{\chi^{**} + 4\lambda^2 \sigma_u^2} \]

\[ \Leftrightarrow \chi^{**} + 4\lambda^2 \sigma_u^2 = 2\lambda. \]

This implies that

\[ \chi^{**} = \frac{1}{2\sigma_u} \left[ (1 - \bar{\gamma}_1)^2 + (1 - \bar{\gamma}_2)^2 \sigma_u^2 \right]^{\frac{1}{2}}; \quad (27) \]

\[ x^{**} = \frac{1 - \bar{\gamma}_1}{2\chi^{**}} \bar{\pi}_1 + \frac{1 - \bar{\gamma}_2}{2\chi^{**}} (\omega_2 - \mu). \quad (28) \]

**Price distribution.** The equilibrium trading strategies described in equations (27) and (28) imply that, for given realizations of \( \pi_1, \omega_2, \) and \( u, \) the price is

\[ p^{**} = \frac{1}{1 + \beta} \left[ (1 - \bar{\gamma}_1) \widehat{\pi}_1 + (1 - \bar{\gamma}_2) \widehat{\pi}_2 - \bar{\alpha} + \lambda (x + u) \right] \]

\[ = \frac{1}{1 + \beta} \left[ (1 - \bar{\gamma}_2) \mu - \bar{\alpha} + \frac{1 - \bar{\gamma}_1}{2} \bar{\pi}_1 + \frac{1 - \bar{\gamma}_2}{2} (\omega_2 - \mu) + \chi^{**} u \right]. \]

The expected price is then

\[ E(p^{**}) = \frac{1}{1 + \beta} \left[ (1 - \bar{\gamma}_2) \mu - \bar{\alpha} + \frac{1 - \bar{\gamma}_1}{2} E(\pi_1) + \frac{1 - \bar{\gamma}_2}{2} [E(\omega_2) - \mu] \right]. \]

If the market conjectures that the firm implements the long-term project is consistent, we have \( E(\pi_1) = 0 \) and \( E(\omega_2) = \mu. \) This implies

\[ E(p^{**}) = \frac{1}{1 + \beta} [(1 - \bar{\gamma}_2) \mu - \bar{\alpha}] . \]

Price volatility is instead
\[ Var(p^{**}) = \frac{1}{(1 + \beta)^2} \left[ \left( \frac{1 - \bar{\gamma}_1}{2} \right)^2 Var(\pi_1) + \left( \frac{1 - \bar{\gamma}_2}{2} \right)^2 Var(\omega_2) + (\lambda^{**})^2 \sigma_u^2 \right] \]

\[ = \frac{1}{(1 + \beta)^2} \left[ \left( \frac{1 - \bar{\gamma}_1}{2} \right)^2 Var(\pi_1) + \left( \frac{1 - \bar{\gamma}_2}{2} \right)^2 Var(\omega_2) + \left( \frac{1 - \bar{\gamma}_2}{2} \right) \cdot 1 + \left( \frac{1 - \bar{\gamma}_2}{2} \right)^2 \sigma_u^2 \right]. \]

If the market conjectures that the firm implements the long-term project is consistent, we have \( Var(\pi_1) = Var(\eta_1) = 1 \) and \( Var(\omega_2) = \sigma_u^2 \). This implies

\[ Var(p^{**}) = \frac{1}{(1 + \beta)^2} \left[ (1 - \bar{\gamma}_1)^2 + (1 - \bar{\gamma}_2)^2 \sigma_u^2 \right]; \]

\[ Cov(\pi_2, p^{**}) = \frac{1 - \bar{\gamma}_2}{2 (1 + \beta)} \sigma_u^2. \]

**Speculator’s expected profit.** Let \( ER^{**} \) denote the speculator’s ex-ante profit from trading in the equilibrium with long-termism. We have

\[ ER^{**} = E \{ x^{**} (\mathcal{V} - p^{**}) \} - (g_1 + g_2) \]

Equations (26) and (28) imply that

\[ \mathcal{V} - p^{**} = (1 - \bar{\gamma}_1) \pi_1 + (1 - \bar{\gamma}_2) (\omega_2 - \mu) - \lambda^{**} (x^{**} + u). \]

\[ = (1 - \bar{\gamma}_1) \pi_1 + (1 - \bar{\gamma}_2) (\omega_2 - \mu) - \frac{1}{2} \bar{\gamma}_1 \pi_1 + \frac{1}{2} \bar{\gamma}_2 (\omega_2 - \mu) - \lambda^{**} u \]

\[ = \frac{1 - \bar{\gamma}_1}{2} \pi_1 + \frac{1 - \bar{\gamma}_2}{2} (\omega_2 - \mu) - \lambda^{**} u. \]

Therefore, we can write:

\[ E \{ x^{**} (\mathcal{V} - p^{**}) \} = E \{ x^{**} (x^{**} \lambda^{**} - \lambda^{**} u) \} \]

\[ = \lambda^{**} Var(x^{**}), \]

since both \( E(x^{**}) \) and \( E(u) \) are equal to zero.
This implies

\[ ER^{**} = \lambda^{**} \left[ \left( \frac{1 - \tilde{\gamma}_1}{2\lambda^{**}} \right)^2 + \left( \frac{1 - \tilde{\gamma}_2}{2\lambda^{**}} \right)^2 \sigma^2 \right] - (g_1 + g_2) \]

\[ = \frac{1}{4} \frac{(1 - \tilde{\gamma}_1)^2 + (1 - \tilde{\gamma}_2)^2 \sigma^2}{2\sigma_u} \left[ (1 - \tilde{\gamma}_1)^2 + (1 - \tilde{\gamma}_2)^2 \sigma^2 \right]^{1/2} - (g_1 + g_2) \]

\[ = \frac{\sigma_u}{2} \left[ (1 - \tilde{\gamma}_1)^2 + (1 - \tilde{\gamma}_2)^2 \sigma^2 \right]^{1/2} - (g_1 + g_2). \]

As we will see, in equilibrium \( \tilde{\gamma}_1 = 0 \) and \( \tilde{\gamma}_2 < \frac{c}{\mu} \). Therefore, we need

\[ \frac{\sigma_u}{2} \left[ 1 + \left( 1 - \frac{c}{\mu} \right)^2 \sigma^2 \right]^{1/2} > g_1 + g_2 \]

\[ \sigma_u^2 > 4 \frac{(g_1 + g_2)^2}{1 + \left( 1 - \frac{c}{\mu} \right)^2 \sigma^2}. \]

Assumption 3 ensures that the above inequality is satisfied.

## B Appendix B

### B.1 Proof of Lemma 1

**Part 1 of Lemma 1**

Suppose the market expects the manager to undertake the short-term project and exert high effort, i.e., \( \omega_1 \sim N(\tilde{\sigma}, \sigma^2) \), \( \omega_2 = 0 \), and \( \tilde{\sigma} = 1 \). From Part 1 of Proposition 1, we have

\[ p^* = \frac{1}{1 + \beta} \left[ 1 - \tilde{\gamma}_1 - \tilde{\alpha} + \frac{1 - \tilde{\gamma}_1}{2} (\tilde{\pi}_1 - 1) + \lambda^* u \right]. \]

First, suppose the market conjecture is correct and the manager chooses indeed the long-term project. This implies \( \omega_1 \sim N(e, \sigma^2) \), \( \omega_2 = 0 \) and, thus, \( \tilde{\pi}_1 \sim N(e, \sigma^2 + 1) \), \( \tilde{\pi}_2 \sim N(0, \sigma^2) \) (since \( \pi_1 = \omega_1 + \eta_1 \) and \( \pi_2 = \omega_2 + \eta_2 \)). In this case, the manager chooses to exert effort \( (e = 1) \) if the following condition is satisfied:
\[
\beta \frac{1 - \overline{\gamma}_1 - \overline{\alpha}}{1 + \beta} - \frac{r}{2} \beta^2 \frac{1}{(1 + \beta)^2} \left[ \left( \frac{1 - \overline{\gamma}_1}{2} \right)^2 \text{Var}(\pi_1 | e = 1) + (\lambda^* \sigma_u)^2 \right] + \\
\frac{E(p^*), \text{ since:}}{E(\omega_1 \overline{\gamma}_1 | e = 1) = 1} \\
\frac{E(\pi_1 | e = 0)}{E(\pi_1 | e = 0)} \\
\frac{\text{Var}(p^*)}{\text{Var}(p^*)} \\
\frac{\gamma_1 \cdot 1 - \frac{r}{2} \overline{\gamma}_1^2 (\sigma^2 + 1) + \gamma_2 \cdot 0 - \frac{r}{2} \sigma^2}{\gamma_2 \sigma^2} - c
\]

The left-hand (right-hand) side of the inequality above represents the manager’s payoﬀ when he exerts \( e = 1 \) (\( e = 0 \)). Notice that \( \text{Var}(\pi_1 | e = 1) = \text{Var}(\pi_1 | e = 0) \), since eﬀort only affects the mean of \( \omega_1 \) but not its volatility. Therefore, deviating to \( e = 0 \) reduces the manager’s expected pay but not its volatility. The inequality above simpliﬁes to

\[
\gamma_1 - c \geq -\beta \frac{1 - \overline{\gamma}_1}{2 (1 + \beta)}. \tag{29}
\]

The inequality in (30) describes the incentive constraint in Part 1.a of Lemma 1.

Suppose now the market conjecture is wrong and the manager undertakes the long-term project instead. This implies \( \omega_1 = 0 \), \( \omega_2 \sim N(\mu e, \sigma^2) \) and, thus, \( \pi_1 \sim N(0, 1) \), \( \pi_2 \sim N(\mu e, \sigma^2 + \sigma^2) \). Notice that the stock price \( p^* \) does not incorporate any information about the true distribution of \( \omega_2 \), since the speculator expects that the ﬁrm take the short-term project and, thus, only acquires information about short-term earnings \( \pi_1 \). In this case, the stock price does not have any eﬀect on the manager’s incentives to exert eﬀort. The manager chooses high eﬀort if the following condition is satisﬁed:

\[
\beta \frac{1 - \overline{\gamma}_1 - \overline{\alpha}}{1 + \beta} - \frac{r}{2} \beta^2 \text{Var}(p^*) + \gamma_1 \cdot 0 - \frac{r}{2} \overline{\gamma}_1^2 + \gamma_2 \mu - \frac{r}{2} \gamma_2^2 (\sigma^2 + \sigma^2) - c \\
\geq \beta \text{Var}(p^*) + \gamma_1 \cdot 0 - \frac{r}{2} \overline{\gamma}_1^2 + \gamma_2 \cdot 0 - \frac{r}{2} \gamma_2^2 (\sigma^2 + \sigma^2). 
\]

The inequality above simpliﬁes to \( \gamma_2 \mu - c \geq 0 \), which describes the incentive constraint in Part 1.b of Lemma 1.
Part 2 of Lemma 1

Suppose the market expects the manager to choose the long-term project and exert high effort, i.e., $\exists_1 = 0, \exists_2 \sim N(\mu \bar{\sigma}, \sigma_2^2)$, and $\bar{\sigma} = 1$. From Part 2 of Proposition 1 we have

$$p^{**} = \frac{1}{1+\beta} \left[ (1-\bar{\gamma}_2) \mu - \bar{\alpha} + \frac{1-\bar{\gamma}_1}{2} \pi_1 + \frac{1-\bar{\gamma}_2}{2} (\omega_2 - \mu) \right].$$

First, suppose the market conjecture is correct and the manager takes indeed the long-term project. This implies $\exists_1 = 0$, $\exists_2 \sim N(e; 2 \bar{\gamma}_2)$, and, thus, $\pi_1 \sim N(0, 1)$, $\pi_2 \sim N(\mu e, \sigma_2^2 + \sigma_3^2)$ (since $\pi_1 = \omega_1 + \eta_1$ and $\pi_2 = \omega_2 + \eta_2$). In this case, the manager chooses to exert effort ($e = 1$) if the following condition is satisfied:

$$E[x^{**} \lambda^{**}|e=1] - \frac{r^2}{2} \beta^2 \frac{1}{(1+\beta)^2} Var(p^{**}) + E(p^{**}) \text{ when } e=1$$

$$+ \gamma_1 \cdot 0 - \frac{r}{2} \bar{\gamma}_1^2 + \gamma_2 \mu - \frac{r}{2} \bar{\gamma}_2^2 (\sigma_2^2 + \sigma_3^2) - c$$

$$\geq \beta \left[ (1-\bar{\gamma}_2) \mu - \bar{\alpha} - \frac{1-\bar{\gamma}_2}{2} \mu \right]$$

$$E(p^{**}) \text{ when } e=0$$

$$- \frac{r^2}{2} \beta^2 \frac{1}{(1+\beta)^2} Var(p^{**})$$

$$+ \gamma_1 \cdot 0 - \frac{r}{2} \bar{\gamma}_1^2 + \gamma_2 \cdot 0 - \frac{r}{2} \bar{\gamma}_2^2 (\sigma_2^2 + \sigma_3^2).$$

The left-hand (right-hand) side of the inequality above represents the manager’s payoff when he exerts $e = 1$ ($e = 0$). Like before, effort only affects the mean of $\omega_2$ but not its volatility and, thus, $Var(p^{**})$ is the same on both sides of the inequality. The inequality above simplifies to

$$\gamma_2 \mu - c \geq -\beta \frac{1-\bar{\gamma}_2}{2 (1+\beta)} \mu. \quad (30)$$

The inequality in (30) describes the incentive constraint in Part 2.b of Lemma 1.

Suppose now the market conjecture is wrong and the manager undertakes the short-term project instead. This implies $\omega_1 \sim N(e, \sigma_2^2), \omega_2 = 0$ and, thus, $\pi_1 \sim N(e, \sigma_2^2 + 1), \pi_2 \sim N(0, \sigma_2^2)$. The stock price $p^{**}$ incorporates information about both $\pi_1$ and $\omega_2$. Therefore,
the speculator’s expected demand is

\[ E(x^{**}) = \frac{1 - \gamma_1}{2\lambda^{**}} e^{(x)} + \frac{1 - \gamma_2}{2\lambda^{**}} (-\mu). \]

The manager chooses \( e = 1 \) if the following condition is satisfied:

\[
\begin{align*}
&\beta \left( (1 - \gamma_2) \mu - \bar{\alpha} + \frac{1 - \gamma_1}{2} - \frac{1 - \gamma_2}{2} \mu \right) - \frac{r}{2} \beta^2 \frac{1}{(1 + \beta)^2} \text{Var}(p^{**}) + \\
&\gamma_1 \cdot 1 - \frac{r}{2} \gamma_1^2 \left( \sigma_o^2 + 1 \right) + \gamma_2 \cdot 0 - \frac{r}{2} \gamma_2^2 \sigma_2^2 - c
\end{align*}
\]

The inequality above simplifies to

\[ \gamma_1 + \beta \cdot \frac{1 - \gamma_1}{2(1 + \beta)} - c \geq 0, \]

which describes the incentive constraint in Part 2.a of Lemma 1.

**B.2 Proof of Proposition 2**

**Optimal contract for the short-term project (Part 1 of Proposition 2)**

Shareholders take the market’s conjectures about the contract as given when solving for the optimal contract. The optimal contract for the short-term project solves:

\[
\min_{\beta \geq 0, \gamma_1 \geq 0} \beta^2 \text{Var}(p^*) + \gamma_1^2 \text{Var}(\omega_1 + \eta_1) + 2\beta\gamma_1 \text{Cov}(\omega_1 + \eta_1, p^*)
\]

subject to \( \beta \frac{1 - \gamma_1}{1 + \beta} + \gamma_1 \geq c. \)

First, suppose that the market correctly anticipates the choice of the project, i.e., \( \omega_1 \sim N(1, \sigma_o^2), \omega_2 = 0 \). Given the distribution of \( p^* \) described in Proposition 1, the problem in
\[ (31) \] simplifies to:

\[
\min_{\beta \geq 0, \gamma_1 \geq 0} \gamma_1^2 \left( \sigma_\omega^2 + 1 \right) + \beta^2 \frac{(1 - \bar{\gamma}_1)^2}{2(1 + \beta)^2} \left( \sigma_\omega^2 + 1 \right) + 2\gamma_1 \beta \frac{1 - \bar{\gamma}_1}{2(1 + \beta)} \left( \sigma_\omega^2 + 1 \right)
\]

subject to \[ \beta \frac{1 - \bar{\gamma}_1}{2(1 + \beta)} + \gamma_1 \geq c. \] (33)

The Lagrangian function for this problem is

\[
L^* = \gamma_1^2 + \beta^2 \frac{(1 - \bar{\gamma}_1)^2}{2(1 + \beta)^2} + \gamma_1 \beta \frac{1 - \bar{\gamma}_1}{1 + \beta} - \Lambda \left[ \beta \frac{1 - \bar{\gamma}_1}{2(1 + \beta)} + \gamma_1 - c \right].
\]

The first-order conditions are

\[
2\gamma_1 + \beta \frac{1 - \bar{\gamma}_1}{1 + \beta} - \Lambda = 0;
\]

\[
\beta \left( \frac{1 - \bar{\gamma}_1}{1 + \beta} \right)^2 + \gamma_1 \frac{1 - \bar{\gamma}_1}{1 + \beta} - \Lambda \frac{1 - \bar{\gamma}_1}{2(1 + \beta)} = 0.
\]

Notice that equation (34) can be rearranged as

\[
2\beta \frac{1 - \bar{\gamma}_1}{1 + \beta} + 2\gamma_1 - \Lambda = 0.
\]

Therefore, we can rewrite the first order conditions as

\[
2\gamma_1 + \beta \frac{1 - \bar{\gamma}_1}{1 + \beta} - \Lambda = 0; \]

(35)

\[
2\beta \frac{1 - \bar{\gamma}_1}{1 + \beta} + 2\gamma_1 - \Lambda = 0.
\]

(36)

The first order conditions in (35) and (36) together imply:

\[
2\gamma_1 + \beta \frac{1 - \bar{\gamma}_1}{1 + \beta} - \Lambda = 2\beta \frac{1 - \bar{\gamma}_1}{1 + \beta} + 2\gamma_1 - \Lambda \iff \beta = 0,
\]

since \( \bar{\gamma}_1 \leq c < 1 \).

Therefore, the optimal contract for the short-term project features \( \beta^* = \gamma_2^* = 0 \); the value of \( \gamma_1^* \) is pinned down by the incentive constraint, i.e., \( \gamma_1^* = c \), and the fixed component of pay \( \alpha^* \) is chosen so that the manager’s participation constraint in equation (12) is binding.

Now, suppose the market (mistakenly) expects the firm to implement the long-term project and, thus, \( \omega_1 = 0, \omega_2 \sim N(\mu, \sigma_\omega^2) \). The true distributions are instead \( \omega_1 \sim N(1, \sigma_\omega^2) \),
\( \omega_2 = 0 \). From Proposition \( \Pi \) we have:

\[
p^{**} = \frac{1}{1 + \beta} \left[ (1 - \bar{\eta}_2) \mu - \bar{\alpha} + \frac{1 - \bar{\eta}_1}{2} \pi_1 + \frac{1 - \bar{\eta}_2}{2} (\omega_2 - \mu) + \lambda^{**} u \right],
\]

and, thus,

\[
\text{Var} (p^{**}) = \frac{1}{(1 + \beta)^2} \left[ \left( \frac{1 - \bar{\eta}_1}{2} \right)^2 \left( 1 + \sigma_\omega^2 \right) + \left( \frac{1 - \bar{\eta}_1}{2} \right) \cdot 1 + \left( \frac{1 - \bar{\eta}_2}{2} \right)^2 \sigma_\omega^2 \right];
\]

\[
\text{Cov} (\pi_1, p^{**}) = \frac{1 - \bar{\eta}_1}{2 (1 + \beta)} (\sigma_\omega^2 + 1).
\]

In this case, the problem in (31) simplifies to

\[
\min_{\beta \geq 0, \gamma_1 \geq 0} \gamma_1^2 (\sigma_\omega^2 + 1) + \beta^2 \left( \frac{1 - \bar{\eta}_1}{2} \right)^2 \left[ 1 + \frac{1 + \left( \frac{1 - \bar{\eta}_2}{1 - \bar{\eta}_1} \right)^2 \sigma_\omega^2}{\text{Var}(\pi_1)} \right] + 2 \gamma_1 \beta \frac{1 - \bar{\eta}_1}{2 (1 + \beta)} (\sigma_\omega^2 + 1) \quad (37)
\]

subject to \( \beta \frac{1 - \bar{\eta}_1}{2 (1 + \beta)} + \gamma_1 \geq c. \) \( (38) \)

The Lagrangian function for this problem is

\[
L^{st} = \gamma_1^2 (\sigma_\omega^2 + 1) + \beta^2 \left( \frac{1 - \bar{\eta}_1}{2} \right)^2 \left( \sigma_\omega^2 \Sigma + 1 \right) + \gamma_1 \beta \frac{1 - \bar{\eta}_1}{1 + \beta} (\sigma_\omega^2 + 1) - \Lambda \left[ \beta \frac{1 - \bar{\eta}_1}{2 (1 + \beta)} + \gamma_1 - c \right].
\]

The first-order conditions are

\[
2 \gamma_1 (\sigma_\omega^2 + 1) + \beta \frac{1 - \bar{\eta}_1}{1 + \beta} (\sigma_\omega^2 + 1) - \Lambda = 0;
\]

\[
\beta \left( \frac{1 - \bar{\eta}_1}{1 + \beta} \right)^2 (\sigma_\omega^2 \Sigma + 1) + \gamma_1 \frac{1 - \bar{\eta}_1}{1 + \beta} (\sigma_\omega^2 + 1) - \Lambda \frac{1}{2} \frac{1 - \bar{\eta}_1}{1 + \beta} = 0. \quad (39)
\]

Notice that equation (39) can be rearranged as

\[
2 \beta \frac{1 - \bar{\eta}_1}{1 + \beta} (\sigma_\omega^2 \Sigma + 1) + 2 \gamma_1 (\sigma_\omega^2 + 1) - \Lambda = 0.
\]
Therefore, we can rewrite the first order conditions as

\[
2\gamma_1 (\sigma_\omega^2 + 1) + \beta \frac{1 - \frac{1}{\beta}}{1 + \beta} (\sigma_\omega^2 + 1) - \Lambda = 0; \\
2\beta \frac{1 - \frac{1}{\beta}}{1 + \beta} (\sigma_\omega^2 \Sigma + 1) + 2\gamma_1 (\sigma_\omega^2 + 1) - \Lambda = 0.
\]

(40)  (41)

The first order conditions in (40) and (41) together imply:

\[
2\gamma_1 (\sigma_\omega^2 + 1) + \beta \frac{1 - \frac{1}{\beta}}{1 + \beta} (\sigma_\omega^2 + 1) - \Lambda = 2\beta \frac{1 - \frac{1}{\beta}}{1 + \beta} (\sigma_\omega^2 \Sigma + 1) + 2\gamma_1 (\sigma_\omega^2 + 1) - \Lambda
\]

\[
\Leftrightarrow \beta \frac{1 - \frac{1}{\beta}}{1 + \beta} (\sigma_\omega^2 + 1) - 2\beta \frac{1 - \frac{1}{\beta}}{1 + \beta} (\sigma_\omega^2 \Sigma + 1) = 0
\]

\[
\Leftrightarrow \beta \left[-1 + \sigma_\omega^2 (1 - 2\Sigma)\right] = 0.
\]

Since $\Sigma > \frac{1}{2}$ and, thus, $1 - 2\Sigma < 0$, we must have $\beta = 0$ for the above equation to be satisfied. Therefore, the optimal contract for the short-term project features $\beta^* = \gamma_2^* = 0$ regardless of the market conjecture about project choice.

**Optimal contract for the long-term project (Part 2 of Proposition 2)**

The optimal contract for the long-term project depends on whether the stock price incorporates information about $\omega_2$.

If the speculator does not acquire information about $\omega_2$ (this occurs when the speculator expects that the short-term project is implemented; see Proposition 1), we have $\gamma_1^* = \beta^* = 0$; $\gamma_2^*$ is then pinned down by the incentive constraint, i.e., $\gamma_2^* = \frac{c}{\mu}$.

If the speculator acquires information about $\omega_2$, the optimal contract solves:

\[
\min_{\beta \geq 0, \gamma_2 \geq 0} \gamma_2^2 (\sigma_\omega^2 + \sigma_2^2) + \beta^2 \frac{(1 - \frac{1}{\beta})^2 + (1 - \frac{1}{\beta})^2 \sigma_\omega^2}{2 \left(1 + \beta\right)^2} + 2\gamma_1\beta \frac{1 - \frac{1}{\beta}}{2 \left(1 + \beta\right) \sigma_\omega^2} \left\{\begin{array}{c}
\text{Var}(\rho^{\mu \mu}) \\
\text{Cov}(\omega_2 + \eta_2, \rho^{\mu \mu})
\end{array}\right\} \\
\text{subject to } \beta \frac{1 - \frac{1}{\beta}}{2 \left(1 + \beta\right)} + \gamma_2 \geq \frac{c}{\mu}.
\]

(42)  (43)

The Lagrangian function for the problem is

\[
\mathcal{L}^{\mu} = \gamma_1^2 (\sigma_\omega^2 + \sigma_2^2) + \beta^2 \frac{(1 - \frac{1}{\beta})^2 + (1 - \frac{1}{\beta})^2 \sigma_\omega^2}{2 \left(1 + \beta\right)^2} + 2\gamma_1\beta \frac{1 - \frac{1}{\beta}}{2 \left(1 + \beta\right) \sigma_\omega^2} - \Lambda \left[\beta \frac{1 - \frac{1}{\beta}}{2 \left(1 + \beta\right)} + \gamma_1 - \frac{c}{\mu}\right].
\]
The first-order conditions are
\[ 2\gamma_2 (\sigma_\omega^2 + \sigma_2^2) + \beta \frac{1 - \gamma_2}{1 + \beta} \sigma_\omega^2 - \Lambda = 0; \tag{44} \]
\[ 2\beta \frac{(1 - \gamma_1)^2 + (1 - \gamma_2)^2 \sigma_\omega^2}{2 (1 + \beta)^2} + 2\gamma_2 \frac{1 - \gamma_2}{2 (1 + \beta)} \sigma_\omega^2 - \Lambda \frac{1 - \gamma_2}{2 (1 + \beta)} = 0. \tag{45} \]

Equation (45) simplifies to
\[ 2\beta \frac{1}{1 + \beta} \left[(1 - \gamma_1)^2 + (1 - \gamma_2)^2 \sigma_\omega^2\right] + 2\gamma_2 (1 - \gamma_2) \sigma_\omega^2 - \Lambda (1 - \gamma_2) = 0 \]
\[ \Leftrightarrow 2\beta \left[(1 - \gamma_1)^2 + (1 - \gamma_2)^2 \sigma_\omega^2\right] + 2\gamma_2 (1 - \gamma_2) \sigma_\omega^2 = \Lambda (1 - \gamma_2). \tag{46} \]

Using equations (44) and (46), we have
\[ 2\gamma_2 (1 - \gamma_2) \left(\sigma_\omega^2 + \sigma_2^2\right) + \bar{\beta} (1 - \gamma_2) \sigma_\omega^2 = 2\beta \left[(1 - \gamma_1)^2 + (1 - \gamma_2)^2 \sigma_\omega^2\right] + 2\gamma_2 (1 - \gamma_2) \sigma_\omega^2 \]
\[ \Leftrightarrow 2\gamma_2 (1 - \gamma_2) \sigma_2^2 = \bar{\beta} \left[2 (1 - \gamma_1)^2 + (1 - \gamma_2)^2 \sigma_\omega^2\right]. \tag{47} \]

The incentive constraint in (43) is binding and simplifies to
\[ \beta \frac{1 - \gamma_2}{2 (1 + \beta)} + \gamma_1 = \frac{c}{\mu} \Leftrightarrow \bar{\beta} (1 - \gamma_2) = 2 \left(\frac{c}{\mu} - \gamma_2\right). \tag{48} \]

Combining equations (47) and (48), we have
\[ 2\gamma_2 (1 - \gamma_2) \sigma_2^2 = \left(\frac{\frac{c}{\mu} - \gamma_2}{1 - \gamma_2}\right) \left[2 (1 - \gamma_1)^2 + (1 - \gamma_2)^2 \sigma_\omega^2\right] \]
\[ \Leftrightarrow \gamma_2 \sigma_2^2 - \left(\frac{c}{\mu} - \gamma_2\right) \left[2 \left(\frac{1 - \gamma_1}{1 - \gamma_2}\right)^2 + \sigma_\omega^2\right] = 0. \tag{49} \]

An optimal value of \( \gamma_2 \) must then satisfy \( \Gamma(\gamma_2) = 0. \)

We have \( \Gamma(0) < 0 \) and \( \Gamma \left(\frac{c}{\mu}\right) > 0; \) this implies that \( \gamma_2^{\ddagger} \in \left(0, \frac{c}{\mu}\right) \) and, thus, that \( \beta^{\ddagger} > 0 \) - given \( \gamma_2^{\ddagger}, \beta^{\ddagger} \) is given by the incentive constraint (48). Notice also that \( \Gamma \) is always increasing in \( \gamma_2. \) This implies that \( \gamma_2 \) such that \( \Gamma(\gamma_2) = 0 \) is unique and, therefore, the optimal contract \( (\gamma_2^{\ddagger}, \beta^{\ddagger}) \) is unique.
Equation (49) can be rearranged as

\[
\gamma_2^{\dagger} = \frac{c}{\mu} \cdot \frac{2 \left( \frac{1 - \gamma_1}{1 - \gamma_2} \right)^2 + \sigma^2}{2 \left( \frac{1 - \gamma_1}{1 - \gamma_2} \right)^2 + \sigma^2 + \sigma^2}.
\]

Notice that \( \gamma_2^{\dagger} \) is decreasing in \( \sigma_2^2 \) and we have \( \gamma_2^{\dagger} < \frac{c}{\mu} \).

**Optimal Contract with Consistent Conjectures.** Equation (49) characterizes the optimal value of \( \gamma_2 \) for given market’s conjectures about the contract, i.e., for given \( (\beta, \gamma_1, \gamma_2) \). In equilibrium, this conjectures need to be consistent. Therefore, we need to evaluate equation (49) and the incentive constraint (48) at \( \gamma_2 = \gamma_2^{**}, \gamma_1 = \gamma_1^{**} = 0, \) and \( \gamma_2 = \gamma_2^{**} \).

When \( \beta = \beta, \gamma_1 = 0, \) and \( \gamma_2 = \gamma_2, \) the two equations (49) and (48) become

\[
\gamma_2 \sigma_2^2 - \left( \frac{c}{\mu} - \gamma_2 \right) \left[ \frac{2}{(1 - \gamma_2)^2 + \sigma^2} \right] = 0; \quad (50)
\]

\[
\frac{\beta}{1 + \beta} (1 - \gamma_2) = 2 \left( \frac{c}{\mu} - \gamma_2 \right). \quad (51)
\]

As before, we have \( \Gamma^{**} (0) < 0 \) and \( \Gamma^{**} \left( \frac{c}{\mu} \right) > 0 \). In what follows, I show that \( \Gamma^{**} \) is increasing in \( \gamma_2 \), which implies that the equilibrium value \( \gamma_2^{**} \) is unique. We have

\[
\frac{\partial \Gamma^{**}}{\partial \gamma_2} = \sigma_2^2 + \left[ \frac{2}{(1 - \gamma_2)^2 + \sigma^2} \right] - \left( \frac{c}{\mu} - \gamma_2 \right) \frac{4}{1 + \beta} \frac{1 - \gamma_2^2}{(1 - \gamma_2)^3}
\]

\[
= \sigma_2^2 + \sigma^2 + \frac{2}{(1 - \gamma_2)^2} - \frac{\beta}{1 + \beta} \frac{2}{(1 - \gamma_2)^2}
\]

\[
= \sigma_2^2 + \sigma^2 + \frac{2}{(1 - \gamma_2)^2} \left( 1 - \frac{\beta}{1 + \beta} \right) > 0.
\]

Given \( \gamma_2^{**} \), the value of \( \beta^{**} \) is pinned down by the incentive constraint in (51), i.e.,

\[
\frac{\beta}{1 + \beta} (1 - \gamma_2^{**}) - 2 \left( \frac{c}{\mu} - \gamma_2^{**} \right) = 0.
\]

\( \Psi(\beta) \)

An equilibrium value of \( \beta \) must satisfy \( \Psi(\beta) = 0. \)
Notice that we have \( \Psi(0) < 0 \) and \( \Psi \) always increasing in \( \beta \). Therefore, the value \( \beta^{**} \) that satisfies \( \Psi(\beta^{**}) = 0 \) is unique.

Finally, \( \gamma_2^{**} \) decreases with \( \mu \). This is helpful in the proof of Proposition 3 in Appendix C. By totally differentiating equation (50) we obtain

\[
\frac{\partial \Gamma^{**}}{\partial \gamma_2} d\gamma_2^{**} + \frac{\partial \Gamma^{**}}{\partial \mu} d\mu = 0 \iff \frac{d\gamma_2^{**}}{d\mu} = -\frac{\frac{\partial \Gamma^{**}}{\partial \mu}}{\frac{\partial \Gamma^{**}}{\partial \gamma_2}}.
\]

Given that \( \Gamma^{**} \) is increasing in both \( \gamma_2 \) and \( \mu \), \( \frac{d\gamma_2^{**}}{d\mu} < 0 \).

C Appendix C

C.1 Proof of Lemma 2

The following equation characterizes the threshold \( \overline{\mu} \):

\[
\mu - \frac{r}{2} \left( \frac{c}{\mu} \right)^2 \left( \sigma_w^2 + \sigma_2^2 \right) - 1 = 1 - \frac{r}{2} c^2 \left( \sigma_w^2 + 1 \right)
\]

\[
\iff \mu - \frac{r}{2} \left( \frac{c}{\mu} \right)^2 \left( \sigma_w^2 + \sigma_2^2 \right) - 1 + \frac{r}{2} c^2 \left( \sigma_w^2 + 1 \right) = 0 \quad (52)
\]

Notice that the left-hand side of equation (52) is negative when \( \mu = 1 \), since \( \sigma_2^2 > 1 \) (Assumption 1). Therefore, we must have \( \overline{\mu} > 1 \). Moreover, it is always strictly increasing in \( \mu \). This implies that the threshold \( \overline{\mu} \) is unique.

C.2 Proof of Proposition 3

I prove Proposition 3 in two steps. First, I show that firm value always increases with \( \mu \) in the equilibrium with long-termism. Second, I characterize the new threshold value \( \underline{\mu} \) and show that \( \underline{\mu} \) is strictly lower than \( \overline{\mu} \).

Step One: Monotonicity

As we discuss in Appendix B.2, the optimal long-term contract solves the problem

\[
\min_{\beta \geq 0, \gamma_2 \geq 0} \gamma_2^2 \left( \sigma_w^2 + \sigma_2^2 \right) + \left( \frac{\beta}{1 + \beta} \right)^2 \frac{1}{2} \left[ (1 - \overline{\tau}_1)^2 + (1 - \overline{\tau}_2)^2 \sigma_w^2 \right] + \gamma_2 \frac{\beta}{1 + \beta} (1 - \overline{\tau}_2) \sigma_w^2 \quad (53)
\]

subject to \( \frac{\beta}{2} \frac{1 - \overline{\tau}_2}{1 + \beta} + \gamma_2 \geq \frac{c}{\mu} \).
The incentive constraint is always binding under an optimal contract, so we can use it to reduce the choice variables in the problem. The incentive constraint then implies
\[ \frac{\beta}{1 + \beta} = \frac{\frac{c}{\mu} - \gamma_2}{1 - \gamma_2}. \]

The problem in (53) then simplifies to
\[ \min_{\gamma_2 \geq 0} \gamma_2^2 \left( \sigma_\omega^2 + \sigma_2^2 \right) + 2 \left( \frac{c}{\mu} - \gamma_2 \right)^2 \left[ (1 - \gamma_1)^2 + (1 - \gamma_2)^2 \sigma_\omega^2 \right] + 2 \gamma_2 \left( \frac{c}{\mu} - \gamma_2 \right) \left( 1 - \gamma_2 \right) \sigma_\omega^2 \]
\[ \Leftrightarrow \min_{\gamma_2 \geq 0} \gamma_2^2 \left( \sigma_\omega^2 + \sigma_2^2 \right) + 2 \left( \frac{c}{\mu} - \gamma_2 \right)^2 \left[ \left( \frac{1 - \gamma_1}{1 - \gamma_2} \right)^2 + \sigma_\omega^2 \right] + 2 \gamma_2 \left( \frac{c}{\mu} - \gamma_2 \right) \sigma_\omega^2. \tag{54} \]

The objective in (54) is the manager’s risk-premium in the long-term contract. Let me denote by \( R^* \) this risk-premium. How does \( R^* \) change, in equilibrium, when \( \mu \) changes? By totally differentiating \( R^* \) we obtain
\[ \frac{dR^*}{d\mu} = \frac{\partial R^*}{\partial \gamma_2} \frac{\partial \gamma_2}{\partial \mu} + \frac{\partial R^*}{\partial \gamma_2} \frac{\partial \gamma_2}{\partial \mu} + \frac{\partial R^*}{\partial \mu}. \]

In equilibrium, \( \gamma_2 \) is chosen optimally; therefore, we have \( \frac{\partial R^*}{\partial \gamma_2} = 0. \) From equation (54), \( \frac{\partial R^*}{\partial \mu} \) is evidently negative. Notice also that, in equilibrium, \( \frac{\partial \gamma_2}{\partial \mu} < 0 \) (see Appendix B.2) and, since \( \gamma_2 = \gamma_2 \) in equilibrium, we have \( \frac{\partial \gamma_2}{\partial \mu} < 0. \) We have
\[ \frac{\partial R^*}{\partial \gamma_2} = 2 \left( \frac{c}{\mu} - \gamma_2 \right)^2 \left( 1 - \gamma_1 \right)^2 > 0. \]

Therefore, we have \( \frac{dR^*}{d\mu} < 0 \) and, thus, firm value is always increasing in \( \mu \) in an equilibrium with long-termism.

**Step Two: Threshold Values**

Let \((R^*, R^+)\) denote the agent’s risk-premium implied by, respectively, the optimal long-term and short-term contracts in the benchmark model. Notice that, while \( R^+ \) depends on the value of \( \mu \), via the IC constraint, \( R^* \) does not. The threshold \( \mu^* \) is characterized by the

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\(^{18}\)Notice that differentiating the objective in (54) with respect to \( \gamma_2 \) yields the same condition for the optimal value of \( \gamma_2 \) as in equation (49).
following equality:

$$\overline{\mu} = c - \mathcal{R}^\dagger (\overline{\mu}) = 1 - c - \mathcal{R}^*$$

$$\iff \overline{\mu} = 1 - \mathcal{R}^* + \mathcal{R}^\dagger (\overline{\mu}).$$

The threshold $\mu$ is instead characterized by the following equality:

$$\underline{\mu} - c - \mathcal{R}^{**} (\mu) = 1 - c - \mathcal{R}^*$$

$$\iff \underline{\mu} = 1 - \mathcal{R}^* + \mathcal{R}^{**} (\mu).$$

I want to show that $\underline{\mu} < \overline{\mu}$. I prove this by contradiction. Suppose that the contrary is true, i.e., $\underline{\mu} \geq \overline{\mu}$. This implies

$$\underline{\mu} \geq \overline{\mu} \iff \mathcal{R}^{**} (\underline{\mu}) \geq \mathcal{R}^\dagger (\overline{\mu}).$$

Note that $\underline{\mu} > \overline{\mu}$ cannot be true by the optimality of the contract. Given that the incentive constraint is relaxed when $\mu$ increases, and given that this constraint is binding at optimum, we cannot have that $\mathcal{R}^{**} (\underline{\mu}) > \mathcal{R}^\dagger (\overline{\mu})$ when $\mu > \overline{\mu}$. Therefore, we are left with the case $\underline{\mu} = \overline{\mu}$, which implies $\mathcal{R}^{**} (\underline{\mu}) = \mathcal{R}^\dagger (\overline{\mu} = \underline{\mu})$. This would imply that when $\mu = \underline{\mu}$, both contracts $(\gamma^*_2, \beta^{**})$ and $\gamma^\dagger_2$ are optimal, which contradicts the uniqueness of $(\gamma^*_2, \beta^{**})$. Therefore, we must have $\underline{\mu} < \overline{\mu}$.

### C.3 Proof of Lemma 3

I prove Lemma 3 in two steps. First, I show that $\Delta$ does not affect the speculator’s optimal strategies. Therefore, it does not affect the characterization of the equilibrium. Second, I show that the speculator prefers the equilibrium with long-termism if $\Delta$ is sufficiently large.

The first part of the proof is trivial. Consider the case when the market-maker and the speculator expect the manager to undertake the short-term project (i.e., $\overline{\omega}_1 \sim \mathcal{N} (\mu, \sigma_\omega^2)$, $\overline{\omega}_2 = 0$; Part 1 of Proposition 1). The speculator’s optimal trading strategy solves the following problem:

$$\max_x (x + \Delta) \mathbb{E} \{ V - p \mid \pi_1 \} \quad (55)$$

Notice that the problem in (55) is equivalent to the problem $\max_x x \mathbb{E} \{ V - p \mid \pi_1 \}$, which characterizes the speculator’s trading strategy in the baseline model where $\Delta = 0$.\footnote{We have $\mathcal{R}^{**} (\mu) \leq \mathcal{R}^\dagger (\mu) < \mathcal{R}^\dagger (\overline{\mu})$ if $\mu > \overline{\mu}$, since the principal can always ignore $p$ in the contract. Therefore, he pays the agent at most $\mathcal{R}^\dagger (\mu)$ in risk-premium when $p$ is contractible.} Therefore, the optimal trading strategy $x^*$ is the same.
This implies that the equilibrium trading strategies (for both the market-maker and the speculator) are the same. As a consequence, the speculator’s profits from trades are the same. The same idea holds for the case when the firm is expected to invest in the long-term project.

Given that the equilibrium trading strategies are the same, the speculator’s profits from trades are also the same. As a result, \( \Delta \) does not affect her incentives to acquire information either. However, we have \( E(x^*) = E(x^{**}) = 0 \). This implies that, after trading takes place, the speculator’s expected net position in the firm remains \( \Delta \). As a result, \( \Delta \) has an effect on the speculator’s preferences over the two equilibria.

Let \( \hat{V}^* \) denote the firm’s expected value under short-termism; \( \hat{V}^{**} \) instead denotes the firm’s expected value under long-termism. Thus, we have \( \hat{V}^{**} > \hat{V}^* \). We can write the speculator’s expected payoff in the two equilibria as follows:

\[
\begin{align*}
\hat{E}R^* &= \Delta \hat{V}^* + \frac{\sigma u}{2} \left[ (1 - \eta_1)^2 (\sigma_\omega^2 + 1) \right]^{\frac{1}{2}} - g_1, \\
\hat{E}R^{**} &= \Delta \hat{V}^{**} + \frac{\sigma u}{2} \left[ (1 - \eta_1)^2 + (1 - \eta_2)^2 \sigma_\omega^2 \right]^{\frac{1}{2}} - (g_1 + g_2).
\end{align*}
\]

Notice that since \( \hat{V}^{**} > \hat{V}^* \), the difference \( \hat{E}R^* - \hat{E}R^{**} \) decreases with \( \Delta \). Therefore, there always exist \( \Delta \) large enough such that \( \hat{E}R^* < \hat{E}R^{**} \).

\[\text{C.4 Proof of Proposition 4}\]

The proof of Proposition 4 follows the same steps as the analysis of the equilibrium in the baseline model.

I begin with characterizing the equilibrium with short-termism. In this case, the equilibrium price function is

\[
p' = \frac{1}{1 + \beta} \left[ 1 - \eta_1 - \alpha + \frac{1 - \eta_1}{2} (\omega_1 - 1) + \lambda' u \right],
\]
and, thus,

\[
Var (p') = \frac{1}{(1 + \beta)^2} \left[ \left( \frac{1 - \gamma_1}{2} \right)^2 - \frac{\sigma_\omega^2}{Var(\omega_1)} + \left( \frac{1 - \gamma_1}{2} \right)^2 \sigma_\omega^2 \right] = \left( \frac{1 - \gamma_1}{1 + \beta} \right)^2 \frac{\sigma_\omega^2}{2};
\]

\[
Cov (\pi_1, p') = \frac{1 - \gamma_1}{2 (1 + \beta)} \sigma_\omega^2.
\]

The optimal contract for the short-term project then solves:

\[
\begin{align*}
\min_{\beta \geq 0, \gamma_1 \geq 0} & \quad \beta^2 Var (p') + \gamma_1^2 Var (\pi_1) + 2\beta \gamma_1 Cov (\pi_1, p') \\
\text{subject to} & \quad \beta \frac{1 - \gamma_1}{1 + \beta} + \gamma_1 \geq c.
\end{align*}
\]

The Lagrangian function for this problem is

\[
L^{st} = \gamma_1^2 (\sigma_\omega^2 + 1) + \beta^2 \left( \frac{1 - \gamma_1}{2} \right)^2 \sigma_\omega^2 + 2\gamma_1 \beta^2 + \frac{1 - \gamma_1}{2 (1 + \beta)} \sigma_\omega^2 - \Lambda \left[ \beta \frac{1 - \gamma_1}{2 (1 + \beta)} + \gamma_1 - c \right].
\]

The first-order conditions are

\[
\begin{align*}
2\gamma_1 (\sigma_\omega^2 + 1) + \beta \frac{1 - \gamma_1}{1 + \beta} \sigma_\omega^2 - \Lambda &= 0; \\
2\beta^2 \left( \frac{1 - \gamma_1}{2} \right)^2 \sigma_\omega^2 + 2\gamma_1 \beta^2 + \frac{1 - \gamma_1}{2 (1 + \beta)} \sigma_\omega^2 - \Lambda \frac{1 - \gamma_1}{2 (1 + \beta)} &= 0 \\
\iff 2\beta \frac{1 - \gamma_1}{1 + \beta} \sigma_\omega^2 + 2\gamma_1 \sigma_\omega^2 - \Lambda &= 0.
\end{align*}
\]

The first order conditions above together imply:

\[
\begin{align*}
2\beta^2 \frac{1 - \gamma_1}{1 + \beta} \sigma_\omega^2 + 2\gamma_1 \sigma_\omega^2 &= 2\gamma_1 (\sigma_\omega^2 + 1) + \beta \frac{1 - \gamma_1}{1 + \beta} \sigma_\omega^2 \\
\iff \beta \frac{1 - \gamma_1}{2 (1 + \beta)} \sigma_\omega^2 &= 2\gamma_1.
\end{align*}
\]

Using the incentive constraint in program (56) gives

\[
(c - \gamma_1) \sigma_\omega^2 - 2\gamma_1 = 0 \iff \gamma_1' = \frac{\sigma_\omega^2}{\sigma_\omega^2 + 2c}
\]

53
Given $\gamma'_1$, $\beta'$ makes the incentive constraint - evaluated at $\gamma_1 = \gamma'_1$ and $\beta = \beta'$ - binding. Therefore, we have both $\gamma'_1$ and $\beta'$ strictly positive in equilibrium.

The rest of the proof follows the same steps as in the baseline model and is thus omitted.