Intermediary-Based Equity Term Structure

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Abstract

New empirical facts show that equity term premium is counter-cyclical, while the term structure of equity yield is procyclical and switches sign between expansions and recessions. We decompose the term structure of equity yield into an equity term premium and a mean reversion component about the expected changes in future yields to understand this seemingly contradictory evidence. Although the first component is counter-cyclical, we show the second mean reversion component dominantly drives the procyclical fluctuations of the overall equity yield curve. We propose a financial intermediary-based asset pricing model to quantitatively account for both facts simultaneously. In our model, the mean reversion component is endogenously driven by the time-varying tightness of the intermediaries’ leverage constraint. We demonstrate that the cyclical pattern of equity term structure imposes a strong discipline on the speed of mean reversion of discount rate for any standard asset pricing models. In the standard calibration of long-run risks model (Bansal and Yaron, 2004) and habit model (Campbell and Cochrane, 1999), the mean reversion speed of discount rate is too slow to account for a negative correlation between equity yield curve and equity term premium.

Keywords: equity term structure, financial intermediary, mean reversion, discount rate

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1 Introduction

In this paper, we study the cyclical variation of the term structure of equity returns. New empirical facts show that the term structure of equity yield is highly procyclical, that is, it is positively sloped in expansions and negatively sloped in recessions, while the equity term premium is counter-cyclical. Through offering an equity yield decomposition framework, we decompose the term structure of equity yield into an equity term premium and a mean reversion component about the expected changes in future yields. We show that, in order to reconcile the seemingly contradictory negative relation between equity yield curve and equity term premium, the second mean reversion component has to be strong enough to drive the procyclical fluctuations of the overall equity yield curve, although the first component still remains counter-cyclical. We propose a financial intermediary-based asset pricing model to quantitatively account for the cyclical pattern of equity term structure, as well as a large set of conventional asset pricing moments, and consumption and dividend dynamics. In our model, the mean reversion component is endogenously driven by the time-varying tightness of intermediaries’ leverage constraint. We also demonstrate that the cyclical pattern of equity term structure imposes a strong discipline on the speed of mean reversion of discount rate for any standard asset pricing models. In the standard calibration of long-run risks model (Bansal and Yaron, 2004) and habit model (Campbell and Cochrane, 1999), the mean reversion speed of discount rate is too slow to account for such a negative correlation between equity yield curve and equity term premium.

By way of background, the previous research on the equity term structure has focused on its unconditional average slope. Van Binsbergen et al. (2012) and Van Binsbergen and Koijen (2017) document that the term structure is downward sloping on average and argue “dividend strip data facts are difficult to reconcile with traditional macro-finance models.” However, Bansal et al. (2019) argue that in the data the term structure could be upward sloping when accounting for illiquidity. The more robust empirical evidence is about the conditional equity term structure, our main focus of this paper. Both Van Binsbergen et al. (2013) and Bansal et al. (2019) document that the equity yield curve is procyclical, in particular, it is positively sloped in expansions and negatively sloped in recessions. Meanwhile, Gormsen (2018) documents that the equity term structure of holding-period returns is counter-cyclical, which suggests the price fluctuations are driven mainly by long-maturity risks, only less by short-maturity risks. These two empirical facts are seemingly contradictory to each other. The main focus of our paper is to understand this seemingly contradictory negative correlation between equity yield curve and equity term premium.

We first provide a decomposition framework of (forward) equity yield in an analogous
way to the term structure of interest rate literature, for instance, Cochrane and Piazzesi (2005). We write the (forward) equity yield with n-period to maturity as the average of expected future one-period (forward) equity yields plus the equity term premium (Proposition 1). Further, as in Proposition 2, we decompose the (forward) equity yield slope into an equity term premium component plus a mean reversion component about expected changes in (forward) equity yields. Unconditionally, if we take average on both sides of our decomposition, we show that average (forward) equity yield slope is equal to average equity term premium, which is consistent with the prior work on average term structure slope. However, conditionally, this equivalence is not true due to the existence of the second mean reversion component. As a matter of fact, the second mean reversion component, which we highlight in this paper, is important for us to understand the negative relation between equity yield curve and equity term premium.

In the data, we observe a steeply downward sloping slope of equity yield in the global financial crisis of 2007-2009. One might conjecture that it is because equity term premium becomes significantly negative, that is, short maturity dividends become particularly risky and require particularly high returns. However, the counter-cyclical equity term premium tells us that opposite is the case: in bad times, it is long-maturity claims that have relatively high returns. Our decomposition framework gives a strong prediction that the downward sloping equity yield curve in bad times must be a result of the mean reversion component, that is, investors expected the yields to mean revert back down quickly. In another word, our decomposition framework tells us that, in order to reconcile the seemingly contradictive negative relation between equity yield curve and equity term structure, it must be the case that the second mean reversion component dominantly drives the procyclical fluctuations of the overall term structure of equity yield, while the first component still remains counter-cyclical.

How to understand the mean reversion of (forward) equity yields? Based on Equation (6), we show that it must either come from the mean reversion of maturity-specific risk premium or the mean reversion of expected dividend growth. Since the latter is empirical measurable, therefore, the cyclical pattern of expected changes in future (forward) equity yield we emphasize here mainly imposes a discipline of the mean reversion speed in risk premium (discount rate). In his review article, Cochrane (2017) emphasizes the new facts in equity term structure have presented a new testing ground for existing asset pricing models. We show that the mean-reversion speeds of discount rate, i.e., the time-varying aggregate volatility in long-run risks model (Bansal and Yaron, 2004) and the time variation in effective risk aversion in habit model (Campbell and Cochrane, 1999), are not fast enough to make the equity yield slope to be procyclical.
In this paper, we propose a financial intermediary based asset pricing model, to quantitatively account for cyclical patterns of equity term structure, as well as a large set of conventional asset pricing moments, and consumption and dividend dynamics. We embed a financial intermediary sector with a leverage constraint à la Gertler and Kiyotaki (2010) into an endowment economy. The model features a calibrated financial sector, recursive preferences, and an independently and identically distributed consumption growth process. The leverage constraint makes intermediary equity capital (net worth) to be an important state variable that affects asset prices and helps to understand a wide variety of dynamic asset pricing phenomena. In terms of computation, rather than a log-linear approximation method, we use a global method that allows for occasionally binding constraint to solve the model, and show the global method is critical for quantifying asset pricing implications.

The key model ingredient is that we build a stylized leverage constraint faced by financial intermediary into an otherwise standard endowment economy. As in Gertler and Kiyotaki (2010), a limited enforcement argument that financial intermediary can divert a fraction of bank assets and default on deposits provides a microfoundation for the leverage constraint. In particular, the debt financing capacity to an intermediary is proportional to the equity capital of the intermediary times a leverage multiple. In this setup, the intermediary net worth strongly affects asset prices through an adverse dynamic feedback effect: a negative consumption shock lowers the intermediary net worth, increases the probability that constraint becomes binding in the future, and therefore reduces the borrowing capacity of the intermediary sector today and in the future. Lower borrowing capacity results in lower demand for risky assets. In the equilibrium, the intermediary sector still holds all the risky assets. To clear the market, the asset price has to fall, and risk premium has to rise. The resulting fall in asset price further lowers the net worth. An initial small i.i.d. consumption shock is endogenously amplified through this propagation mechanism.

Importantly, such a leverage constraint opens up an endogenous channel of countercyclical market price of the consumption shock and therefore generate a strong enough mean reversion of risk premium, needed to reconcile the cyclical pattern of equity term structure. As the financial intermediary sector becomes more financially constrained, the marginal value of net worth shapely increases, and becomes much more sensitive to aggregate consumption growth shocks. As a result, the risk price is endogenously increasing with the tightness of the leverage constraint, at a increasing rate (that is, a convex relationship between risk price and the tightness of constraint). This is the key to generate a strong enough mean reversion effect of discount rate that is required to reconcile the negative relation between (forward) equity yield and equity term premium. In particular, in recessions when the intermediary net worth is very low, due to the above-mentioned convexity pattern, a small mean reversion
of net worth leads to a large reduction in risk price, corresponding to a significant mean reversion of risk premium (discount rate).

The leverage constraint also have other important asset pricing implications. First, it effectively introduces a wedge between interest rates on interbank and household loans. As a distinct implication from the model, the loan spread widens significantly in the credit crunch which features a large drop in intermediary net worth. This pattern is consistent with the evidence that high TED spread\(^1\) coincides with low price-dividend ratio and high stock market volatility.

Second, the leverage constraint also delivers countercyclical equity premium and stock market volatility, even though consumption growth is homoscedastic. The equilibrium asset prices are more sensitive to the fundamental shocks when the intermediary net worth is low. As the financial intermediary sector becomes more financially constrained, both the exposure of market return to consumption shock (i.e., return beta) and the market price of the consumption shock increase, and thus contribute to a higher equity premium. In the model, price-dividend ratio and leverage ratio of the aggregate intermediary sector predict long-horizon equity returns. Both the slope coefficients and \(R^2\) line up with the data relatively well at all horizons. And the model also captures the volatility feedback effect; that is, a consumption shock, as a negative innovation to market return, is a positive innovation to return volatility.

Quantitatively, through a careful calibration exercise, we demonstrate such a simple model endogenously generates fruitful implications. Our model matches well conventional asset pricing moments, including a sizable and time varying equity premium, procyclical price-dividend ratio variations, countercyclical return volatility, countercyclical interest spread (TED spread). More importantly, under the context of this study, our model also quantitatively reproduces key moments on equity term structure, including a slightly positive unconditional equity term structure, a procyclical equity yield slope and its sign-switching feature, the countercyclical equity term premium, as well as matching the correlation of equity yield slope with intermediary leverage and interest rate spread.

Besides the equity term structure literature as reviewed above, our model is also connected to several strands of literature. First, this paper is directly related to Maggiori (2012) and He and Krishnamurthy (2013) on financial intermediary and asset pricing. As a continuous time adaptation of Gertler and Kiyotaki (2010) type of leverage constraint into an endowment economy, Maggiori (2017) is a special case of the model in this paper, in which the constraint

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\(^1\)TED spread is measured by the spread between 3-month LIBOR rate in U.S. dollars and 3-month U.S. government treasury bill rate.
never binds in the equilibrium. Thus, it has neither implications for interest rate spread, nor implications of occasionally binding constraint on asset pricing. In He and Krishnamurthy (2013), the financial intermediary faces an equity financing constraint, rather than a debt financing constraint. The most important distinctive contribution of our paper is that we emphasize the implication of financial intermediary frictions on the cyclical pattern of equity term structure.

Second, the paper also relates to the theoretical literature on intermediary frictions. There are two broad classes of theories: leverage-constraints theories and equity risk-capital constraints. Both theories start with the assumption that intermediaries are constrained in raising more equity. They share two common predictions: First, intermediary equity (or net worth) is the key state variable that affects asset prices. Second, the effect of intermediary equity on asset prices is nonlinear, with a larger effect when the intermediary equity is low. The leverage-constraints models include Geanakoplos and Fostel (2008), Adrian and Shin (2010) and Brunnemeier and Pedersen (2009), Danielsson et al. (2011), Geanakoplos (2012), and Adrian and Boyarchenko (2012). Gertler and Kiyotaki (2010) type of frictions lies in the first category. He and Krishnamurthy (2013) and Brunnemeier and Pedersen (2009) are examples of equity risk-capital models. The goal of this paper is different from the theoretical literature to propose alternative micro-foundations for financial frictions, rather I focus on the quantitative asset pricing implications of a stylized type of leverage constraint as in Gertler and Kiyotaki (2010), which has been widely studied in the macroeconomic and policy related literature.

More broadly, this paper is related to the literature in macroeconomics studying the effects of financial frictions on aggregate activity, including Kiyotaki and Moore (1997), Carlstrom and Fuerst (1997) and Bernanke and Gertler (1989), among others. These papers focus on the credit frictions faced by non-financial borrowers. Gertler and Kiyotaki (2010) introduces a leverage constraint between household and financial intermediary, also see Gertler et al. (2011), and Gertler and Karadi (2012), among others. The equilibrium in these works is derived by log-linearizing around the steady state and assuming the constraint is always binding. Instead, we use a global method to solve the model, and emphasize that accounting for occasionally binding constraint is very important for quantifying asset pricing implications of the model. Our work contributes to the literature by arguing that quantitative analysis on macroeconomic effects and policy evaluations of financial frictions should take into account the importance of occasionally binding constraint on asset price dynamics, which lie in the center of the propagation mechanism of financial frictions.

The remainder of the paper is organized as follows: we present a decomposition frame-
work of (forward) equity yield curve and present evidence about the cyclical pattern of equity term structure. In Section 3, we present a financial intermediary based asset pricing model, and outline model solution, computation and discuss some analytical results in asset pricing. Section 4 presents quantitative implications of our model. Section 5 concludes. Model derivations, data sources and computation details are provided in the Appendix.

2 An Equity Yield Curve Decomposition Approach

In this section, we provide some fundamental relations about equity prices, equity yield, and dividend strip returns. We offer a decomposition framework, which is analogous to the term structure of interest rate literature, for instance, Cochrane and Piazzesi (2005). In particular, we decompose the term structure slope of (forward) equity yield into two components: a equity term premium component, which is related to the term slope of risk premium, and a expected change in (forward) equity yield component. We show that unconditionally, the term structure slopes of (forward) equity yield and dividend strip returns contain the same information. However, conditionally, the determinants of these two term structure slopes are very different.

2.1 Definitions

Let $S_t$ denote the price of a claim on all future dividends. Then, $S_t$ can be written as

$$S_t = \sum_{n=1}^{\infty} P_{n,t}, \quad (1)$$

where $P_{n,t}$ is the price of a claim on dividend at time $t+n$. Such a claim is often called “dividend strip” or “zero-coupon equity”.

One-period return on dividend strip is defined as

$$R_{n,t+1} = \frac{P_{n-1,t+1}}{P_{n,t}}, \quad (2)$$

which is the return to buy a maturity-$n$ dividend strip at time $t$ at the price of $P_{n,t}$, and sell it off as a maturity-$(n-1)$ dividend strip at time $t+1$ at the price of $P_{n-1,t+1}$. 
The hold-to-maturity return \( R_{n,t+n}^H \) is defined as
\[
R_{n,t+n}^H = \frac{D_{t+n}}{P_{n,t}},
\]
which is the return to buy maturity-\( n \) dividend strip at time \( t \) at the price of \( P_{n,t} \), and hold it till maturity by obtaining \( D_{t+n} \).

It is also instructive to define equity yield for maturity \( n \) as
\[
e_{n,t} = \frac{1}{n} \ln \left( \frac{D_t}{P_{n,t}} \right).
\]

While the hold-to-maturity return \( R_{n,t+n}^H \) is only measurable at time \( t + n \), the equity yield can be computed at time \( t \). Denote the \( n \) period average log return on an \( n \) period dividend strip hold-to-maturity return as \( r_{n,t+n}^H = \frac{1}{n} \ln( R_{n,t+n}^H ) \), we have the relation
\[
e_{n,t} = \frac{1}{n} \ln \left( \frac{D_t}{P_{n,t}} \right) = E_t \left[ \frac{1}{n} \ln \left( \frac{D_{t+n}}{P_{n,t}} \right) - \frac{1}{n} \ln \left( \frac{D_{t+n}}{D_t} \right) \right] = E_t \left[ r_{n,t+n}^H - g_{n,t}^d \right] = y_{n,t} + \theta_{n,t} - g_{n,t}^d
\]
where \( y_{n,t} \) is the nominal bond yield, \( \theta_{n,t} \) is a maturity-specific risk premium, and \( g_{n,t}^d \) is the expected average log dividend growth rate from \( t \) to \( t + n \). Equation (5) shows that equity yields and expected hold-to-maturity returns only differ in the expected average dividend growth rate. Equity yields are determined by three components: the maturity-\( n \) nominal bond yield, the maturity-\( n \) risk premium yield and the expected dividend growth rate. Ceteris paribus, a higher bond yield or risk premium yield lowers the current strip price, which leads to a higher equity yield, while a higher expected dividend growth leads to a lower equity yield.

In practice, the contract on dividend strips are quoted on futures prices. An index dividend future is an agreement where, at maturity, the buyer pays the futures price \( F_{n,t} \), which is determined today, and the seller pays the dollar amount of dividends \( D_{t+n} \) in exchange. By no arbitrage the futures price is given by \( F_{n,t} = P_{n,t} \exp(ny_{n,t}) \), and the forward equity

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yield, defined as \( e_{n,t}^f = \frac{1}{n} \frac{D_n}{F_{n,t}} \), has the following relation with equity yield

\[
e_{n,t}^f = e_{n,t} - y_{n,t}
= \theta_{n,t}^r - \theta_{n,t}^d.
\]  

(6)

Equation (6) shows that the forward equity yields and equity yields only differs in the nominal bond yield. By teasing out the effect from nominal bond yield, forward equity yields actually help us to focus on the property of equity-specific risk premium.

In Van Binsbergen et al. (2012) and Van Binsbergen and Koijen (2017), they use dividend strip returns to show that the term structure slope of risk premium is downward sloping. However, the dividend strip return is subject to measurement issue. For example, Bansal et al. (2019) point out that because the bid-ask spread in dividend strip prices is large, the monthly dividend strip returns are imprecisely measured. They argue that the economic information contained in the equity yields are substantially more robust to the liquidity issue. The literature also documents some clear cyclical patterns of the term structure slope. On the one hand, Van Binsbergen et al. (2013) and Bansal et al. (2019) show that the equity yield slope is pro-cyclical. On the other hand, Gormsen (2018) shows that the term structure slope of dividend strip return is counter-cyclical.

In the following sections, we provide decomposition method to clarify the relationship between the term structure slope of equity yield and dividend strip return, and reconcile the seemingly contradictive empirical evidence. We will also discuss the economic forces that drive the unconditional and conditional term structure slope.

### 2.2 Decompose the Equity Yield

The expected dividend strip return consists of two components

\[
E_t(r_{n,t+1}) = \xi_{n,t} + \zeta_{n,t},
\]  

(7)

where \( \xi_{n,t} \) is a maturity-\( n \) equity risk premium and \( \zeta_{n,t} \) is the expected return on a zero-coupon bond with the same maturity.

The term structure slope of dividend strip return from maturity-\( m \) to \( n \) can be written as

\[
E_t(r_{n,t+1} - r_{m,t+1}) = \underbrace{\xi_{n,t} - \xi_{m,t}}_{\text{equity term premium}} + \underbrace{\zeta_{n,t} - \zeta_{m,t}}_{\text{bond term premium}}.
\]  

(8)
Equation (8) shows that the slope of dividend strip return is determined by the term premium from equity and bond. In appendix A.1, we derive corresponding terms for the equity term premium in a pricing equation. We also discuss the implications of leading asset pricing models for the equity term premium.

The dividend strip returns are one period returns, thus their term structure slope only involves the difference in their riskiness. However, equity yields measure expected return over different horizons, therefore, their term structure slope not only reflects the difference in riskiness but also the expected changes in riskiness along the time dimension. Analogous to bond yield (Cochrane and Piazzesi, 2005), we can write long-maturity equity yield as a series of expected short-term equity yield plus term premium. Specifically, we have the representation in proposition 1.

**Proposition 1.** The equity yield can be represented as expected short-term yield plus term premium

\[
e_{n,t} = \frac{1}{n} E_t \left( \sum_{i=0}^{n-1} e_{1,t+i} \right) + \frac{1}{n} E_t \left[ \sum_{i=0}^{n-1} (\xi_{n-i,t+i} - \xi_{1,t+i} + \zeta_{n-i,t+i} - \zeta_{1,t+i}) \right].
\]

Similarly, The term structure slope of forward equity yield can be represented as expected short-term forward equity yield plus equity term premium:

\[
e^f_{n,t} = \frac{1}{n} E_t \left( \sum_{i=0}^{n-1} e^f_{1,t+i} \right) + \frac{1}{n} E_t \left[ \sum_{i=0}^{n-1} (\xi_{n-i,t+i} - \xi_{1,t+i}) \right].
\]

**Proposition 2.** The term structure slope of equity yield can be decomposed into a term premium component and a expected changes in yields component:

\[
e_{n,t} - e_{1,t} = \frac{1}{n} \sum_{i=1}^{n} \left[ (\xi_{i,t} - \xi_{1,t}) + (\zeta_{i,t} - \zeta_{1,t}) \right] + \frac{1}{n} \sum_{i=1}^{n} (i - 1) \left[ E_t (e_{i-1,t+1} - e_{i-1,t}) \right].
\]

Similarly, The term structure slope of forward equity yield can be decomposed into a equity
term premium component and an expected changes in yields component:

\[
e_{n,t} - e_{1,t} = \frac{1}{n} \sum_{i=1}^{n} [\xi_{i,t} - \xi_{1,t}] + \frac{1}{n} \sum_{i=1}^{n} (i - 1) \left[ E_t \left( e_{i-1,t+1}^f \right) - e_{i-1,t}^f \right].
\]  

(12)

Proof. See Appendix A.2.

Proposition 2 shows that in addition to the information about term premium which is reflected in the term structure slope of dividend strip return, equity yield also contains information about the expected changes in equity yield. In the following analysis, we will see that the expected changes component does not affect the yield slope unconditionally, but it is the key to resolve the discrepancy between cyclical behavior of strip return slope and equity yield slope.

2.2.1 Unconditional Term Structure

If we take an unconditional expectation of equation (11), we have

\[
E(e_{n,t} - e_{1,t}) = \frac{1}{n} \sum_{i=1}^{n} E \left[ (\xi_{i,t} - \xi_{1,t}) + (\zeta_{i,t} - \zeta_{1,t}) \right].
\]  

(13)

Comparing to the unconditional term structure slope of dividend strip return computed from equation (8)

\[
E(r_{n,t} - r_{1,t}) = E \left[ (\xi_{n,t} - \xi_{1,t}) + (\zeta_{n,t} - \zeta_{1,t}) \right],
\]  

(14)

we will see that the unconditional slope of equity yield is just a weighted average of the dividend strip return slope. In this sense, the unconditional term structure slope of dividend strip return and equity yield carry the same information. To the extent that equity yields are less contaminated by the liquidity issue, it should be a better target when studying the unconditional property of the term structure of risk premium.

2.2.2 Conditional Term Structure

The counter-cyclical equity term premium and pro-cyclical equity yield curve documented by the literature seem to be contradictory. However, once we have the decomposition relation in proposition 2 and take the expected changes in equity yields component into account, the discrepancy can be reconciled. From equation (12), as long as the pro-cyclical expected
changes component which dominates the effect from the variation in equity term premium, a counter-cyclical equity term premium and pro-cyclical equity yield slope can co-exist.

The cyclical pattern of equity term premium and equity yield slope impose a strong discipline for the mean reversion speed in discount rate. In recessions, the long-maturity equity yield is high relative to the short yield. This does not mean short-maturity dividend strips earn higher risk premium than long-maturity dividend strips. Rather, it indicates that the equity yields are expected to decrease in a very high speed that prevail over the upward-sloping tendency induced by term premium. From equation (6), the forward equity yield will be upward-sloping (downward-sloping) whenever the risk premium yield is upward-sloping (downward-sloping) or the expected average dividend growth rate is going to decrease (increase). Because the expected dividend growth is measurable, we can back out the discount rate information from forward equity yield. This provides a new testing ground for the existing asset pricing models.

To formally compare data and model performance, we lay down several testable regressions from Proposition 2 equation (12). First, we study the cyclicality of equity yield slope by regressing the difference between long- on short-maturity forward equity yield on contemporaneous dividend to price ratio.

\[ e_{n,t}^f - e_{1,t}^f = \alpha_0 + \alpha_1 (d_t - p_t) + \epsilon_t. \] (15)

In the empirical exercise, we explicitly add back the effect from difference in expected dividend growth \( g_{n,t}^d - g_{1,t}^d \) as shown in equation (6) so that the cyclciality of equity yield slope purely comes from the fluctuation of risk premium yield \( \theta_{n,t} - \theta_{1,t} \).

Second, we regress the realized return difference between index and short-maturity dividend strip return on the ex-ante dividend price ratio. This regression allows us to look at the cyclicality of equity term premium.

\[ r_{m,t+1} - r_{1,t+1}^f = \beta_0 + \beta_1 (d_t - p_t) + \epsilon_{t+1}, \] (16)

in which \( r_{m,t+1} \) denotes the return on market index and \( r_{1,t+1}^f \) denotes the one-period return on a maturity-1 dividend future.

Third, we run an expectation hypothesis regression of the form

\[ \frac{1}{n} \sum_{i=1}^{n} (i-1) \left[ e_{i-1,t+1}^f - e_{i-1,.,t}^f \right] = \gamma_0 + \gamma_1 (e_{n,t}^f - e_{1,t}^f) + \eta_{t+1}, \] (17)
where the left hand side variable is a weighted average of realized changes in equity yield which is a proxy for the expected changes in yield

\[
\frac{1}{n} \sum_{i=1}^{n} (i - 1) \left[ E_t \left( e_{t-1,t+1}^f - e_{t-1,t}^f \right) \right]
\]

in equation (12). We also adjust the corresponding terms for expected dividend growth so that we can focus on the discount rate channel.

Regression (17) solely relies the accounting identity decomposition, which allows us to study the relationship between the equity yield and term premium in a model-free framework. If equity yield is only driven by the expected changes in future yield, i.e., the expectation hypothesis holds, the regression coefficient \( \gamma_1 = 1 \). However, we expect the expectation hypothesis to fail because equity term premium also fluctuates over time. The relation between equity yield and term premium can be inferred from the regression coefficient \( \hat{\gamma}_1 \). Combining the decomposition equation (12) and regression equation (17) gives (suppressing constant)

\[
\frac{1}{n} \sum_{i=1}^{n} [\xi_{i,t} - \xi_{1,t}] = (1 - \hat{\gamma}_1)(e_{n,t}^f - e_{1,t}^f),
\]

In this context, the important feature is whether equity yields move by more or less than suggested by the expectation hypothesis. If \( \hat{\gamma}_1 > 1 \), term premium and expected changes work in the opposite direction, and the effect from expected changes dominates, leading to a negative relation between the equity yield curve and the term premium. If \( \hat{\gamma}_1 < 1 \), the equity yield curve and the term premium are positively related. If \( 0 < \hat{\gamma}_1 < 1 \), it indicates that term premium and expected changes work in the same direction, while if \( \hat{\gamma}_1 < 0 \), term premium and expected changes work in the opposite direction, and the effect from term premium dominates.

### 2.2.3 A New Testing Ground for Asset Pricing Models: Theory vs. Evidence

In his review article, Cochrane (2017) emphasizes the new facts in equity term structure have presented a new testing ground for existing asset pricing models. In this section, We also demonstrate that the cyclical pattern of equity term structure imposes a strong discipline on the speed of mean reversion of discount rate for any standard asset pricing models. In the standard calibration of long-run risks model (Bansal and Yaron, 2004) and habit model (Campbell and Cochrane, 1999), the mean reversion speed of discount rate is too slow to account for such negative correlation between equity yield curve and equity term premium.

Table 1 Panel A shows the average equity term premium and regression results in data. Our results are based on S&P 500, EuroStoxx 50, Nikkei 225 dividend future contracts. The data covers the period from December 2004 to February 2017. We report the sample averages of strip return difference between the 5-year and 1-year dividend future as well as
the monthly regression coefficients for equation (15)-(17). The standard errors are Newey and West (1987) corrected for 12 lags.

The average 5-year minus 1-year dividend future return are positive for U.S. and negative for Europe and Japan, but the standard errors for the estimate are huge, making them insignificant from zero. The results for cyclical patterns of equity yield and term premium are quite clear and consistent across countries. First, the regression coefficient for equation (15) are all negative, suggesting that equity yield slope is pro-cyclical. Second, from the estimates of equation (16), ex-ante dividend to price ratio can positively predict future return difference between long- and short-maturity dividend strips, which indicate that the term premium are counter-cyclical. Last, and most importantly, the expectation hypothesis regression (17) implies that equity yields move by less than suggested by the expectation hypothesis. Specifically, the point estimate for U.S. is close to one, which indicates that the fluctuation of equity yield are mostly driven by the expected changes in discount rates. The regression coefficient for other two countries are all well above one.

Taking all together, these empirical evidence reveals a coherent story that equity term premium and expected changes in yield move in the opposite direction, but effect from expected changes are much greater which dominates the fluctuation of equity yield. In recessions, the long-maturity dividend strips are more risky, which makes the equity yield curve tend to be positively sloped. However, investors also expected the high discount rates to mean revert back, which makes the yield curve tend to be downward-sloping. The empirical evidence suggests that mean-reversion speed of discount rates needs to be large enough so that the second effect would predominate and make the equity yield pro-cyclical.

We relate the empirical findings to leading asset pricing models through simulations. We run 10,000 simulations with 1,000 years of artificial data, and calculate the median, 5 percentile and 95 percentile estimates from annual regressions. Panel B of Table 1 reports the results from habit model by Campbell and Cochrane (1999) and long-run risk model by Bansal and Yaron (2004).

Habit model generates positive equity term premium because long-maturity dividend claims are more exposed to discount rate shocks. In the habit model, discount rate shocks and dividend growth shocks are negatively correlated, therefore, investors require additional premium to hold dividend claims. Moreover, because discount rate shocks are persistent, long-maturity dividend strip prices respond more, which leads to a higher exposure.

Habit model generates time-varying risk premium through the changes in investors’ effective risk aversion. The same force also drives the cyclicality of equity term premium. Because risk price is counter-cyclical and long-maturity claims are more exposed to shocks,
the equity term premium is counter-cyclical. This intuition is confirmed in the simulation as we can see in Table 1 the coefficient estimates $\alpha_1$ is greater than zero. However, the equity yield regression (15) and expectation hypothesis regression (17) have the wrong sign. This indicate that indeed, the term premium and expected change work in the opposite direction, however, the speed of mean-reversion for risk prices is not fast enough to offset the effect from the increased term premium, making the equity yield counter-cyclical.

**Long-run risk model** also has an upward sloping equity term structure. The term structure arises because growth shocks are persistent and the effect accumulates on the long-maturity dividend claims. In long-run risk models, time-varying volatility is the key to generate a time-varying risk premium. This also drive the fluctuation of term premium because the change in volatility is also associated with long-run shocks to which long-maturity dividend claims have higher exposure. The simulations in Table 1 also confirm that equity term premium is countercyclical. However, long-run risk also fail to match the data in the equity yield regression (15) and expectation hypothesis regression (17) due to the same reason as habit model. To obtain a high risk premium, the volatility is designed to be very persistent. This result in a very slow mean-reversion speed and, in turn, a counter-cyclical equity yield.

**Other models** such as variable disaster model by Gabaix (2012) or the model by Lettau and Wachter (2007) also can not generate the correct dynamic pattern. In the disaster model, because dividend shocks and disaster shocks are independent, there is no equity term premium. In Lettau and Wachter (2007), long-maturity dividend claims are less risky because of a long-run insurance mechanism. However, with a counter-cyclical risk premium, the term premium will be pro-cyclical rather than counter-cyclical. In fact, any models with negative equity term premium and counter-cyclical risk price will all produce a pro-cyclical term premium, which is at odds with data. One exception is the regime-switching model in Bansal et al. (2019). Essentially, the model achieve a sign-switching yield slope by mitigating the equity term premium effect from the standard calibration of habit and long-run risk model, and increase the mean-reversion of risk price.

## 3 An Intermediary-based Asset Pricing Model

### 3.1 Motivating Evidence

In the data, the financial intermediary leverage ratio is closely related to the term structure slope of forward equity yield. Figure 1 shows both the realized and the estimated term structure slope of forward equity yield, measured as the difference between 5-year and 1-
year forward equity yield. The fitted values are obtained from a simple linear regression of forward equity yield slope on contemporaneous financial intermediary leverage. In U.S. data, the realized term structure of forward equity yield has a big dip in the financial crisis period and the sign of the slope turn negative. A simple linear regression with only financial leverage as explanatory variable tracks the realized slope very well: the fitted value is able to generate a big dip of term slope in the recent financial crisis as well as drops in previous recession periods.

Motivated by the empirical evidence, we study the equity term structure implications of a financial intermediary model. The model not only performs well in matching the traditional asset pricing moments such as equity premium and risk-free rate, but can also produce a counter-cyclical equity term premium and pro-cyclical equity yield. In the model, long-maturity dividend claims are more exposed to financial shocks, thus, are more risky. The financial accelerator effect induced by the leverage constraint generates a counter-cyclical risk price, which in turn give rise to a counter-cyclical term premium. On the other hand, thanks to the leverage and expected return dynamics, the mean-reversion of risk price is endogenous and strong enough. Consequently, the expected change effect dominates the term premium effect, which help to generate a pro-cyclical equity yield and a negative yield slope in recessions.

3.2 Model Setup

We embed a financial intermediary sector with a leverage constraint à la Gertler and Kiyotaki (2010) into a standard Lucas (1978) endowment economy.

There are three sectors in the economy, namely, households, financial intermediaries (banks), and non-financial firms. We assume households cannot invest directly in the risky asset market by holding the equity of non-financial firms. There is a limited market participation, also see Mankiw and Zeldes (1991), Basak and Cuoco (1998), or Vissing-Jorgensen (2002). Instead, households can only save through bank deposits. In addition, Households also own the whole banking sector. Banks borrow short-term debt from households\footnote{To motivate a limited enforcement argument later, it is best to think of banks only obtaining deposits from households who do not own them.}, and invest in the equity of the firms.

Time is discrete and infinite, $t = 0, 1, 2$,. The non-financial firms in this economy are modeled as in a Lucas (1978) tree economy which pays aggregate output every period. The aggregate output is denoted by $Y_0, Y_1, Y_2$,. The growth rate of the output process $g_{t+1}$ is
given by
\[ g_{t+1} = \frac{Y_{t+1}}{Y_t} = \exp\left(\mu_y + \sigma \varepsilon_{Y,t+1}\right), \]
in which \( \varepsilon_{y,t+1} \) follows an i.i.d. standard normal distribution. The parameter \( \sigma \) captures the aggregate consumption volatility.

We use \( Q_t \) to denote the price of the Lucas tree at period \( t \), and thus the total return on the Lucas tree, \( R_{b,t+1} \), is defined as
\[ R_{b,t+1} = \frac{Q_{t+1} + Y_{t+1}}{Q_t}. \]

3.2.1 Households

There is a unit mass of identical households who makes intertemporal consumption and saving decisions. We collapse all households into a single representative household. He is infinitely lived and endowed with recursive utility as in Epstein and Zin (1989):
\[ U_t = \left\{ (1 - \beta)C_t^{1 - \frac{1}{\psi}} + \beta \left( E_t[U_{t+1}^{1 - \gamma}] \right)^{\frac{1 - \frac{1}{\psi}}{1 - \gamma}} \right\}^{\frac{1}{1 - \frac{1}{\psi}}}. \]

where \( C_t \) is the period \( t \) consumption.

The household can only save through a deposit account with banks. Let \( \{\pi_t\}_{t=0}^{\infty} \) denote the stream of income that the household receives from the ownership of banks, and \( R_{D,t} \) denote the one-period realized gross return for the bank deposits. A set of budget constraints is described as the following:
\[ C_0 + B_0 = \pi_0, \]
\[ C_t + B_t = B_{t-1}R_{D,t} + \pi_t, \quad t \geq 1. \]

In the above formulation, the household receives a stream of income, \( \{\pi_t\}_{t=0}^{\infty} \) and makes his consumption and saving decisions. \( C_t \) is the period \( t \) consumption choice, and \( B_t \) is the amount he deposits in the one-period defaultable bond, which has gross realized return \( R_{D,t+1} \) in the next period. \( \pi_t \) is the amount of wealth transferred from the banking sector to the household at period \( t \). Technically, the \( \{\pi_t\}_{t=0}^{\infty} \) sequence is constructed so that it can be easily verified that \( C_t = Y_t \) satisfies the budget constraint.
3.2.2 Financial Intermediaries

A bank uses its net worth and deposits raised from households to invest the equity of the firms. At period $t$, the bank started with net worth $n_t$ and chooses hold $s^b_t$ shares of the stock. The bank has to borrow $s^b_tQ_t - n_t$ from the household in order to finance the purchase of the stock. Let $b_t$ denote the amount of loan. The flow-of-funds constraint implies:

$$s^b_tQ_t = n_t + b_t. \quad (23)$$

Equation (24) is the law of motion of net worth. At period $t + 1$, each share of Lucas tree pays $Q_{t+1} + Y_{t+1}$. If the bank is still solvent, it will repay $b_t$ amount of loan to households at a fixed rate $\bar{R}_t$; if it goes bankrupt, the bank’s net worth become zero and households get whatever is left.

$$n_{t+1} = \max \left( s^b_t(Q_{t+1} + Y_{t+1}) - b_t\bar{R}_t, 0 \right) \quad (24)$$

Denote $\mathbf{I}^b_{D,t}$ as the default indicator of an individual bank, $p_t$ as the default probability, and $M_{t+1}$ as households’ stochastic discount factor. The deposit rate $\bar{R}_t$ must satisfy the Euler equation

$$(1 - p_t)E_t \left[ M_{t+1}\bar{R}_t|\mathbf{I}^b_{D,t} = 0 \right] + p_tE_t \left[ M_{t+1} \frac{s^b_t(Q_{t+1} + Y_{t+1})}{b_t}|\mathbf{I}^b_{D,t} = 1 \right] = 1, \quad (25)$$

The first term of equation (25) is the discounted value of loan if the bank is solvent in next period, and the second term represents the value of loan if bank defaults.

In addition to the budget constraint, banks also subject to a participation constraint, motivated by a limited enforcement argument in Gertler and Kiyotaki (2010). At each period, the banker has an opportunity to divert a $\theta$ fraction of bank assets at its market price and default on its debt. And the depositors can only recover $(1 - \theta)$ fraction of bank asset, due to limited enforcement. Because the depositors recognize the bank’s incentive to divert funds, they will restrict the amount they lend. In this way a participation constraint arises: we need to make sure that the value of the bank must exceed the banker’s outside option.

$$V_t \geq \theta s^b_tQ_t, \quad (26)$$

in which $V_t$ measures the discounted value of payoffs from operating honestly and $\theta$ is the fraction of asset that a banker can divert. In the presence of this financial constraint, banks may want to accumulate cash to grow out of the constraint. To rule out this possibility, we assume that in each period, a fraction $\lambda$ of the bank is forced to liquidate, in which case,
their net worth is paid out as dividend. The remaining fraction $1 - \lambda$ will survive to the net period. The liquidation probability is i.i.d. across banks and time.

A bank maximize its franchise value subject to the constraint (23), (24), (25) and (26). Specifically, the problem can be written recursively as:

$$
V_t = \max_{s_t} (1 - p_t) E_t \left\{ M_{t+1} \left[ \lambda n_{t+1} + (1 - \lambda) V_{t+1} \right] | I_{D,t+1} = 0 \right\}
$$

subject to

$$
n_{t+1} = s_t^b (Q_{t+1} + Y_{t+1}) - (s_t^b Q_t - n_t) \overline{R}_{t+1}
$$

$$
V_t \geq \theta s_t^b Q_t
$$

$$
(1 - p_t) E_t \left[ M_{t+1} \overline{R}_{t+1} | I_{D,t} = 0 \right] + p_t E_t \left[ M_{t+1} s_t^b (Q_{t+1} + Y_{t+1}) | I_{D,t} = 1 \right] = 1.
$$

(27)

### 3.2.3 Evolution of Aggregate Bank Net Worth

We can aggregate across banks to obtain the evolution of aggregate bank net worth. In equilibrium, the banking sector hold all the risky assets. Therefore, the aggregate bank holding of Lucas tree $S_t^b = 1$. From the flow-of-funds constraint (23) we have

$$
Q_t = N_t + B_t,
$$

in which, $N_t$ is the aggregate bank net worth and $B_t$ is the aggregate debt.

In each period, households use a fraction $\delta$ of the Lucas tree to set up new banks, as assumed in Gertler and Kiyotaki (2010). Because we focus on a symmetric equilibrium, all the banks have the same leverage ratios and will default at the same time, i.e., bank defaults are systemic. If the banking sector is solvent, the aggregate net worth of old banks next period is just the value of Lucas tree minus the promised debt repayment, $(Q_{t+1} + Y_{t+1}) - (Q_t - N_t) \overline{R}_{t+1}$. With $\lambda$ fraction of old banks liquidated, we have

$$
N_{t+1} = (1 - \lambda) \left[ (Q_{t+1} + Y_{t+1}) - (Q_t - N_t) \overline{R}_{t+1} \right] + \delta Q_{t+1}.
$$

If the banking sector become insolvent, the old banks are wiped out and the aggregate bank net worth next period equals to the value of new banks,

$$
N_{t+1} = \delta Q_{t+1}.
$$

Let $I_{D,t}$ denote the default indicator of the whole banking sector, we can summarize the
evolution of aggregate bank net worth as

\[ N_{t+1} = \begin{cases} 
(1 - \lambda) \left[ (Q_{t+1} + Y_{t+1}) - (Q_t - N_t) R_{t+1} \right] + \delta Q_{t+1}, & \text{if } I_{D,t+1} = 0 \\
\delta Q_{t+1}, & \text{if } I_{D,t+1} = 1 
\end{cases} 
\]  

(29)

The aggregate net worth of banking sector is a critical determinant of the slackness of financial constraint, thus, the equilibrium prices. Define bank net worth share as aggregate net worth dividend by total asset value, \( \hat{n}_t = \frac{N_t}{Q_t} \). We show that a Markov equilibrium exists with bank net worth share \( \hat{n}_t \) as state variable. In Appendix B, we provide the details of the model solution and the construction of Markov equilibrium.

3.3 Asset Pricing Implications

3.3.1 The Leverage Constraint

At the equilibrium, by the property of value function (27), the bank’s franchise value can be expressed as \( V_t = n_t \mu_t \), where \( \mu_t \) the marginal value of bank net worth. The participation constraint can be expressed as

\[ \mu_t n_t \geq \theta s_t^b Q_t. \]

Therefore, the participation constraint provides a microfoundation for a leverage constraint:

\[ \frac{Q_t s_t^b}{n_t} \leq \frac{\mu_t}{\theta}, \]

in which a bank’s leverage ratio is defined as its total assets over net worth. In a symmetric equilibrium where all banks choose the same leverage, we can also sum across individual banks to obtain the relation for aggregate leverage,

\[ \frac{Q_t}{N_t} \leq \frac{\mu_t}{\theta}. \]

The maximum leverage ratio depends on the aggregate state variable \( \hat{n}_t \), and is countercyclical, as the shadow price of net worth \( \mu_t \) is high in bad times when net worth is scarce.

Expecting that a bank will be able to abscond with stocks purchased with loans from household, household will require a collateral posted against the loans. Therefore, the participation constraint can be also rewritten/reinterpreted and aggregated as a collateral constraint, as follows:

\[ B_t \leq \left( \frac{\mu_t}{\theta} - 1 \right) N_t. \]  

(30)
On the left hand side of (30), the aggregate loans from household sector, $B_t$, is equal to $Q_t - N_t$. The right hand of (30) is equal to aggregate net worth of the banking sector with a multiplier. It can be considered as the collateral required by the household to post against the loans.

### 3.3.2 Asset Markets

Suppose that there is a retail interbank market where the banks can trade Arrow-Debreu securities among themselves. In addition, the banks have a better enforcement/monitoring technology than households, therefore, the Arrow-Debreu securities are traded frictionless, i.e. no banks can default on them. Two classes of such assets of our interest are discussed in order.

First, risky assets. As in Campbell and Cochrane (1999) and Bansal and Yaron (2004), we model aggregate consumption and aggregate dividend as two separate processes. In particular, the log growth rate of aggregate dividend is specified as:

$$\log \left( \frac{D_{t+1}}{D_t} \right) = \mu_d + \varphi \sigma \varepsilon_{y,t+1} + \varphi_d \sigma \varepsilon_{d,t+1}. $$

in which $\varepsilon_{y,t+1}$ is the consumption shock, and $\varepsilon_{d,t+1}$ is the dividend growth shock that is uncorrelated with consumption growth shock. Both shocks are i.i.d standard Normally distributed. Two additional parameters $\varphi > 1$ and $\varphi_d > 1$ allow us to match the overall volatility of dividends, which is larger than that of consumption in the data, and its correlation with consumption. We use $Q_{d,t}$ to denote the price of the dividend claim, and the market return is thus defined as

$$R_{m,t+1} = \frac{Q_{d,t+1} + D_{t+1}}{Q_{d,t}}.$$  

Second, the interbank loans that lend one unit of net worth today and return $R_{FI,t}$ units in the next period, which $R_{FI,t}$ denotes the gross interest rate.

### 3.3.3 Asset Pricing

In this section, we provide some intuition on the model mechanism that generates the asset price dynamics. For the following discussion, we focus on the case without bank defaults. In our setup, the major driver of asset price dynamics is the financial accelerating effect induced by participation constraint. To avoid the distraction by the additional complication, we ignore the bank defaults for now. The assets traded in the interbank market should be priced by the banks’ stochastic discount factor as in lemma 1.
Lemma 1. The returns, $R_{t+1}$, for any assets that financial intermediary can trade frictionlessly among themselves (i.e. "frictionless" means that bank cannot default on them), including $R_{m,t+1}, R_{y,t+1}$ and $R_{L,t+1}$, must satisfy

$$E \left[ M_{t+1} \{ \lambda + (1 - \lambda) \mu_{t+1} \} R_{t+1} \right] = \Omega_t, \quad (31)$$

in which

$$\Omega_t = \frac{E \left[ M_{t+1} \{ \lambda + (1 - \lambda) \mu_{t+1} \} \right]}{E[M_{t+1}]} + \gamma_t \theta.$$

We use $M_{t+1}^{FI}$ to denote the “augmented stochastic discount factor” implied by bank’s optimization problem,

$$M_{t+1}^{FI} = M_{t+1} \frac{\lambda + (1 - \lambda) \mu_{t+1}}{\Omega_t}, \quad (32)$$

which can price all the assets traded frictionlessly among banks. Beside $M_{t+1}$, the intertemporal marginal rate of substitution of consumption, $\tilde{M}_{t+1}$ also depends on an additional component, $\Phi_{t+1}$, which is defined as:

$$\Phi_{t+1} = \frac{\lambda + (1 - \lambda) \mu_{t+1}}{\Omega_t}. \quad (33)$$

The term, $\lambda + (1 - \lambda) \mu_{t+1}$, is a measure of shadow price of net worth at the next period, which is a weighted average of marginal value of net worth given the bank is forced to liquidate or not. $\Omega_t$ can be interpreted as the risk adjusted present value of investing one unit of net worth for one period. Thus, we can think of the second component, $\Phi_{t+1}$, as the shadow price appreciation from period $t$ to $t+1$, and the augmented stochastic discount factor has the interpretation of the inter-temporal marginal rate of substitution with respect to additional unit of net worth. $M_{t+1}^{FI}$ depends not only on household consumption, but also on intermediary equity capital. The banker dislikes assets with low return when aggregate consumption is low, and when his financial intermediary has low net worth/high debt.

The participation constraint (26) is essential to generate the financial accelerating effect. Absent from this constraint, the marginal value of bank net worth $\mu$ will always equal to 1. To the extent that an additional unit of net worth relaxes the incentive constraint or reduces the possibility of binding constraint in the future, $\mu$ will exceed unity in general. Figure 2 shows the marginal value of bank net worth $\mu$ as a function of state variable bank net worth share $\hat{n}$. The marginal value of net worth increases as the bank net worth decreases. In addition, the marginal value becomes very sensitive to changes of net worth as adverse fundamental shocks hit the banking sector into binding region, $\hat{n} \leq \hat{n}_b$. 

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The non-linear sensitivity of the marginal value of net worth, $\mu_{t+1}$, with respect to a fundamental shock, translates into counter-cyclical exposure of $\Phi_{t+1}$ to the shock, and therefore, generates counter-cyclical market price of risk. Figure 3 plots the volatility of intermediary SDF. Because the consumption volatility is constant by assumption, the increase of SDF volatility in the low net worth region is fully attributed to the financial accelerator component $\Phi_{t+1}$.

The covariance between $\Phi_{t+1}$ and market return $R_{m,t+1}$ is responsible for asset pricing impacts for the additional channel of a leverage constraint. Figure 4 plots the expected returns as functions of net worth share. Indeed, with countercyclical risk price generated by the financial accelerating effect, the expected returns of risky assets, i.e, dividend claim and consumption, are high (low) in recessions (booms) when bank net worth is low (high).

The non-linear sensitivity of the marginal value of net worth also help to endogenously generate counter-cyclical volatility. Figure 5 plots the volatility of returns as functions of net worth share. As the economy moves to the low net worth region, a small change in net worth share leads to a large change in risk prices, which, in turn, leads to large changes in discount rates and asset prices.

Next, we turn to deposit rate $\bar{R}_t$ and inter-bank rate $R_{FI,t}$. These two interest rates are priced by different stochastic discount factors and the interest rate difference is closely related to the slackness of participation constraint. We have $\bar{R}_t = R_{FI,t}$ whenever the participation constraint is not binding, because in this case, the leverage constraint is slack and both loans act as a perfect substitute. We have $\bar{R}_t \leq R_{FI,t}$ when the intermediary sector is constrained. From the demand perspective, inter-bank borrowing is very attractive. It allows banks to invest in the stock without affecting their debt capacity with the household. As a result, all banks want to borrow from each other on the inter-bank market. Market clearing requires interest rate to go up to clear the market. As shown in Figure 4, when the constraint is not binding, i.e. $\hat{n} > \hat{n}_b$, the interbank rate and deposit rate are the same. When the economy goes into the binding region, borrowing more from households is not possible. With higher expected return on bank assets, the additional source of funding from other banks become increasingly valuable such that, in equilibrium, the interbank rate shoots up.

3.3.4 Term Premium and Equity Yield Slope

In addition to generate the stylized patterns on market return and interest rates, more importantly, our model is a potential candidate to explain the stylized fact that: (1) The equity term premium is counter-cyclical. (2) The equity yield curve is pro-cyclical. In
addition, it is upward sloping in expansions and downward sloping in recessions.

In our model, dividend growth are positively related to financial shocks, which give rise to a risk premium. The financial shocks also affect the risk price. A negative financial shock will worsen the balance sheet of banks, which, in turn, will increase the price of risk. Because shocks to risk price is persistent, they will a greater effect on the price of long-maturity dividend strips. This gives a positive equity term premium. As we have discussed in the previous section, because the financial accelerator effect generates a counter-cyclical risk price, the equity term premium will also be counter-cyclical.

In our model setup, because dividend growth is i.i.d and the term structure of risk-free interest rate is flat, the term structure shapes of equity yield, forward equity yield and hold-to-maturity return are the same. They all only reflect the information on discount rate. Therefore, for the sake of simplicity, we only focus on the equity yield for the following discussion. In Figure 6, the black dashed line shows the slope of term structure of equity yields generated by our model. The difference between the 5-year and 1-year equity yield is an increasing function of net worth share. It is negatively sloped in the low net worth region and positively sloped in the high net worth region. The unconditional slope is positive as shown by the horizontal dashed line.

The key element to generate a pro-cyclical equity yield slope is the strong mean-reversion of risk price. From equation (12), the term structure slope of equity yield can be decomposed into a equity term premium component and an expected change in yield component. The first component is counter-cyclical, to have a pro-cyclical yield slope, we need the expected changes to be pro-cyclical and strong enough. Figure 8 plots the one-period equity yield as a function of state variable $\hat{n}$. The one-period equity yield is decreasing with net worth share because risk price is high in bad times and low in good times. If the economy is in recession, as shown in the left point of Figure 8, the net worth is expected to recover and move back to steady state. Therefore, we have $e_{n,t} > E_t(e_{n,t+1})$, which implies that the expected change component is negative. On the other hand, if the economy is in expansion, as shown in the right point of Figure 8, the net worth is expected to gradually decrease to the steady state. By the same logic, this implies that the expected change component is positive. The magnitude of expected change is determined by the mean-reverting speed. A fast mean-reverting speed of risk price makes the expected short term yield quickly revert to the long-run mean and the expected change component relatively large.

The sign switching feature of equity yield is actually a stronger discipline than the pro-cyclical pattern (which can be universally positive or negative). It implies that not only the fluctuations of expected changes should dominate, but also the absolute value. The sign of
yield slope in expansion is unambiguously positive because the expected change component and equity term premium components both imply a positive slope. However, in recession, the expected change component implies a negative slope while the equity term premium component implies a positive one. The sign of the slope depends on which effect dominates.

Figure 6 plots the equity term premium component and expected change component as a functions of state variable bank net worth share \( \hat{n} \). We can see that equity term premium is always positive and decrease from low net worth state to high net worth state. However, in the low net worth region, the expected negative change in risk premium dominate the equity term premium effect and makes the overall term structure of equity yield downward sloping. The strong mean-reverting effect comes from the endogenous fast recovery of bank balance sheet in recessions. To understand this point, we write down the evolution of aggregate bank net worth:

\[
N_{t+1} = (1 - \lambda)N_t \left[ \frac{Q_t}{N_t} R_{b,t+1} - \left( \frac{Q_t}{N_t} - 1 \right) R_{t+1} \right] + \delta Q_{t+1}.
\]

When the bank net worth is low, two effects are in play. First, the expected return on asset \( R_{b,t+1} \) is endogenously high. Second, because the banking sector holds all the risky assets, the aggregate leverage \( \frac{Q_t}{N_t} \) is high. Both effects lead to fast recovery of balance sheet.

In Appendix C, we provide a reduce-form illustration on the importance of mean-reversion speed. Many other leading asset pricing models such as habit and long-run risk also features time varying risk premium. The reason why these model can not generate a negative yield slope in recession is that their mean-reverting speed is too slow that the effect is dominated by term premium.

4 Quantitative analysis of the model

In this section, we calibrate the model at quarterly frequency and evaluate its ability to replicate key moments of both cash flow dynamics and asset returns. We focus on a long sample of U.S. annual data (1930 – 2017), including pre-war data, whenever the data is available. We focus on the benchmark model with recursive preferences, based on calibrated parameters reported in Table 2, and extensively discuss its quantitative asset pricing implications. Appendix C.4 provides more details on the data sources.
4.1 Calibration

In this section, we discuss the parameter values in the benchmark calibration, which are summarized in Table 2.

Following the literature, we set the relative rate of risk aversion, $\gamma$, to be 10, and the elasticity of intertemporal substitution, $\psi$, to be 2. We set the discount factor, $\beta$, to be 0.998 to match the level of risk-free interest rate in the data.

In the log output growth process, the parameters $\mu_y$ and $\sigma$ are calibrated to match the mean and volatility of the consumption growth in the data. Similarly, $\mu_d$ matches the average log dividend growth rate. We calibrate the two additional parameters in the log dividend growth process, $\varphi > 1$ and $\varphi_d > 1$, to match the overall volatility of dividends and its correlation with consumption.

There are three parameters for the financial sector: the annual liquidation/exit probability of banks, $\lambda$; the transfer parameter for new banks, $\delta$, and the fraction of bank asset divertible, $\theta$. I set $\lambda = 0.2$ implying that banks survive for 5 years on average, similar to the number used in Gertler and Kiyotaki (2010). There are no direct empirical counterparts in the data to pin down the rest two parameters, $\delta$ and $\theta$. I choose these two parameters indirectly to match the following two targets: an average leverage ratio of 4 for economy-wide financial intermediary sector, and an average of interest rate spread of 0.22% per annum, consistent with that of TED spread.

As in Gertler and Kiyotaki (2010), the model treats the entire intermediary sector as a group of identical institutions. Note that in the model the capital structure of the intermediary plays a central role in asset prices determination. It is important to match the leverage ratio because it affects how consumption shocks get magnified and the probability of being in the constrained versus unconstrained region. We follow the composition of the financial intermediary sector defined in Adrian, Moench and Shin (2011) to compute the leverage ratio of the aggregate financial intermediary from the Flow of Funds Table 3. The average leverage ratio over the sample period 1945 – 2011 is 3.67. We calibrate the parameters so that the model produces an average leverage ratio of 4.

4.2 Aggregate Moments

We computed the model implied aggregate moments from a long simulation of 10000 annual observations. The results are summarized in Table 3.

---

Designed by the calibration procedure, the model matches the aggregate consumption and dividend dynamics very well. It is noteworthy that by choosing two parameters, $\varphi$ and $\varphi_d$, i.e. the loading of aggregate dividend growth on consumption growth shock and its own shock, the model roughly matches the correlation between consumption and dividend growth, and the overall volatility of dividend process.

We use two asset pricing moments, namely, the leverage ratio and the mean of interest rate spread to calibrate the model. Not surprisingly, the model matches these two moments very well. In the model, when the constraint is not binding, the interest rate spread is equal to zero. Therefore, the binding probability is closely linked to both the mean and volatility of interest rate spread, which substantially raise the bar. Our model can simultaneously achieve both. The model implied volatility of interest rate spread, which is not targeted, is also very close to the data.

The model also performs very well in matching other asset pricing moments which are not targeted in the calibration. First, the model produces a high equity premium 5.89%, and a stock market volatility of 14.5%, which is only slightly lower than a volatility of 19.71% in the data. Second, the model is also able to generate an unconditionally positive equity yield slope. The model implied 5-year minus 1-year equity yield spread is 0.9%, which is a substantial fraction of the 1.2% spread we observe in the data.

However, we also notice that there are some discrepancies between the model implied moments with the data. The model implied model understates the volatility of the log price-dividend ratio. In the model, the standard deviation of the log price-dividend ratio is 0.08, as compared with 0.46 in the annual data. Historical stock prices display low-frequency variation relative to cash flow, which is not captured in the model. The historical standard deviation of log price-dividend ratio is this high in part because stock prices were persistently high at the end of the sample period.

Overall speaking, Table 3 suggests that the model performs relatively well to match both cash flow dynamics and asset pricing moments for U.S. data, given the driving force is an i.i.d. process. We could introduce a predictable component in expected consumption and dividend growth to further improve the persistence and standard volatility of price-dividend ratio.

### 4.3 Conditional Moments

The model also endogenously produces return predictability and time varying volatility. We repeatedly simulate 1000 artificial samples from the model, each with 81 annual observations.
We run predictive regression of return and integrated volatility on lagged pd ratio for each sample, and report the median value, 2.5, 5, 95 and 97.5 percentiles as well as the regression result from the data.

Table 4 reports the results on return predictability. Overall, our model can reproduce the return predictive pattern in the data. The beta coefficients reported and R squared in data are well within the model simulated 5% to 95% confidence interval for both 1-year, 3-year and 5-year regression horizons. In our model, the consumption process is i.i.d, so the return predictability is purely driven by financial friction. As bad shocks hits the financial sector, the marginal value of net worth increases, and the price of risk also increase. Higher discount rates drive down the price to dividend ratio, which produces negative relation of pd ratio and future returns in our model.

Table 5 shows that, in both our model and data, pd ratio can also predict integrated volatility, which is constructed by summing up the demeaned quarterly squared returns:

\[
\text{InteVol}_t = \sqrt{\sum_{i=0}^{3} \left( r_{m,t-i} - \sum_{i=0}^{3} r_{m,t-i}/4 \right)^2}.
\]

Notably, our model is able to generate time-varying volatility of returns without endogenously feed in any time-varying cash flow dynamics. In that sense, it is purely driven by change of discount rates. As we have discussed in Section 3.3.3, in low net worth region, risk prices are more sensitive to consumption shocks. Large discount rate shocks leads to large change of asset prices, which increases the return volatility.

### 4.4 Term Structure Dynamics

Our model is also capable of producing a counter-cyclical equity term premium and pro-cyclical equity yield slope, similar to what we observe in the data. Table 6 shows the estimates of regression (15)-(17) from intermediary model. The simulation confirms that, in our model, equity yield curve is pro-cyclical (a negative \( \hat{\alpha}_1 \)) and equity term premium is counter-cyclical (a positive \( \hat{\beta}_1 \)). More importantly, the \( \hat{\gamma}_1 \) estimates for the expectation hypothesis regression is greater than one and very close to the data counter-part. This indicates that similar to the data, the equity yield slope in our model is also largely driven by the expected change component.

Table 7 panel A reports the average equity yield slope in deep recession, recession and expansion period within the model. We classify the net worth share in the bottom 2% as
deep recession, 2%-15% as recession and the rest as expansion period. Our model is able to produce a strongly downward-sloping term structure of equity yield in deep recessions, a slightly negative yield slope in the recession periods and a positive slope in expansions.

As we have discussed in Section 3.3.4, the main driving force for the yield slope to switch sign is mean-reverting risk price, which is further determined by the slackness of financial constraint. Because interest rate spread is also highly related to the financial constraint, we can use interest rate spread and net worth share as empirical proxies for the slackness of constraint. With that in mind, our model has five unique predictions on the relationship between the interest rate spread, net worth share and term structure slope. Specifically, we will have: (1) Net worth share and interest rate spread are negatively correlated, (2) Net worth share and equity yield are negatively correlated, (3) Net worth share and yield slope are positively correlated, (4) Interest rate spread and equity yield are positively correlated, (5) Interest rate spread and yield slope are negatively correlated. These predictions are all confirmed in the data. In table 7 panel B, we report the correlation computed from the data as well as the ones implied by the model. In fact, the model implied correlations not only have the same sign as the data but the coefficients are also quite close.

The model also predicts that short-maturity yield are much more volatile than long-maturity yield, i.e., the equity yield volatility is downward-sloping. Figure 8 plots the volatility of equity yield as a function of maturity. Intuitively, short-maturity yield largely reflects the current risk price and fluctuate a lot with the healthiness of balance sheet while long-maturity yield reflects the long-run average risk price, which is close to steady state and response little to shocks. When a negative shock hits the banking sector, the binding probability of participation constraint increases. In the short-run, the marginal value of net worth is so high that bankers demand a very high premium to hold risky assets so the short-maturity yield shoots up. However, in the long-run, bank net worth is expected to recover so that the long-maturity yield changes little.

To see how much of the slope variation is attributed to different maturity components, we compute the variance ratio of 5-year equity yield to 1-year equity yield. In addition, because

\[ Cov(\hat{n}, e_5) = Cov(\hat{n}, e_5) - Cov(\hat{n}, e_1), \]

we also compute the covariance ratio \( \frac{Cov(\hat{n}, e_5)}{Cov(\hat{n}, e_1)} \) to see how much of the co-variation between yield slope and state variables is attributed to different maturity components.

Table 7 panel C shows the variance/covariance ratios. We can see that both in the data and our model, short-term maturity yield are much more volatile and fluctuates more with
net worth share or interest rate spread. However, the short-maturity equity yield are much more volatile than what implied by the model. This means our model still fall short of the mean-reversion speed.

5 Conclusion

In this paper, we study the the cyclical variation of the term structure of equity returns. New empirical facts show that the term structure of equity yield is highly procyclical, that is, it is positively sloped in expansions and negatively sloped in recessions, while the equity term premium is counter-cyclical. Through offering a equity yield decomposition framework, we decompose the term structure of equity yield into equity term premium and a mean reversion component about the expected changes in future yields. We show that in order to reconcile the seemingly contradictive negative relation between equity yield curve and equity term premium, the second mean reversion component have to be strong enough to drive the procyclical fluctuations of the overall equity yield term structure, although the first component is still counter-cyclical. We propose a financial intermediary-based asset pricing model to quantitatively account for cyclical patterns of equity term structure, as well as a large set of conventional asset pricing moments, and consumption and dividend dynamics. In our model, the mean reversion component is endogenously driven by the time-varying tightness of the intermediaries’ leverage constraint. We also demonstrate that the cyclical pattern of equity term structure imposes a strong discipline on the speed of mean reversion of discount rate for any standard asset pricing models. In the standard calibration of long-run risks model (Bansal and Yaron, 2004) and habit model (Campbell and Cochrane, 1999), the mean reversion speed of discount rate is too slow to account for such negative correlation between equity yield curve and equity term premium.
References


A Term Premium and Equity Yield

A.1 Term Premium

This section aims to understand the term premium from the basic pricing equation and summarize the term premium implications for various models. From the basic pricing equation, \( E_t(r_{n,t+1}) \) can be computed as:

\[
E_t(r_{n,t+1}) = E_t\left[ \log(R^H_{n,t+n}) - \log(R^H_{n-1,t+n}) \right] \\
= -E_t(m_{t,t+n}) - \frac{1}{2} Var_t(m_{t,t+n} + \Delta d_{t,t+n}) \\
+ E_t(m_{t+1,t+n}) + \frac{1}{2} Var_t(m_{t+1,t+n} + \Delta d_{t+1,t+n}) \\
= -E_t(m_{t+1}) - \frac{1}{2} Var_t(m_{t+1} + m_{t+1,t+n} + \Delta d_{t+1} + \Delta d_{t+1,t+n}) \\
+ \frac{1}{2} Var_t(m_{t+1,t+n} + \Delta d_{t+1,t+n}) \\
= -E_t(m_{t+1}) - \frac{1}{2} Var_t(m_{t+1}) - \frac{1}{2} Var_t(\Delta d_{t+1}) - Cov_t(m_{t+1}, \Delta d_{t+1}) \\
- Cov_t(m_{t+1}, m_{t+1,t+n}) - Cov_t(m_{t+1}, \Delta d_{t+1,t+n}) - Cov_t(m_{t+1,t+n}, \Delta d_{t+1}) \\
- Cov_t(\Delta d_{t+1}, \Delta d_{t+1,t+n}).
\]

(A.1)

where \( m_{t,t+n} \) is the pricing kernel from \( t \) to \( t+n \) and \( \Delta d_{t+1,t+n} \) is the dividend growth from \( t \) to \( t+n \). For the sake of simplicity, we use \( m_{t+m} \) and \( \Delta d_{t+m} \) to denote one-period pricing kernel (dividend growth) from \( t+m-1 \) to \( t+m \).

The slope of dividend strip return from maturity-\( n-1 \) to \( n \) can be computed as:

\[
E_t(r_{n,t+1}) - E_t(r_{n-1,t+1}) = \left[ \begin{array}{c} \\
- Cov_t(m_{t+1}, \Delta d_{t+1,t+n}) - Cov_t(m_{t+1,t+n}, \Delta d_{t+1}) \\
- Cov_t(m_{t+1}, m_{t+1,t+n}) - Cov_t(\Delta d_{t+1}, \Delta d_{t+1,t+n}) \\
- Cov_t(m_{t+1}, \Delta d_{t+1,t+n-1}) - Cov_t(m_{t+1,t+n-1}, \Delta d_{t+1}) \\
- Cov_t(m_{t+1}, m_{t+1,t+n-1}) - Cov_t(\Delta d_{t+1}, \Delta d_{t+1,t+n-1}) \\
- Cov_t(m_{t+1}, \Delta d_{t+n}) - Cov_t(m_{t+n}, \Delta d_{t+1}) \\
- Cov_t(m_{t+1}, m_{t+n}) - Cov_t(\Delta d_{t+1}, \Delta d_{t+n}) \\
\end{array} \right]
\]

(A.2)

The covariance of different period pricing kernels \(-Cov_t(m_{t+1}, m_{t+n})\) reflects the bond term premium \( \zeta_{n,t} - \zeta_{n-1,t} \), and the other terms reflects the equity term premium

\[
\xi_{n,t} - \xi_{n-1,t} = -Cov_t(m_{t+1}, \Delta d_{t+n}) - Cov_t(m_{t+n}, \Delta d_{t+1}) - Cov_t(\Delta d_{t+1}, \Delta d_{t+n})
\]

(A.3)
$Cov_t(\Delta d_{t+1}, \Delta d_{t+n})$ is usually very small, so the equity term premium crucially depends on the cross-covariance between the current pricing kernel and future dividend growth $Cov_t(m_{t+1}, \Delta d_{t+n})$, or the cross-covariance between the future pricing kernel and current dividend growth $Cov_t(m_{t+n}, \Delta d_{t+1})$.

The cross-covariance term $Cov_t(m_{t+1}, \Delta d_{t+n})$ and $Cov_t(m_{t+n}, \Delta d_{t+1})$ actually represent different classes of models:

- $Cov_t(m_{t+1}, \Delta d_{t+n})$: dividend growth is predictable. For example, in long-run risk, long run shock to $m_{t+1}$ also affect future dividend growth, which give a negative covariance and positive term premium. In Ai et al. (2012), long run shock to $m_{t+1}$ depress current investment while increase future investment which in turn makes the expected $\Delta d_{t+2}$ to be lower. This gives a positive covariance and negative equity term premium. While shock run shocks lower the current dividend payouts and makes the expected $\Delta d_{t+2}$ to be higher which produce a positive equity term premium. The relative volatility of the two shocks determines the sign of equity term premium.

- $Cov_t(m_{t+n}, \Delta d_{t+1})$: current dividend shock affect investors’ risk appetite in the future. For example, habit formation and financial intermediary models. A shock to current dividend also change the effective risk aversion, which is persistent and reflected in the pricing kernel going forward. This generate a negative cross-variance term $Cov_t(m_{t+2}, \Delta d_{t+1})$ and a positive term premium.

A.2 Equity Yield

Proof of Proposition 2. Equity yield $e_{n,t}$ involves an average of the yield from $t$ to $t+n$. To make things simpler, we define an auxiliary one-period equity yield

$$\hat{e}_{n,t} = ne_{n,t} - (n - 1)e_{n-1,t} \quad (A.4)$$

to facilitate our analysis. We can draw an analogy to the zero coupon yield curve. The one-period equity yield is similar to forward interest rate except that it is computed from the equity yield curve. From the above definition, we have the relation $e_{n,t} = \frac{1}{n} \sum_{i=1}^{n} \hat{e}_{n,t}$. $\hat{e}_{n,t}$ has the same term structure slope as $e_{n,t}$, but because it only involves the incremental one-period yield from maturity-$(n - 1)$ to $n$, it is much more transparent.

The maturity-$n$ one-period equity yield actually can be decomposed into a maturity-$n$
equity risk premium and expected change in maturity-\(n-1\) yield.

\[
\hat{e}_{n,t} = ne_{n,t} - (n-1)e_{n-1,t}
\]

\[
= ne_{n,t} - (n-1)E_t(e_{n-1,t+1}) + g^d_{1,t} + (n-1)[E_t(e_{n-1,t+1}) - e_{n-1,t}] - g^d_{1,t}
\]

\[
= E_t(r_{n,t+1}) + (n-1)[E_t(e_{n-1,t+1}) - e_{n-1,t}] - g^d_{1,t}
\]

\[
= \xi_{n,t} + \zeta_{n,t} + (n-1)[E_t(e_{n-1,t+1}) - e_{n-1,t}] - g^d_{1,t}
\]

expected return of maturity-\(n\) strip  
expected change in yield  
expected dividend growth

The term structure slope of one-period yield from maturity-\(n-1\) to \(n\) can be calculated as:

\[
\hat{e}_{n,t} - \hat{e}_{n-1,t}
\]

\[
= \xi_{n,t} - \xi_{n-1,t} + \zeta_{n,t} - \zeta_{n-1,t} + (n-1)[E_t(e_{n-1,t+1}) - e_{n-1,t}] - (n-2)[E_t(e_{n-2,t+1}) - e_{n-2,t}]
\]

\[
= [\xi_{n,t} - \xi_{n-1,t} + \zeta_{n,t} - \zeta_{n-1,t}] + [E_t(\hat{e}_{n-1,t+1}) - \hat{e}_{n-1,t}]
\]

\[
\text{term premium} + \text{expected change in yield}
\]

Equation (A.6) actually constitutes a base element of the slope decomposition. Taking a special case, the slope from maturity-1 to \(n\) is:

\[
\hat{e}_{n,t} - \hat{e}_{1,t} = \sum_{i=2}^{n} (\hat{e}_{i,t} - \hat{e}_{i-1,t})
\]

\[
= \sum_{i=2}^{n} [\xi_{i,t} - \xi_{i-1,t} + \zeta_{i,t} - \zeta_{i-1,t}] + \sum_{i=2}^{n} [E_t(\hat{e}_{i-1,t+1}) - \hat{e}_{i-1,t}]
\]

\[
= [\xi_{n,t} - \xi_{1,t} + \zeta_{n,t} - \zeta_{1,t}] + (n-1)[E_t(e_{n-1,t+1}) - e_{n-1,t}]
\]

Then term structure slope of equity yield from maturity-1 to \(n\) can be decomposed as

\[
e_{n,t} - e_{1,t} = \frac{1}{n} \sum_{i=1}^{n} (\hat{e}_{i,t} - \hat{e}_{i-1,t})
\]

\[
= \frac{1}{n} \sum_{i=1}^{n} [(\xi_{i,t} - \xi_{i-1,t}) + (\zeta_{i,t} - \zeta_{i-1,t})] + \frac{1}{n} \sum_{i=1}^{n} (i-1)[E_t(e_{i-1,t+1}) - e_{i-1,t}]
\]

\[
\text{term premium} + \text{expected changes in equity yields}
\]

This completes the proof of equation (11). The bond yield slope also has a similar decomposition:

\[
y_{n,t} - y_{1,t} = \frac{1}{n} \sum_{i=1}^{n} (\zeta_{i,t} - \zeta_{i-1,t}) + \frac{1}{n} \sum_{i=1}^{n} (i-1)[E_t(y_{i-1,t+1}) - y_{i-1,t}]
\]

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If we subtract equation (A.8) by (A.9), we will have the forward equity yield slope decomposition equation (12).

\[B\text{ Model Solution}\]

\[B.1\text{ Solution of Bank’s Problem}\]

Define \(\tilde{M}_{t+1} = M_{t+1} + \frac{c}{Q_t}Q_t\) as the ratio of asset to net worth and \(\mu_t\) the marginal value of bank net worth. Because both the objective and constraints of the bank are constant returns to scale, the bank’s franchise value can be expressed as \(V_t = n_t\mu_t\), and the bank’s optimization is reduced to choosing the leverage ratio \(\phi_t\) to maximize ”Tobin’s Q", \(\mu_t\). Therefore, we can rewrite bank’s maximization problem (27) as

\[\mu_t = \max_{\phi} \int_{\mathcal{F}_N(\phi)} \tilde{M}_{t+1} \left[ \phi(R_{t+1}^b - \tilde{R}_{t+1}(\phi)) + \tilde{R}_{t+1}(\phi) \right] d\tilde{F}, \quad (B.1)\]

subject to the incentive constraint (26)

\[\mu_t \geq \theta\phi, \quad (B.2)\]

with deposit rate satisfying condition (25)

\[\int_{\mathcal{F}_D(\phi)} M_{t+1} \tilde{R}_{t+1}(\phi) d\tilde{F} + \int_{\mathcal{F}_N(\phi)} M_{t+1} \frac{\phi}{\phi - 1} R_{t+1}^b d\tilde{F} = 1, \quad (B.3)\]

in which \(\tilde{F}(g_{t+1})\) is the distribution function of endowment growth rate \(g_{t+1}\), \(\mathcal{F}_N(\phi)\) and \(\mathcal{F}_D(\phi)\) are the non-default and default sets of individual bank with leverage \(\phi\):

\[\mathcal{F}_N(\phi) = \left\{ (g_{t+1}) : R_{t+1}^b \geq \frac{\phi - 1}{\phi} \tilde{R}_{t+1}(\phi) \right\}, \]

\[\mathcal{F}_D(\phi) = \left\{ (g_{t+1}) : R_{t+1}^b < \frac{\phi - 1}{\phi} \tilde{R}_{t+1}(\phi) \right\}. \]

To derive the optimality conditions, we first write down the Lagrangian

\[\int_{\mathcal{F}_N(\phi)} \tilde{M}_{t+1} \left[ \phi(R_{t+1}^b - \tilde{R}_{t+1}(\phi)) + \tilde{R}_{t+1}(\phi) \right] d\tilde{F} + \gamma_t(\mu_t - \theta\phi). \quad (B.4)\]

Denote \(f(g_{t+1})\) as the density function of \(g_{t+1}\), and \(\bar{g}\) as the default threshold growth rate, which satisfy \(R_{t+1}^b(\bar{g}) = \frac{\phi - 1}{\phi} \tilde{R}_{t+1}(\phi)\). Then the first order condition with respect to \(\phi\) can be
computed as

$$\int_{g}^{+\infty} \tilde{M}_{t+1} \left[ (R_{t+1}^b - \bar{R}_{t+1}(\phi)) - (\phi - 1) \frac{d\bar{R}_{t+1}(\phi)}{d\phi} \right] f(g_{t+1})dg_{t+1} -$$

$$\left[ (R_{t+1}(\bar{g}) - \bar{R}_{t+1}(\phi)) + \bar{R}_{t+1}(\phi) \right] f(\bar{g}) \frac{\partial \bar{g}}{\partial \phi} - \gamma_t \theta = 0.$$  

Because $$\phi(R_{t+1}^b(\bar{g}) - \bar{R}_{t+1}(\phi)) + \bar{R}_{t+1}(\phi) = 0$$, we can simplify the above equation to have

$$\int_{\mathcal{F}_N(\phi)} \tilde{M}_{t+1} \left[ (R_{t+1}^b - \bar{R}_{t+1}(\phi)) - (\phi - 1) \frac{d\bar{R}_{t+1}(\phi)}{d\phi} \right] d\tilde{F} = \gamma_t \theta. \quad (B.5)$$

To substitute $$\frac{d\bar{R}_{t+1}(\phi)}{d\phi}$$, differentiate deposit rate equation (B.3) with respect to $$\phi$$

$$\int_{g}^{+\infty} M_{t+1} \frac{d\bar{R}_{t+1}(\phi)}{d\phi} f(g_{t+1})dg_{t+1} + \int_{g}^{\bar{g}} M_{t+1} \frac{-1}{(\phi - 1)^2} R_{t+1}^b f(g_{t+1})dg_{t+1}$$

$$- M_{t+1} \bar{R}_{t+1}(\phi) f(\bar{g}) \frac{\partial \bar{g}}{\partial \phi} + M_{t+1} \frac{\phi}{\phi - 1} R_{t+1}^b(\phi) f(\bar{g}) \frac{\partial \bar{g}}{\partial \phi} = 0.$$  

Then $$\frac{d\bar{R}_{t+1}(\phi)}{d\phi}$$ can be computed as

$$\frac{d\bar{R}_{t+1}(\phi)}{d\phi} = \frac{\int_{\mathcal{F}_D(\phi)} M_{t+1} R_{t+1}^b d\tilde{F}}{(\phi - 1)^2 \int_{\mathcal{F}_N(\phi)} M_{t+1} d\tilde{F}}. \quad (B.6)$$

Plug $$\bar{R}_{t+1}(\phi)$$ and $$\frac{d\bar{R}_{t+1}(\phi)}{d\phi}$$ into the FOC (B.5), we have

$$\int_{\mathcal{F}_N(\phi)} \tilde{M}_{t+1} R_{t+1}^b d\tilde{F} \left( 1 - \int_{\mathcal{F}_D(\phi)} M_{t+1} R_{t+1}^b d\tilde{F} \right) \frac{\int_{\mathcal{F}_N(\phi)} \tilde{M}_{t+1} d\tilde{F}}{\int_{\mathcal{F}_N(\phi)} M_{t+1} d\tilde{F}} = \gamma_t \theta, \quad (B.7)$$

with complementarity slackness condition:

$$\gamma_t > 0, \mu_t - \theta \phi = 0 \text{ or } \gamma_t = 0, \mu_t > \theta \phi \quad (B.8)$$

We also have the envelope condition

$$\mu_t = \frac{\int_{\mathcal{F}_N(\phi)} \tilde{M}_{t+1} d\tilde{F}}{\int_{\mathcal{F}_N(\phi)} M_{t+1} d\tilde{F}} + \gamma_t \mu_t \quad (B.9)$$
B.2 Construction of Markov Equilibrium

A competitive equilibrium is a collection of prices, \( \{Q_t, \bar{R}_t\}_{t=0}^{\infty} \), and quantities \( \{S^b_t, N_t, B_t\}_{t=0}^{\infty} \) that satisfy (1) household utility maximization; (2) bank maximizes its franchise value; (3) market clearing conditions; (4) a set of consistency conditions.

The market clearing conditions include:

\[
C_t = Y_t, \quad (B.10)
\]
\[
S^b_t = 1, \quad (B.11)
\]
\[
Q_t = N_t + B_t, \quad (B.12)
\]

Equation (B.10) is the consumption goods market clear condition. Equation (B.11) is the asset market clear condition, which says that all shares must be held by the banks. Equation (B.12) is the accounting identity.

Define the net worth share of banks as the net worth of banks divided by total assets \( \hat{n}_t \).

We normalize the prices by \( Y_t \) such that all variables are stationary,

\[
\hat{n}_t = \frac{N_t}{Q_t}, \quad q_t = \frac{Q_t}{Y_t}. \quad (B.13)
\]

We use ' to denote next period quantities and prices. Define the default set as

\[
\mathcal{I}_D = \{g_{t+1}|(Y_{t+1} + Q_{t+1}) < \bar{R}_{t+1}(Q_t - N_t)\},
\]

or in normalized form:

\[
\mathcal{I}_D(\hat{n}) = \{g'(1 + q(\hat{n}'))g' < \bar{R}(\hat{n})q(\hat{n})(1 - \hat{n})\}. \quad (B.14)
\]

and the non-default set as \( \mathcal{I}_N = \mathcal{I}_D^c \). \( g \) is the endowment growth process which is defined in (19).

The law of motion for the endogenous variable \( \hat{n}' \) becomes

\[
\hat{n}' = \begin{cases} 
(1 - \lambda) \left[ \frac{1 + q(\hat{n})}{q(\hat{n})} - \frac{q(\hat{n})}{g q(\hat{n})} (1 - \hat{n}) \bar{R}(\hat{n}) \right] + \delta, & \text{if } g' \in \mathcal{I}_N \\
\delta, & \text{if } g' \in \mathcal{I}_D
\end{cases} \quad (B.15)
\]

With the market clearing condition (B.10), we can construct the normalized utility of the
household as the fixed point of
\[ u = \{(1 - \beta) (g'^{1 - \frac{1}{\psi}} + \beta (E[(ug'^{1 - \gamma})^{1 - \frac{1}{\psi}}])^{1 - \frac{1}{\psi}}. \]  

(B.16)

The stochastic discount factors are simply
\[ M' = \beta (g')^{-\frac{1}{\psi}} \left[ \frac{1}{E[(g')^{1-\gamma}]^{1-\frac{1}{\psi}}} \right]^{\frac{1}{\psi}-\gamma} \]  
\[ \tilde{M}' = M' [\lambda + (1 - \lambda) \mu(\tilde{n})]. \]  

(B.17)

**Proposition 3.** *Markov equilibrium* Suppose there exists equilibrium functionals \( \{\mu(\tilde{n}), q(\tilde{n}), \bar{R}(\tilde{n}), \gamma(\tilde{n})\} \) satisfying the following set of functional equations:

\[
\int_{\mathcal{I}_N(\tilde{n})} \tilde{M}' (1 + q(\tilde{n}')) g' d\tilde{F} - \left[ 1 - \int_{\mathcal{D}(\tilde{n})} M' (1 + q(\tilde{n}')) g' d\tilde{F} \right] \frac{\int_{\mathcal{I}_N(\tilde{n})} \tilde{M}' d\tilde{F}}{\int_{\mathcal{I}_N(\tilde{n})} M' d\tilde{F}} = \gamma(\tilde{n}) \theta \\
\gamma(\tilde{n}) \left( \mu(\tilde{n}) - \frac{\theta}{\tilde{n}} \right) = 0 \\
\mu(\tilde{n}) = \frac{\int_{\mathcal{I}_N(\tilde{n})} \tilde{M}_{t+1} d\tilde{F}}{\int_{\mathcal{I}_N(\tilde{n})} M_{t+1} d\tilde{F}} + \gamma(\tilde{n}) \mu(\tilde{n}) \\
\bar{R}(\tilde{n}) = \frac{1}{\int_{\mathcal{I}_N(\tilde{n})} M' d\tilde{F}} \left[ 1 - \frac{1}{1 - \tilde{n}} \int_{\mathcal{D}(\tilde{n})} M' (1 + q(\tilde{n}')) g' d\tilde{F} \right]
\]

where the law of motion of \( \tilde{n} \) is given by (B.15), the distribution of \( g' \) is given by (19), the default set \( \mathcal{D}(\tilde{n}) \) is defined in (B.14), and the stochastic discount factors \( M' \) and \( \tilde{M}' \) are defined in (B.17).

**Proof.** We have derived the bank optimality conditions in Appendix B.1. If we apply the market clearing condition (B.11) (B.12) to replace the individual bank leverage multiplier with aggregate leverage, \( \phi = \frac{1}{\tilde{n}} \), and individual bank default set with banking sector default set, \( \mathcal{I}_N(\phi) = \mathcal{I}_N(\tilde{n}) \), the F.O.C (B.5), complementary slackness condition (B.8), envelope condition (B.9) and deposit rate equation (B.3) directly correspond to the four functional equations in Proposition 3. \( \Box \)
B.3 Computation Algorithm

To compute the equilibrium, we introduce two ancillary variables $\nu(\hat{n})$ and $\rho(\hat{n})$. The equilibrium functional $\{\nu(\hat{n}), \rho(\hat{n}), \mu(\hat{n}), q(\hat{n}), R(\hat{n})\}$ which satisfies

$$\nu(\hat{n}) = \frac{\int_{I_N(\hat{n})} M' d\tilde{F}}{\int_{I_N(\hat{n})} M' d\tilde{F}}$$

$$\int_{I_N(\hat{n})} \frac{M'(1 + q(\hat{n}'))g'}{p(\hat{n})} d\tilde{F} - \left[ 1 - \int_{I_D(\hat{n})} M'(1 + q(\hat{n}'))g' \frac{d\tilde{F}}{p(\hat{n})} \right] \nu(\hat{n}) = 0$$

$$\mu(\hat{n}) = \begin{cases} \frac{\nu(\hat{n})}{\hat{n}}, & \text{if } \hat{n} \geq \frac{\theta}{\nu(\hat{n})} \\ \frac{\theta}{\hat{n}}, & \text{if } \hat{n} < \frac{\theta}{\nu(\hat{n})} \end{cases}$$

$$q(\hat{n}) = \begin{cases} \rho(\hat{n}), & \text{if } \hat{n} \geq \frac{\theta}{\nu(\hat{n})} \\ \frac{\nu(\hat{n}) \rho(\hat{n})}{\theta + \nu(\hat{n})(1 - \hat{n})}, & \text{if } \hat{n} < \frac{\theta}{\nu(\hat{n})} \end{cases}$$

$$R(\hat{n}) = \frac{1}{\int_{I_N(\hat{n})} M' d\tilde{F}} \left[ 1 - \frac{1}{1 - \hat{n}} \int_{I_D(\hat{n})} M'(1 + q(\hat{n}'))g' \frac{d\tilde{F}}{q(\hat{n})} \right],$$

and $\gamma(\hat{n}) = 1 - \frac{\nu(\hat{n})}{\rho(\hat{n})}$ constitute a same equilibrium as in Proposition 3. The expression for $q(\hat{n})$ and $\mu(\hat{n})$ in the binding region can be derived as follows:

Suppose we are in the region of $\hat{n} < \frac{\theta}{\nu(\hat{n})}$. From F.O.C (B.5) and Envelope condition (B.9), we have

$$q(\hat{n}) = p(\hat{n}) \frac{\nu(\hat{n})}{\nu(\hat{n}) + \gamma(\hat{n}) \theta}$$

$$\mu(\hat{n}) = \nu(\hat{n}) + \gamma(\hat{n}) \mu(\hat{n}),$$

and the binding constraint gives:

$$\mu(\hat{n}) = \frac{\theta}{\hat{n}}. \quad \text{(B.20)}$$

Plug equation (B.20) into the equation equation (B.19) we can obtain

$$q(\hat{n}) = \frac{\nu(\hat{n}) p(\hat{n})}{\theta + \nu(\hat{n})(1 - \hat{n})}. \quad \text{(B.21)}$$

The state variable for the equilibrium functionals is net worth share $\hat{n}$, whose value falls in a closed interval $[\delta, 1]$. We use the following algorithm to compute the equilibrium functionals.

1. Give an initial guess of the functionals $\{\nu_0(\hat{n}), p_0(\hat{n}), \mu_0(\hat{n}), q_0(\hat{n}), R_0(\hat{n})\}$, the law of motion for $\hat{n}$, which is a mapping from $\hat{n}$ to $\hat{n}'$, and the default policy $I_{D,0}(\hat{n})$.

2. Given functionals $\{\nu_i(\hat{n}), p_i(\hat{n}), \mu_i(\hat{n}), q_i(\hat{n}), R_i(\hat{n})\}$, law of motion, default policy, com-
pute \( \{ \nu_{i+1}({\tilde{n}}), p_{i+1}({\tilde{n}}), \mu_{i+1}({\tilde{n}}), q_{i+1}({\tilde{n}}), \tilde{R}_{i+1}({\tilde{n}}) \} \) with equations (B.18).

3. Update the law of motion and default policy with equation (B.15), (B.14).

4. Compute \( \max (\| \mu_{i+1}({\tilde{n}}) - \mu_i({\tilde{n}}) \|_\infty), \| q_{i+1}({\tilde{n}}) - q_i({\tilde{n}}) \|_\infty) \), if it is less than 1e-8 then stop; else, return to step 2.

C A Reduced-form Model

C.1 Dividend process and pricing kernel

In this section, we provide a reduced-form model to illustrate the intuition about time-varying equity yield slope. Assume that the dividend growth follow an i.i.d process. The log growth rate can be written as:

\[
\Delta d_{t+1} = \mu + \sigma \eta_{t+1}
\]  
(C.1)

There is a state variable \( x_t \) which determines risk price of \( \eta_{t+1} \) shock. It follows an mean-reverting AR(1) process:

\[
x_{t+1} = \rho x_t + \varepsilon_{t+1}
\]  
(C.2)

\( \eta_t \) and \( \varepsilon_t \) are standard normal shocks, they have covariance \( Cov(\eta_t, \varepsilon_t) = \psi \). The reduced form log pricing kernel is:

\[
m_{t+1} = -r_f - \frac{1}{2} \lambda(x_t)^2 - \lambda(x_t)\eta_{t+1}
\]  
(C.3)

where \( \lambda(x_t) \) is the risk price of \( \eta \). We assume \( \lambda(x_t) \) is a decreasing and linear function of \( x_t \), i.e., high values of \( x_t \) are associated with good states.

\[
\lambda(x_t) = a_0 - a_1 x_t,
\]  
(C.4)

where \( a_0 \) and \( a_1 \) are positive coefficients.

In this economy, the risk-free rate is a constant, \( r_f \). Asset’s risk premium is determined by the exposure to \( \eta_{t+1} \) shock and its associated risk price \( \lambda(x_t) \). We allow the shocks to dividend and the shocks to risk price to be correlated so that the model is flexible enough to account for many economic environments.
C.2 Dividend Strip Price

Given the dividend growth process and the pricing kernel, the \( n \)-period log dividend strip price to dividend ratio, which is defined as \( \text{pd}_{t,n} \equiv p_{t,n} - d_t \), can be computed recursively as:

\[
\text{pd}_{t,n} = \log E_t [\exp \{m_{t+1} + \Delta d_{t+1} + \text{pd}_{t+1,n-1}\}].
\] (C.5)

Conjecture that the log pd ratio is a linear function of state variable, \( \text{pd}_{t,n} = A_{0,n} + A_{1,n}x_t \).

Plug into equation (C.5), we have

\[
\text{pd}_{t,n} = \log E_t [\exp \{(-r_f + \mu) - \frac{1}{2}\lambda(x_t)^2 + [\sigma - \lambda(x_t)] \eta_{t+1} + A_{0,n-1} + A_{1,n-1}x_{t+1}\}]
\]

\[
= (-r_f + \mu) - \frac{1}{2}\lambda(x_t)^2 + A_{0,n-1} + A_{1,n-1}\rho x_t + \frac{1}{2} Var_t ([\sigma - \lambda(x_t)] \eta_{t+1} + A_{1,n-1}x_{t+1})
\]

\[
= (-r_f + \mu) + \frac{1}{2}\sigma^2 + A_{0,n-1} + A_{1,n-1}\rho x_t - \lambda(x_t)\sigma + A_{1,n-1}^2 + [\sigma - \lambda(x_t)] A_{1,n-1}\psi
\]

\[
= A_{0,n-1} + A_{1,n-1} [(\sigma - a_0)\psi] + A_{1,n-1}^2 + (-r_f + \mu) + \frac{1}{2}\sigma^2 - a_0\sigma + [A_{1,n-1}(\rho + a_1\psi) + a_1\sigma] x_t.
\] (C.6)

Equation (C.6) holds for any \( x_t \), so it implies that

\[
A_{1,n} = A_{1,n-1} + a_1\sigma
\]

\[
A_{0,n} = A_{0,n-1} + A_{1,n-1} [(\sigma - a_0)\psi] + A_{1,n-1}^2 + (-r_f + \mu) + \frac{1}{2}\sigma^2 - a_0\sigma
\] (C.7)

With the initial condition \( A_{1,0} = 0, A_{0,0} = 0 \), we have

\[
A_{1,n} = \frac{(\rho + a_1\psi)^n - 1}{\rho + a_1\psi - 1} a_1\sigma
\]

\[
A_{0,n} = \sum_{i=1}^{n-1} (A_{1,i} [(\sigma - a_0)\psi] + A_{1,i}^2) + n(-r_f + \mu) + \frac{n}{2}\sigma^2 - na_0\sigma
\] (C.8)

C.3 Term Structure of Equity yield

As we have discussed in appendix A.2, the one-period equity yield \( \hat{e}_{n,t} \) defined in equation (A.4) has the same term structure slope as equity yield. Apply equation (C.8), we can express
\( \hat{e}_{n,t} \) as
\[
\hat{e}_{n,t} = n\epsilon_{n,t} - (n - 1)\epsilon_{n-1,t} = -pd_{n,t} + pd_{n-1,t}
\]
\[
= - A_{0,n} + A_{0,n-1} + ( - A_{1,n} + A_{1,n-1})x_t
\]
\[
= - (\rho + a_1 \psi)^{n-1} \frac{1}{\rho + a_1 \psi - 1} a_1 \sigma [ (\sigma - a_0)\psi] - \frac{[(\rho + a_1 \psi)^{n-1} - 1]^2}{(\rho + a_1 \psi - 1)^2} a_1^2 \sigma^2 + \]
\[
r_f - \mu - \frac{1}{2} \sigma^2 + a_0 \sigma - (\rho + a_1 \psi)^{n-1} a_1 \sigma x_t.
\]
(C.9)

For the following analysis, we ignore the Jensen’s inequality terms with magnitude \( \sigma^2 \), which are very small and have little effect on the shape of the term structure. With the decomposition equation (A.6), the conditional slope of one-period yield can be written as:
\[
\hat{e}_{n,t} - \hat{e}_{n-1,t} = \left[ \xi_{n,t} - \xi_{n-1,t} + \zeta_{n,t} - \zeta_{n-1,t} \right] + \left[ E_t (\hat{e}_{n-1,t+1} - \hat{e}_{n-1,t}) \right]
\]
\[
\approx (\rho + a_1 \psi)^{n-2} \left[ \psi a_1 \sigma (a_0 - a_1 x_t) + a_1 \sigma (1 - \rho) x_t \right].
\]
(C.10)

The unconditional slope is
\[
E(\hat{e}_{n,t} - \hat{e}_{n-1,t}) \approx (\rho + a_1 \psi)^{n-2} a_1 a_0 \psi \sigma.
\]
(C.11)

We restrict the parameters in the region \( \rho + a_1 \psi > 0 \).

Equation (C.10) and (C.11) summarize the yield slope implication of this stylized model. The expected term is always pro-cyclical: without the term premium effect, it imply a upward (downward) sloping term structure in expansion, i.e., \( x_t > 0 \) (recession, i.e., \( x_t < 0 \)), and the slope becomes flatter as the horizon \( n \) increases. The shape of curve also depends the mean reversion speed. A faster mean reversion speed (low \( \rho \)) imply a steeper curve at the short end while relatively flatter curve at the far end. In contrast, if mean reversion speed is slow (high \( \rho \)), the slope on the short end will be relatively gentle, but it become flatter more slowly as the horizon increases. The term premium critically depends on the correlation parameter \( \psi \): a positive \( \psi \) implies a positive and counter-cyclical term premium while a negative \( \psi \) implies a negative and pro-cyclical term premium.

The interesting case is \( \psi > 0 \). It leaves the cyclical and sign of term premium determined: it depends on relative importance of term premium and mean-reversion speed \( \rho \). If the mean-reversion speed is slow, \( \rho > 1 - a_1 \psi \), the equity yield slope will be counter-cyclical. If the mean-reversion speed is fast, \( \rho < 1 - a_1 \psi \), the equity yield slope will be pro-cyclical. The negative equity yield slope we observe in the data actually impose a stronger discipline for
the mean-reversion. For example, if we expect a negative equity yield slope when \( x < -\bar{x} \) (\( \bar{x} > 0 \)), we need a faster mean-reversion speed \( \rho < 1 - a_1 \psi - a_0 \psi / \bar{x} \).

**C.4 Data Sources**

**Forward Equity Yield:** The forward equity yield data is constructed from the futures contracts of the dividend strip, from a major financial institution that is active in dividend strips markets. The data set covers the period from December 2004 to February 2017 at daily frequency and it includes futures contracts on index dividend from S&P 500, Eurostoxx and Nikkei.

**Leverage Ratio:** We target average aggregate financial intermediary leverage for our model calibration. We follow Adrian, Moench and Shin (2011)’s composition of the aggregate financial intermediary sector. From Flow of Funds Table in U.S. we aggregate the assets and liabilities of each component, and then compute the aggregate leverage ratio based on:

\[
\text{Leverage}_t = \frac{\text{Aggregate Financial Assets}_t}{\text{Aggregate Financial Assets}_t - \text{Aggregate Liabilities}_t}
\]

For the regression analysis, we opt to use broker-dealer holding company leverage in He, Kellya and Manela (2017). The data is constructed by matching the New York Fed primary dealer list with CRSP/Compustat and Datastream data on their publicly traded holding companies.

**TED Spread:** Computed by the difference between annualized 3-month LIBOR rate and 3-month T-bill rate. Both series are from FRED dataset.

**Consumption:** Per capita consumption data are from the National Income and Product Accounts (NIPA) annual data reported by the Bureau of Economic Analysis (BEA). The data are constructed as the sum of consumption expenditures on nondurable goods and services (Table 1.1.5, lines 5 and 6) deflated by corresponding price deflators (Table 1.1.9, lines 5 and 6).

**Dividend:** The dividend process is constructed from VWRETD and VWRETX, i.e. the value weighted return on NYSE/AMEX including and excluding dividends, taken from CRSP. The construction of price-dividend ratio follows the data appendix in Bansal, Khatchatrian and Yaron (2005).

**Market Return:** Nominal market return is the value weighted return on NYSE/AMEX including dividends taken from CRSP. The real market return is computed by deflating the nominal return by corresponding price deflators (Table 1.1.9, lines 5 and 6).
**Risk-free Rate:** The nominal risk-free rate is measured by the annual 3-month T-Bill return. The real risk-free rate is computed by subtracting the nominal risk-free rate by expected inflation, a procedure detailed in Beeler and Campbell (2012).

**Integrated Volatility:** Integrated variance is the sum of squared daily stock returns on NYSE/AMEX. Integrated volatility is the square root of integrated variance. The daily value weighted return data on NYSE/AMEX including dividends are taken from CRSP.
## D Tables and Figures

### Table 1: Theories versus Stylized Facts

| Panel A: Data |  
|---|---|---|---|---|
| $E(r_{5,t} - r_{1,t})$ | $\alpha_1$ | $\beta_1$ | $\gamma_1$ |  
| **S&P500** | coeff | 0.020 | -0.435 | 0.289 | 1.009 |  
| | stdev | 0.159 | 0.044 | 0.106 | 0.315 |  
| **EuroStoxx50** | coeff | -0.049 | -0.319 | 0.165 | 1.466 |  
| | stdev | 0.270 | 0.066 | 0.117 | 0.275 |  
| **Nikkei225** | coeff | -0.012 | -0.097 | 0.283 | 1.395 |  
| | stdev | 0.318 | 0.066 | 0.150 | 0.263 |  

| Panel B: Model |  
|---|---|---|---|
| $E(r_{5,t} - r_{1,t})$ | $\alpha_1$ | $\beta_1$ | $\gamma_1$ |  
| **Habit** | median | 0.009 | 0.002 | 0.109 | -0.581 |  
| | 5pctl | 0.008 | 0.001 | 0.086 | -1.389 |  
| | 95pctl | 0.010 | 0.003 | 0.136 | 0.306 |  
| **Long-run risk** | median | 0.022 | 0.024 | 0.083 | -0.058 |  
| | 5pctl | 0.016 | 0.018 | 0.068 | -0.333 |  
| | 95pctl | 0.034 | 0.034 | 0.100 | 0.209 |  

This table shows average equity term premium and regression results of both data and model. Panel A reports the results in data. We run the following three regressions:

\[
e_{5,t} - e_{1,t} = \alpha_0 + \alpha_1(d_t - p_t) + \epsilon_t, \\
\gamma_i = \gamma_0 + \gamma_1(e_{5,t} - e_{1,t}) + \eta_{t+12}, \\
E(r_{m,t+12} - r_{1,t+12} = \beta_0 + \beta_1(d_t - p_t) + \epsilon_{t+12}.
\]

\(r_{m,t+12}\) is the twelve-month forward return to index. The maturity are measured in years. All regression are based on monthly rolling regressions. Standard errors are Newey-West corrected with 12 lags. Equity yields and strip returns are based on S&P 500, EuroStoxx 50 and Nikkei 225 index dividend future contracts. The data covers the period from Dec 2004 to Feb 2017. Panel B reports the simulation results from habit model by Campbell and Cochrane (1999) and long-run risk model by Bansal and Yaron (2004). We run 10000 simulations with 1000 years of artificial data for each sample path, and calculate the median, 5 percentile and 95 percentile estimates from annual regressions.
Table 2: **Parameter Values in the Benchmark Calibration**

<table>
<thead>
<tr>
<th>Summary of Calibration Parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Preference parameters</strong></td>
<td></td>
</tr>
<tr>
<td>Risk aversion</td>
<td>$\gamma$ 10</td>
</tr>
<tr>
<td>Intertemporal elasticity of substitution</td>
<td>$\psi$ 2</td>
</tr>
<tr>
<td>Discount factor</td>
<td>$\beta$ 0.998</td>
</tr>
<tr>
<td><strong>Consumption Dynamics</strong></td>
<td></td>
</tr>
<tr>
<td>Mean of consumption growth</td>
<td>$\mu_y$ 1.85</td>
</tr>
<tr>
<td>Volatility of consumption growth</td>
<td>$\sigma$ 2.10</td>
</tr>
<tr>
<td><strong>Dividend Dynamics</strong></td>
<td></td>
</tr>
<tr>
<td>Capital share</td>
<td>$\mu_d$ 1.05</td>
</tr>
<tr>
<td>Dividend loading on consumption growth shocks</td>
<td>$\varphi$ 3</td>
</tr>
<tr>
<td>Dividend loading on dividend specific shocks</td>
<td>$\varphi_d$ 4</td>
</tr>
<tr>
<td><strong>Financial Intermediary parameters</strong></td>
<td></td>
</tr>
<tr>
<td>Fraction of asset divertible</td>
<td>$\theta$ 0.4</td>
</tr>
<tr>
<td>Fraction of wealth transferred to new banks</td>
<td>$\delta$ 0.034</td>
</tr>
<tr>
<td>Exit probability of banks</td>
<td>$\lambda$ 0.2</td>
</tr>
</tbody>
</table>

This table reports the parameter values used for benchmark calibration. Growth rates and volatility are annualized and reported in percentage terms.
Table 3: **Aggregate Moments Based on Benchmark Calibration**

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Aggregate Quantities</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average consumption growth</td>
<td>$E(\Delta c)$</td>
<td>1.88</td>
</tr>
<tr>
<td>Volatility of consumption growth</td>
<td>$\sigma(\Delta c)$</td>
<td>2.09</td>
</tr>
<tr>
<td>Average dividend growth</td>
<td>$E(\Delta d)$</td>
<td>1.60</td>
</tr>
<tr>
<td>Volatility of dividend growth</td>
<td>$\sigma(\Delta d)$</td>
<td>10.92</td>
</tr>
<tr>
<td>Corr of consumption and investment</td>
<td>$corr(\Delta c, \Delta d)$</td>
<td>0.53</td>
</tr>
<tr>
<td>Leverage ratio of financial intermediary</td>
<td>$E(\frac{Q}{N})$</td>
<td>3.67</td>
</tr>
<tr>
<td><strong>Panel B: Asset Prices</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equity premium</td>
<td>$E(R_m - R_f)$</td>
<td>5.88</td>
</tr>
<tr>
<td>Volatility of equity return</td>
<td>$\sigma(R_m)$</td>
<td>19.71</td>
</tr>
<tr>
<td>Equity yield slope 5yr-1yr</td>
<td>$E(e_5 - e_1)$</td>
<td>1.20</td>
</tr>
<tr>
<td>Average log PD ratio</td>
<td>$E(p - d)$</td>
<td>3.42</td>
</tr>
<tr>
<td>Volatility of PD ratio</td>
<td>$\sigma(p - d)$</td>
<td>0.46</td>
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<tr>
<td>Average of Interest spread</td>
<td>$E(R_{FI} - R_f)$</td>
<td>0.22</td>
</tr>
<tr>
<td>Volatility of Interest spread</td>
<td>$\sigma(R_{FI} - R_f)$</td>
<td>0.98</td>
</tr>
<tr>
<td>Risk-free rate</td>
<td>$E(R_f)$</td>
<td>0.89</td>
</tr>
</tbody>
</table>

This table presents descriptive statistics for aggregate consumption growth, dividends, prices, the interest rate spread (i.e. the spread between interest rates for interbank and household loans). The data are real, sampled at an annual frequency and cover the sample period from 1930 to 2011, whenever the data are available. The sample period for leverage ratio is from 1945 to 2011. The sample period for interbank interest rate is from 1986 to 2011. The “Model” panel presents the corresponding moments implied by the model.
Table 4: **Return Predictability**

<table>
<thead>
<tr>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.50%</td>
<td>5%</td>
</tr>
</tbody>
</table>

**Panel A: Predictive Slopes**

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Predictive Slopes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1yr</td>
<td>-0.66 -0.59 -0.29 -0.01 0.04 -0.07</td>
</tr>
<tr>
<td>3yr</td>
<td>-1.12 -1.03 -0.51 0.04 0.17 -0.23</td>
</tr>
<tr>
<td>5yr</td>
<td>-1.38 -1.23 -0.62 0.14 0.28 -0.42</td>
</tr>
</tbody>
</table>

**Panel B: Predictive R2s**

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Predictive R2s</th>
</tr>
</thead>
<tbody>
<tr>
<td>1yr</td>
<td>0.00 0.00 0.03 0.11 0.13 0.12</td>
</tr>
<tr>
<td>3yr</td>
<td>0.00 0.00 0.04 0.16 0.19 0.10</td>
</tr>
<tr>
<td>5yr</td>
<td>0.00 0.00 0.04 0.16 0.20 0.16</td>
</tr>
</tbody>
</table>

Notes - This table provides evidence on predictability of future excess return by log price-dividend ratio. The entries correspond to regressing

\[ r_{t+1}^e + r_{t+2}^e + \ldots + r_{t+j}^e = \alpha(j) + B(j)x_t + v_{t+j} \]

where \( r_{t+1}^e \) is the excess return, \( j \) denotes the forecast horizon in years. \( x_t \) denotes log price-dividend ratio. The entries for the model are based on 1000 simulations each with 81 annual observations. Standard errors are Newey-West corrected using 10 lags.

Table 5: **Volatility Predictability**

<table>
<thead>
<tr>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.50%</td>
<td>5%</td>
</tr>
</tbody>
</table>

**Panel A: Predictive Slopes**

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Predictive Slopes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1yr</td>
<td>-0.36 -0.33 -0.19 -0.04 -0.02 -0.10</td>
</tr>
<tr>
<td>3yr</td>
<td>-0.39 -0.37 -0.18 -0.03 0.00 -0.13</td>
</tr>
<tr>
<td>5yr</td>
<td>-0.38 -0.34 -0.15 0.02 0.05 -0.11</td>
</tr>
</tbody>
</table>

**Panel B: Predictive R2s**

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Predictive R2s</th>
</tr>
</thead>
<tbody>
<tr>
<td>1yr</td>
<td>0.00 0.00 0.08 0.21 0.24 0.10</td>
</tr>
<tr>
<td>3yr</td>
<td>0.00 0.00 0.06 0.21 0.25 0.08</td>
</tr>
<tr>
<td>5yr</td>
<td>0.00 0.00 0.04 0.18 0.21 0.06</td>
</tr>
</tbody>
</table>

Notes - This table provides evidence on predictability of future integrated volatility by log price-dividend ratio. The entries correspond to regressing

\[ InteVol_{t+1} + InteVol_{t+2} + \ldots + InteVol_{t+j} = \alpha(j) + B(j)x_t + v_{t+j} \]

where \( InteVol_{t+1} \) is the integrated volatility, \( j \) denotes the forecast horizon in years. \( x_t \) denotes log price-dividend ratio. The entries for the model are based on 1000 simulations each with 81 annual observations. Standard errors are Newey-West corrected using 10 lags.
Table 6: Financial Intermediary Model: Expectation Hypothesis Regression

Panel A: Data

<table>
<thead>
<tr>
<th></th>
<th>E(r_{5,t} - r_{1,t})</th>
<th>α_1</th>
<th>β_1</th>
<th>γ_1</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P500</td>
<td>coeff</td>
<td>0.020</td>
<td>-0.435</td>
<td>0.289</td>
</tr>
<tr>
<td></td>
<td>stdev</td>
<td>0.159</td>
<td>0.044</td>
<td>0.106</td>
</tr>
<tr>
<td>EuroStoxx50</td>
<td>coeff</td>
<td>-0.049</td>
<td>-0.319</td>
<td>0.165</td>
</tr>
<tr>
<td></td>
<td>stdev</td>
<td>0.270</td>
<td>0.066</td>
<td>0.117</td>
</tr>
<tr>
<td>Nikkei225</td>
<td>coeff</td>
<td>-0.012</td>
<td>-0.097</td>
<td>0.283</td>
</tr>
<tr>
<td></td>
<td>stdev</td>
<td>0.318</td>
<td>0.066</td>
<td>0.150</td>
</tr>
</tbody>
</table>

Panel B: Financial Intermediary Model

<table>
<thead>
<tr>
<th>Intermediary model</th>
<th>E(r_{5,t} - r_{1,t})</th>
<th>α_1</th>
<th>β_1</th>
<th>γ_1</th>
</tr>
</thead>
<tbody>
<tr>
<td>median</td>
<td>0.014</td>
<td>-0.096</td>
<td>0.081</td>
<td>1.397</td>
</tr>
<tr>
<td>5pctl</td>
<td>0.012</td>
<td>-0.116</td>
<td>0.055</td>
<td>1.229</td>
</tr>
<tr>
<td>95pctl</td>
<td>0.015</td>
<td>-0.078</td>
<td>0.106</td>
<td>1.582</td>
</tr>
</tbody>
</table>

This table shows average equity term premium and regression results of both data and model. Panel A reports the results in data. We run the following three regressions:

\[ e_{5,t}^f - e_{1,t}^f = \alpha_0 + \alpha_1 (d_t - p_t) + \epsilon_t, \]
\[ r_{m,t+12} - r_{1,t+12}^f = \beta_0 + \beta_1 (d_t - p_t) + \epsilon_{t,t+12}, \]
\[ \frac{1}{5} \sum_{i=1}^{5} (i - 1) [e_{i-1,t+12}^f - e_{i-1,t}^f] = \gamma_0 + \gamma_1 (e_{5,t}^f - e_{1,t}^f) + \eta_{t,t+12}. \]

\( r_{m,t+12} \) is the twelve-month forward return to index. The maturity are measured in years. All regression are based on monthly rolling regressions. Standard errors are Newey-West corrected with 12 lags. Equity yields and strip returns are based on S&P 500, EuroStoxx 50 and Nikkei 225 index dividend future contracts. The data covers the period from Dec 2004 to Feb 2017. Panel B reports the simulation results from financial intermediary model. We run 10000 simulations with 1000 years of artificial data for each sample path, and calculate the median, 5 percentile and 95 percentile estimates from annual regressions.
Table 7: Net Worth Share, Interest Spread and Term Structure of Equity Yield

<table>
<thead>
<tr>
<th>Panel A: Term Structure Slope</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity yield slope: deep recession</td>
<td>$E(e_5 - e_1</td>
<td>\text{Deep Recession})$</td>
</tr>
<tr>
<td>Equity yield slope: recession</td>
<td>$E(e_5 - e_1</td>
<td>\text{Recession})$</td>
</tr>
<tr>
<td>Equity yield slope: expansion</td>
<td>$E(e_5 - e_1</td>
<td>\text{Expansion})$</td>
</tr>
</tbody>
</table>

| Panel B: Correlations |  |  |
|-----------------------|--------|
| Net worth share and liquidity premium | $corr(\hat{n}, R_{FI} - R_f)$ | -0.51 |
| Net worth share and 1-yr equity yield | $corr(\hat{n}, e_1)$ | -0.92 |
| Net worth share and equity yield slope | $corr(\hat{n}, e_5 - e_1)$ | 0.79 |
| Interest spread and 1-yr equity yield | $corr(R_{FI} - R_f, e_1)$ | 0.73 |
| Interest spread and equity yield slope | $corr(R_{FI} - R_f, e_5 - e_1)$ | -0.82 |

| Panel C: Variance/Covariance Ratio |  |  |
|------------------------------------|--------|
| Variance ratio equity yield 5-yr/1-yr | $\frac{Var(e_5)}{Var(e_1)}$ | 0.09 |
| Net worth share & equity yield 5-yr/1-yr | $\frac{Cov(\hat{n}, e_5)}{Cov(\hat{n}, e_1)}$ | 0.24 |
| Interest spread & equity yield 5-yr/1-yr | $\frac{Cov(R_{FI} - R_f, e_5)}{Cov(R_{FI} - R_f, e_1)}$ | 0.25 |

Panel A shows the conditional equity yield slope in deep recession, recession and expansion. Panel B shows the correlations between net worth share, interest rate spread and equity yield slope. Panel C shows the variance/covariance ratio for 5-year and 1-year equity yield. The data are sampled at the annual frequency, ranging from 1945 to 2011. Data constructions are described in the Appendix C.4.
Table 8: Composition of Aggregate Financial Intermediary Sector

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>FINBANK</td>
<td><strong>Banks</strong></td>
</tr>
<tr>
<td>CBSI</td>
<td>Charted depository institutions, excluding credit unions</td>
</tr>
<tr>
<td>CU</td>
<td>Credit unions</td>
</tr>
<tr>
<td>FINPI</td>
<td><strong>Pension Funds and Insurances</strong></td>
</tr>
<tr>
<td>PCIC</td>
<td>Property-casualty insurance companies</td>
</tr>
<tr>
<td>LIC</td>
<td>Life insurance companies</td>
</tr>
<tr>
<td>PPF*</td>
<td>Private pension funds</td>
</tr>
<tr>
<td>SLGERF*</td>
<td>State &amp; local government employee retirement funds</td>
</tr>
<tr>
<td>FGRF*</td>
<td>Federal government retirement funds</td>
</tr>
<tr>
<td>FINMF</td>
<td><strong>Mutual Funds</strong></td>
</tr>
<tr>
<td>MMMF*</td>
<td>Money market mutual funds</td>
</tr>
<tr>
<td>MF*</td>
<td>Mutual funds</td>
</tr>
<tr>
<td>CEF*</td>
<td>Closed-end funds and exchange-traded funds</td>
</tr>
<tr>
<td>SHADBANK</td>
<td><strong>Shadow Banks</strong></td>
</tr>
<tr>
<td>MORTPOOL*</td>
<td>Agency- and GSE-backed mortgage pools</td>
</tr>
<tr>
<td>ABS</td>
<td>Issuers of asset-backed securities</td>
</tr>
<tr>
<td>FINCO</td>
<td>Finance companies</td>
</tr>
<tr>
<td>FUNDCORP</td>
<td>Funding corporations</td>
</tr>
<tr>
<td>SBRDLR</td>
<td>Security brokers and dealers</td>
</tr>
</tbody>
</table>

Notes - This Table is based on the definitions in Adrian, Moench and Shin (2010). The component intermediaries denoted by “*” means they are only financed by equity.
Figure 1: Term Structure Slope of Forward Equity Yield

This figure shows both the realized and the estimated term structure slope of forward equity yield, measured as the difference between 5-year and 1-year forward equity yield. The fitted slope values are obtained from estimating the linear regression:

\[ e_{5,t}^f - e_{1,t}^f = \phi_0 + \phi_1 lev_t + \varepsilon_t, \]

in which \( lev_t \) is financial intermediary leverage ratio. The shaded area is NBER recession.
Figure 2: Marginal Value of Bank Net Worth

This figure shows the marginal value of bank net worth as a function of state variable bank net worth share \( \hat{n} \). \( \hat{n}_{ss} \) denotes the average net worth share in this economy, suggested by a long simulation from the model. \( \hat{n}_b \) denotes the binding threshold. \( \hat{n} \leq \hat{n}_b \) is the region at which the constraint is binding. The parameters are based on the benchmark calibration summarized in table 2.

Figure 3: Volatility of SDF

This figure shows the volatility of intermediary stochastic discount factor as a function of state variable bank net worth share \( \hat{n} \). \( \hat{n}_{ss} \) denotes the average net worth share in this economy, suggested by a long simulation from the model. \( \hat{n}_b \) denotes the binding threshold. \( \hat{n} \leq \hat{n}_b \) is the region at which the constraint is binding. The parameters are based on the benchmark calibration summarized in table 2.
Figure 4: Expected Returns of Risky Assets and Two Interest Rate

This figure shows the expected return of dividend claim and consumption claim, interbank borrowing rate and deposit rate as as functions of state variable bank net worth share \( \hat{n} \). \( \hat{n}_{ss} \) denotes the average net worth share in this economy, suggested by a long simulation from the model. \( \hat{n}_b \) denotes the binding threshold. \( \hat{n} \leq \hat{n}_b \) is the region at which the constraint is binding. The parameters are based on the benchmark calibration summarized in table 2.

Figure 5: Volatility of Returns

This figure shows the volatility of return on dividend claim and consumption claim as as functions of state variable bank net worth share \( \hat{n} \). \( \hat{n}_{ss} \) denotes the average net worth share in this economy, suggested by a long simulation from the model. \( \hat{n}_b \) denotes the binding threshold. \( \hat{n} \leq \hat{n}_b \) is the region at which the constraint is binding. The parameters are based on the benchmark calibration summarized in table 2.
Figure 6: **Term Structure Slope of Equity Yield: Decomposition**

This figure shows the decomposition of term structure slope of equity yield from equation (11). The term slope are decomposed into equity term premium and expected change in risk premium. Both are plotted as a functions of state variable bank net worth share $\hat{n}$. The arrows show the direction to which the state variable is expected to move. $\hat{n}_{ss}$ denotes the average net worth share in this economy, suggested by a long simulation from the model. $\hat{n}_b$ denotes the binding threshold. $\hat{n} \leq \hat{n}_b$ is the region at which the constraint is binding. The parameters are based on the benchmark calibration summarized in table 2.
Figure 7: **One-period Equity Yield**

This figure shows the one-period equity yield as a function of state variable bank net worth share $\hat{n}$. The arrows show the direction to which the state variable is expected to move. $\hat{n}_{ss}$ denotes the average net worth share in this economy, suggested by a long simulation from the model. $\hat{n}_b$ denotes the binding threshold. $\hat{n} \leq \hat{n}_b$ is the region at which the constraint is binding. The parameters are based on the benchmark calibration summarized in table 2.

![1-Period Equity Yield graph](image)

Figure 8: **Volatility of Equity Yield**

This figure shows the volatility of equity yield as a function of maturity. The model implied volatility is calculated from a long simulation of 10000 annual observations. The parameters are based on the benchmark calibration summarized in table 2.

![Volatility of Equity Yield graph](image)