Is hedging for believers? The role of expectations in optimal production and hedging decisions

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Abstract

We study theoretically how firms incorporate their market view into production and hedging decisions. Several motivating examples suggest that optimism reduces the demand for hedging while ambiguity raises it when the firm’s market view is favorable. We analyze the production and hedging decisions of Sandmo’s (1971) competitive firm in a general model of smooth ambiguity aversion (Klibanoff et al., 2005). We distinguish between concordant and discordant uncertainty depending on whether the profitability and behavioral effects of ambiguity go in the same or opposite direction. We then identify restrictions on the firm’s ambiguity preferences that allow for clear comparative static effects of optimism, pessimism, and greater ambiguity on production and hedging. Our results explain how differences in market views and ambiguity preferences generate cross-sectional variation in the demand for hedging and shed new light on so-called “selective hedging.”

Keywords: prudence · smooth ambiguity · hedging

JEL-Classification: D21 · D24 · D81

WORK IN PROGRESS
1 Introduction

“Our positive view on the gold price led us to accelerate the elimination of these contracts ahead of the schedule we had established.”

Aaron Regent, Barrick’s President and CEO, Press Release December 1, 2009

“. . . few companies regularly use derivatives to take a “naked” speculative position on FX rates or commodity prices, most corporate derivatives users appear to allow their views of future interest rates, exchange rates, and commodity prices to influence their hedge ratios.”

Stulz (1996)[p. 8]

Corporate hedging refers to the use of financial instruments such as forwards, futures, swaps, and options to reduce the volatility of firm value. This volatility originates from uncertainty over prices including commodity prices, exchange rates, and interest rates. Several determinants of a firm’s demand for hedging have been discussed in the literature: taxes, financial distress, and agency costs, among others [Nance et al. 1993]. Survey evidence regarding derivative usage and risk management practice reveals that firms may allow their market view to influence their hedging decisions (see Dolde 1993 [Bodnar et al. 1998 [Brown et al. 2006]), which has been coined “selective hedging,” see Working (1961) and Stulz (1996). In this paper, we provide a theoretical model to understand how market views affect production and hedging decisions in a context of ambiguity.

At the surface, the relationship between beliefs and production and hedging decisions may seem trivial. Take the case of a commodity producer, say a gold-mining firm, contemplating the effect of gold price uncertainty on its profitability. In the absence of a hedging program, an expectation of high market prices should raise production whereas a higher level of uncertainty should decrease it. When the firm hedges, we would expect production to be carried out at the efficient level. In this case, we would similarly expect that a belief of high market prices will lower the demand for hedging whereas greater uncertainty should stimulate hedging. We provide several closed-form examples, which corroborate this basic economic intuition.

As we will see, these simple intuitions may not carry over to more sophisticated frameworks of decision-making under ambiguity. We use Neilson’s (2010) simplified axiomatic characterization of smooth ambiguity preferences (see Klibanoff et al., 2005) to study the behavior of a risk- and ambiguity-averse firm owner, who chooses production and hedging such as to maximize his expected valuation of firm profits. This model of competitive firm behavior was introduced by Sandmo (1971) and adapted to hedging decisions by Holthausen (1979). It is most suitable for closely-held firms but it encompasses risk neutrality as a special case. While the bulk of the ambiguity aversion literature is rooted in individual decision-making under uncertainty (see Trautmann and van de Kuilen 2013, for a survey), ambiguity aversion has also been found in experimental market settings involving sophisticated decision makers (Sarin and Weber 1993), and in surveys of business owners and managers (Viscusi and Chesson, 1999; Chesson and Viscusi, 2003).
In our general model, we first analyze production decisions absent any hedging program. We distinguish between concordant and discordant uncertainty depending on whether the effects of uncertainty on profitability and production are aligned or opposing. This distinction separates a negative from a positive effect of ambiguity and ambiguity aversion on production at the extensive margin. When it comes to optimism, pessimism and greater ambiguity, we uncover conflicting effects related to the firm’s sensitivity to ambiguity. We identify restrictions on ambiguity preferences that allow to recover the intuitive results that optimism increases production, pessimism reduces it, and greater ambiguity reduces it as well. When hedging is available, we obtain the acclaimed Separation Theorem because production is carried out at the efficient level regardless of the firm’s beliefs. Ambiguity still affects the firm’s demand for hedging, and as in the case of production, all effects are intuitive at the extensive margin but require additional assumption at the intensive margin.

Several behavioral studies of the demand for hedging exist in the literature, and we draw comparisons with our approach whenever appropriate. [Shi and Irwin (2005)] take a Bayesian perspective while [Tuthill and Frechette (2004)] examine hedging with futures and options under rank dependent expected utility. [Jacobs et al. (2018)] investigate the role of reference points, both theoretically and empirically, while [Korn and Rieger (2019)] introduce regret aversion. In the context of ambiguity, [Wong (2015a)] studies the value of hedging for beliefs that are unbiased whereas [Lien and Yu (2017)] have some results on optimism and pessimism for hedging using Chateauneuf et al.’s (2007) of neo-additive capacities under Choquet expected utility. This framework does not fully separate tastes from beliefs, which is why smooth ambiguity preferences are more common in applications.

Our results answer the question how firms incorporate their market view into production and hedging decisions. We first show that the classification into concordant and discordant uncertainty demarcates those cases where we intuit optimism to lower or increase the demand for hedging, respectively. Then we identify an empirically testable restriction on the intensity of relative ambiguity aversion that demarcates cases where this intuition prevails from those where conflicting effects may predominate. Differences across these conditions between firms may then explain cross-sectional variation in the demand for hedging and contribute to a better understanding of selective hedging.

Our paper proceeds as follows. In the next section, we outline the model and explain how we incorporate uncertainty. We then discuss two closed-form examples to develop the basic intuition. Section 3 develops the comparative statics of unhedged production decisions whereas Section 4 analyzes the demand for hedging. A final section concludes, and all technical proofs are gathered in the appendix.
2 The model

2.1 Preliminaries

We consider the competitive firm of Sandmo (1971) and study its production and hedging behavior in the decision-theoretic context of Klibanoff et al.’s (2005) and Neilson’s (2010) smooth model of ambiguity aversion. Specifically, we will focus on the firm’s expectations and how they affect production and hedging decisions.

There is one period with two dates, 0 and 1. The firm produces a single commodity at date 0 according to a deterministic cost function \( c(q) \) where \( q \geq 0 \) denotes the output level. Zero production is costless, \( c(0) = 0 \), initiating production is costless at the margin, \( c'(0) = 0 \), and the firm’s technology exhibits positive and increasing marginal cost, that is, \( c'(q) > 0 \) and \( c''(q) > 0 \) for all \( q > 0 \). At date 1, the firm sells its entire output \( q \) at the prevailing spot price \( \tilde{p} \), where the tilde represents a random variable. This spot price is ex ante unknown but lies within a price range of \([p, \bar{p}]\) with \( 0 < p < \bar{p} \). We normalize the discount rate to zero and compound all payoffs and costs to date 1.

To manage the price risk associated with the sales of the commodity, the firm can engage in hedging. It can trade commodity forwards at date 0, which guarantee a given price \( p^f \in (p, \bar{p}) \) at date 1 for each unit of the commodity delivery. For analytical tractability, we assume commodity forwards to be infinitely divisible and denote by \( x \) the number of units of the commodity sold forward at date 0 (or purchased forward for negative values of \( x \)). For a price \( \tilde{p} = p \), this results in the following profit for the firm at date 1:

\[
\Pi(q, x|p) = pq + (p^f - p)x - c(q).
\]

The focus in this paper is on the firm’s expectations. The firm faces ambiguity in the sense that the price distribution is not objectively known at date 0 but rather given by a family of cumulative distribution functions (CDFs), denoted by \( \{F(p|\theta)\}_{\theta} \). The support of each CDF is contained in \([p, \bar{p}]\), and the firm’s second-order probability distribution ("belief") over \( \theta \) is given by the CDF \( G(\theta) \) with support \([\underline{\theta}, \bar{\theta}]\). Given the firm’s belief, it expects the following commodity price at date 1:

\[
p_{\text{exp}}^G = \int_{\underline{\theta}}^{\bar{\theta}} \int_{p}^{\bar{p}} p \, dF(p|\theta) \, dG(\theta).
\]

The firm can then compare its spot price expectation to the available forward price. We introduce the following terminology.

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1 For example, the firm might use historical data to identify a parametric class of CDFs for the price risk (say, truncated lognormal) and then estimate its parameters. This yields confidence intervals around the point estimates, and each parameter combination from the confidence intervals renders a different CDF. Belief \( G \) then represents how the firm weighs the different possible CDFs and how much emphasis it places on parameter estimates closer to versus further from the point estimate.
Definition 1. We say the firm’s market view relative to the forward price is favorable if \( p_G^{exp} > p_f \), neutral if \( p_G^{exp} = p_f \) and unfavorable if \( p_G^{exp} < p_f \).

Differences between the firm’s expected spot price and the available forward price can arise for several reasons. Assume for now that the objective spot price distribution is given by \( F_o(p) \), resulting in an expected spot price of \( p_o = \int p \, dF_o(p) \). The CDF \( F_o(p) \) may or may not be part of \( \{ F(p|\theta) \}_\theta \). Furthermore, assume that the spot price distribution of the underlying of the forward contract is given by \( F_u(p) \) with associated expected spot price of \( p_u = \int p \, dF_u(p) \). We then arrive at the following decomposition:

\[
\begin{split}
    p_G^{exp} - p_f &= \left( p_G^{exp} - p_o \right)_{\text{market view}} + \left( p_o - p_u \right)_{\text{basis risk}} + \left( p_u - p_f \right)_{\text{futures market}}.
\end{split}
\]  

(1)

The first term represents the firm’s market view because it measures to what extent the firm expects a higher or lower spot market price relative to the objective distribution. Firms may simply not know the objective distribution and therefore hold favorable or unfavorable market views. Even if they do, they may not necessarily agree with the consensus forecast or be subject to information rigidity and update their beliefs only slowly (see Mankiw and Reis, 2002; Reis, 2006). The second term captures basis risk because the commodity produced by the firm may not be perfectly aligned with the underlying of the futures contract. This can cause differences in expected spot prices. The third term is related to the operation of the futures market. If it is unbiased, the forward price coincides with the expected spot price of the underlying \( (p_f = p_u) \). The other cases are normal backwardation \( (p_f < p_u) \) and contango \( (p_f > p_u) \), see Benninga et al. (1984) and Briys et al (1993). In our paper, we focus on the role of the firm’s expectations for production and hedging decisions, which corresponds to the market view component. For the interpretation of our results it is, however, helpful to keep the other factors in mind.

The firm’s risk preferences over profits are represented by an increasing and concave von Neumann-Morgenstern utility function \( u(\Pi) \), that is, \( u'(\Pi) > 0 \) and \( u''(\Pi) \leq 0 \). If the firm were certain for the price risk distribution to be \( F(p|\theta) \) for a \( \theta \in [\underline{\theta}, \overline{\theta}] \), the associated expected utility is given by:

\[
U(q, x|\theta) = \int \Pi(q, x|p) \, dF(p|\theta).
\]

The ambiguity experienced by the firm translates into uncertainty over the level of expected utility, which we indicate in our notation by letting \( U \) depend on \( \theta \). To model the firm’s sensitivity to ambiguity, we utilize Neilson’s (2010) simplified axiomatic approach towards the smooth ambiguity model pioneered by Klibanoff et al. (2005). Specifically, \( \varphi(U) \) denotes the firm’s ambiguity function, which we assume to be increasing and concave, \( \varphi'(U) > 0 \) and

\(^2\) See Akerlof and Dickens (1982) for an economic analysis of cognitive dissonance.
\( \varphi''(U) \leq 0 \). The firm’s objective then takes the following form:

\[
\max_{q \geq 0, x} V(q, x|G) = \int_{\theta} \varphi(U(q, x|\theta)) \, dG(\theta).
\]  

Concavity of \( \varphi \) represents the notion of ambiguity aversion because the firm is better off under the expected price distribution (that is, under \( \int_{\theta} F(p|\theta) \, dG(\theta) \)) than facing the uncertainty over the possible distributions. Technically, this is a direct consequence of Jensen’s inequality. Our notation shows that the firm’s objective depends on its belief \( G \) about price uncertainty.

In Appendix A.1, we demonstrate that concavity of \( u \) and \( \varphi \) imply that the firm’s objective is concave in \((q, x)\).

We organize the paper around three questions. In a first step, we will investigate the effect of the firm’s expectations on optimal production decisions absent any hedging program (that is, \( q \) is endogenous and \( x = 0 \)). In a second step, we will determine the role of expectations for optimal hedging decisions when production occurs at a fixed level (that is, \( q \) is fixed and \( x \) is endogenous). Finally, we will analyze the impact of the firm’s expectation when production and hedging decisions are integrated (that is, both \( q \) and \( x \) are endogenous). We will see that, as a consequence of the separation theorem, the third question turns out to be a special case of the second one.

To carry out our analysis, we will often impose some additional structure on the family of CDFs representing the possible price distributions. Specifically, we use Jindapon and Neilson’s (2007) approach, which we formulate as our first assumption:

**A1.** There are two CDFs \( F \) and \( \bar{F} \) on \([p, \bar{p}]\) such that the family \( \{F(p|\theta)\}_\theta \) is the affine hull of \( \{F(p), \bar{F}(p)\} \), that is,

\[
F(p|\theta) = \frac{\theta - \theta_n}{\bar{\theta} - \theta} \cdot F(p) + \frac{\theta - \theta}{\bar{\theta} - \theta_n} \cdot \bar{F}(p), \quad \text{for } \theta \in (\theta_n, \bar{\theta}), p \in [p, \bar{p}].
\]

If \( \bar{F} \) is preferred over \( F \) by a decision-maker with utility function \( u \), then parameter \( \theta \) ranks the family of CDFs in ascending order with higher values of \( \theta \) corresponding to better price distributions. If the expected price under \( F \) is below the forward price (i.e., \( \int_{\bar{p}}^p p \, dF(p) < p_f \)) and the expected price under \( \bar{F} \) is above the forward price (i.e., \( \int_{p}^\bar{p} p \, d\bar{F}(p) > p_f \)), then there is a unique \( \theta^* \in (\theta_n, \bar{\theta}) \) such that the expected price under \( F(p|\theta^*) \) coincides with the forward price (i.e., \( \int_{\bar{p}}^p p \, dF(p|\theta^*) = p_f \)). In this case, any belief with an expected value of \( \theta^* \) represents a neutral market view while any belief with a higher (resp. lower) expected value than \( \theta^* \) represents a favorable (resp. unfavorable) market view. We also point out that the firm’s exposure to price uncertainty is endogenous because it depends on its production and hedging decisions. Therefore, a preference of \( \bar{F} \) over \( F \), that is, \( \int_{p}^\bar{p} u(p) \, d\bar{F}(p) > \int_{\bar{p}}^p u(p) \, dF(p) \), may or may not induce a preference of \( \Pi(q, x|\theta) \) over \( \Pi(q, x|\theta_n) \) for a given level of production \( q > 0 \) and hedging \( x \). In Appendices B.1 and B.2 we identify conditions under which a preference between \( \bar{F} \) and \( F \) allows us to order \( \Pi(q, x|\theta) \) and \( \Pi(q, x|\theta_n) \).
To study the role of expectations in our model, we consider stochastic changes of the firm’s belief. If \( \theta \) ranks the distributions from low to high in terms of their implied expected utility for the firm, we can represent increased optimism by a belief that considers high \( \theta \)-values as more likely and increased pessimism by a belief that considers low \( \theta \)-values as more likely. If \( \theta \) ranks the implied expected utility levels in descending order instead, the definitions of increased optimism and pessimism need to be reversed to remain meaningful. We can also model greater ambiguity via a Rothschild and Stiglitz (1970) increase in risk in the firm’s belief. We summarize these considerations in the following definition.

**Definition 2.** Consider a firm with belief \( G \).

a) The firm becomes more optimistic (resp. pessimistic) if \( G \) undergoes a first-order stochastically dominant (FSD) shift towards \( \theta \)-values with high (resp. low) expected utility.

b) The firm perceives greater ambiguity if \( G \) undergoes a Rothschild and Stiglitz (1970) increase in risk.

### 2.2 A closed-form example

To illustrate the model and develop some expectations, we analyze an exponential-power specification for \((u, \varphi)\) combined with normality assumptions on the possible price distributions and the firm’s belief. These assumptions admit a closed-form solution of the problem (see Gollier, 2011). We rewrite the firm’s profit as follows:

\[
\Pi(q, x|p) = p(q - x) + pfx - c(q).
\]

The first term denotes the firm’s unhedged production, which is sold at the prevailing spot price, whereas the second term denotes the firm’s hedged production, which is sold at the forward price. The firm’s risk preferences exhibit constant absolute risk aversion (CARA),

\[
u(\Pi) = -\frac{1}{A} \exp(-A \cdot \Pi) \quad \text{with} \ A > 0.
\]

Furthermore, we assume that prices are normally distributed with expected value \( \theta \) and the same variance \( \sigma^2 \), that is, \( \tilde{p} \sim N(\theta, \sigma) \). In this case, the Arrow-Pratt approximation is exact and we arrive at the following expression for the firm’s expected utility:

\[
U(q, x|\theta) = -\frac{1}{A} \exp \left(-A \left[ \theta(q - x) + pfx - c(q) - \frac{1}{2}A(q - x)^2\sigma^2 \right]\right).
\]

The firm faces uncertainty about the mean $\theta$ of the price distribution. Following [Gollier (2011)], we assume that the expected price is itself normally distributed, $\tilde{\theta} \sim N(\mu, s)$, and that the firm’s ambiguity preferences exhibit constant relative ambiguity aversion,
\[
\varphi(U) = -\frac{(-U)^{1+\gamma}}{1 + \gamma} \text{ with } \gamma > 0.
\]

This specification takes into account that expected utility is negative in our example. The ambiguity function is increasing and concave in its domain and displays decreasing absolute ambiguity aversion (see [Baillon and Placido, 2019]).

In this example, the different possible CDFs for the price distribution are ordered according to FSD with higher values of $\theta$ representing a better distribution. Based on its belief, the firm expects a commodity price of $p^{exp} = \mu$ at date 1 so that $\mu > p^f$ represents a favorable, $\mu = p^f$ a neutral and $\mu < p^f$ an unfavorable market view, respectively. The normality of the firm’s belief gives us a parametric handle on optimism, pessimism and the level of ambiguity because changes in $\mu$ represent FSD shifts whereas changes in $s$ correspond to changes in risk. In the extreme case of $s = 0$, the firm is certain that the price distribution is given by $N(\mu, \sigma)$ whereas higher values of $s$ represent less and less confidence regarding the true price distribution. Iso-elastic ambiguity preferences coupled with a normally distributed expected price allow us to solve for the firm’s objective function. We arrive at
\[
V(q, x) = -\frac{1}{1 + \gamma} \left( \frac{1}{A} \right)^{1+\gamma} \times \exp \left( -A(1 + \gamma) \left[ \mu(q-x) + p^f x - c(q) - \frac{1}{2} A(q-x)^2 \left( \sigma^2 + (1 + \gamma)s^2 \right) \right] \right).
\]

We denote by
\[
\Pi^{CE}(q, x) = \mu(q-x) + p^f x - c(q) - \frac{1}{2} A(q-x)^2 \left( \sigma^2 + (1 + \gamma)s^2 \right)
\]
the firm’s certainty-equivalent profit, which takes into account the firm’s risk and ambiguity attitude and the different sources of uncertainty it faces. The maximizers of $V$ coincides with the maximizers of $\Pi^{CE}$ because $V$ is an increasing transformation of $\Pi^{CE}$.

In the absence of a hedging program (i.e., $x = 0$), the level of production that maximizes the firm’s certainty-equivalent profit is implicitly defined by
\[
\mu = Aq^0 \left( \sigma^2 + (1 + \gamma)s^2 \right) + c'(q^0),
\]
where superscript zero indicates the fact that no hedging takes place. The firm’s risk and ambiguity aversion distort the optimal output below the level that would equalize the marginal cost with the expected spot price. Furthermore, it is straightforward to show that optimal
production is decreasing in the firm’s ambiguity aversion, increasing in the firm’s optimism, decreasing in the firm’s pessimism, and decreasing in the level of uncertainty.\footnote{Denoting the second-order condition by \( \Pi_{yy}^{CE} (q^0, 0) = -c'' (q^0) - Aq^0 \left( \sigma^2 + (1 + \gamma) s^2 \right) \), which is negative, the implicit function rule renders
\[
\frac{\partial q^0}{\partial \gamma} = Aq^0 s^2 / \Pi_{yy}^{CE} (q^0, 0) < 0, \quad \frac{\partial q^0}{\partial \mu} = -1 / \Pi_{yy}^{CE} (q^0, 0) > 0 \quad \text{and} \quad \frac{\partial q^0}{\partial s^2} = Aq^0 (1 + \gamma) / \Pi_{yy}^{CE} (q^0, 0) < 0.
\]}

In a next step, we consider a fixed level of production \( q > 0 \) and solve for the firm’s optimal hedging decision. Maximizing \( \Pi^{CE} (q, x) \) with respect to \( x \) yields
\[
x^* = q + \frac{p^f - \mu}{A (\sigma^2 + (1 + \gamma) s^2)},
\]

Firms with a neutral market view hedge their entire production (\( x^* = q \)) whereas firms with a favorable or unfavorable market view underhedge (\( x^* < q \)) or overhedge (\( x^* > q \)), respectively. This helps illustrate why the ranking of the different possible price distributions depends on the firm’s exposure to price uncertainty, which in turn depends on the firm’s decisions. If a firm hedges its entire production, its profit no longer depends on the distribution of the spot price and any distribution is just as good as the other. If a firm underhedges, its profit is positively associated with the realized spot price because the firm is still selling some of its production on the market. However, if a firm overhedges, its profit is negatively associated with the realized spot price because in order to fulfill those forward contracts that are not covered out of own production, the firm needs to buy some of the commodity at date 1 on the market and therefore benefits from lower rather than higher prices.

So a higher \( \mu \) represents increased optimism and a lower \( \mu \) increased pessimism if \( \mu > p^f \), that is, if the firm holds a favorable market view. When the firm’s market view is unfavorable (\( \mu < p^f \)), matters are reversed and a higher \( \mu \) corresponds to increased pessimism and a lower \( \mu \) to increased optimism. The interpretation of the uncertainty parameter \( s \) does not depend on the firm’s market view. We find that the firm’s demand for hedging is increasing (resp. constant, decreasing) in ambiguity aversion and the level of uncertainty for firms with a favorable (resp. neutral, unfavorable) market view. Increased optimism reduces the demand for hedging under a favorable market view but increases it under an unfavorable market view. The reverse is the case for increased pessimism.\footnote{Both the signs of
\[
\frac{\partial x^*}{\partial \gamma} = - \frac{s^2 (p^f - \mu)}{A (\sigma^2 + (1 + \gamma) s^2)^2} \quad \text{and} \quad \frac{\partial x^*}{\partial s^2} = - \frac{(1 + \gamma) (p^f - \mu)}{A (\sigma^2 + (1 + \gamma) s^2)^2}
\]
depend on the sign of \( (p^f - \mu) \), with a positive (resp. zero, negative) effect whenever \( (p^f - \mu) < (\text{resp.} =, >) 0 \).

The sign of
\[
\frac{\partial x^*}{\partial \mu} = - \frac{1}{A^2 (\sigma^2 + (1 + \gamma) s^2)^2}
\]
is always negative but the interpretation of a directional change in \( \mu \) depends on the firm’s market view.}

Figure 1 provides an illustration of Eq. (3).
To the right of $\mu = p^f$ increased optimism reduces the demand for hedging and even more so the lower the level of ambiguity faced by the firm. To the left of $\mu = p^f$ increased optimism increases the demand for hedging, and the effect is again more pronounced at low levels of ambiguity. So the firm’s hedging decisions are more sensitive to optimism and pessimism the lower the level of ambiguity because intuitively the firm is more confident in its beliefs when ambiguity is low rather than high.

Figure 1: Optimal hedging and ambiguity

Note: The underlying parameters are $q = 1,000$, $p^f = 10$, $\sigma = 0.2$ and $\mu \in [8,12]$. We use $s = 0.5$ and $s = 0.3$ to distinguish between a high and low level of ambiguity. We set $A = 0.01$ for the coefficient of absolute risk aversion and $\gamma = 2$ for the coefficient of relative ambiguity aversion.

Finally, when production and hedging decisions are integrated, we obtain the following first-order condition for optimal production:

$$\mu - c'(q^*) - A(q^* - x^*) \left( \sigma^2 + (1 + \gamma)s^2 \right) = 0.$$  

Substituting the optimal demand for hedging $x^*$ from Eq. (3), this simplifies to $c'(q^*) = p^f$, which is the celebrated separation theorem. The firm’s optimal level of production only depends on the prevailing forward price and is independent of risk and ambiguity preferences as well as beliefs. All comparative static results about the optimal demand for hedging continue to hold.

### 2.3 Another example

While the firm was uncertain about the expected spot price in the previous example, we now modify the assumptions to analyze uncertainty about the riskiness of the spot price distribution. Specifically, prices are normally distributed according to $\bar{p} \sim N(\mu, \sqrt{\theta})$. Under
the CARA assumption, the firm’s expected utility is then given by
\[
U(q, x|\theta) = -\frac{1}{A} \exp \left( -A \left[ \mu(q-x) + p^f x - c(q) \right] \right) \cdot \exp \left( \frac{1}{2} A^2(q-x)^2 \theta \right).
\]

For analytical tractability, we let the uncertain variance \(\tilde{\theta}\) take two different values with equal chance, \(\sigma^2 + s\) and \(\sigma^2 - s\) for \(s \in (0, \sigma^2)\). This results in two possible price distributions, \(N(\mu, \sqrt{\sigma^2 + s})\) and \(N(\mu, \sqrt{\sigma^2 - s})\), where the first one is riskier than the second one in the sense of Rothschild and Stiglitz (1970). Changes in \(\sigma^2\) can be interpreted as changes in optimism or pessimism while \(s\) is a measure of uncertainty (notice that \(\text{Var}(\tilde{\theta}) = s^2\)).

Under iso-elastic ambiguity preferences, we obtain the following objective function:
\[
V(q, x) = -\frac{1}{1+\gamma} \left( \frac{1}{A} \right)^{1+\gamma} \exp \left( -A(1+\gamma) \left[ \mu(q-x) + p^f x - c(q) \right] \right)
\times \left( \frac{1}{2} \exp \left( \frac{1}{2} A^2(1+\gamma)(q-x)^2(\sigma^2-s) \right) + \frac{1}{2} \exp \left( \frac{1}{2} A^2(1+\gamma)(q-x)^2(\sigma^2+s) \right) \right).
\]

This implies a certainty-equivalent profit of
\[
\Pi^{CE}(q, x) = \mu(q-x) + p^f x - c(q) - \frac{1}{2} A(q-x)^2\sigma^2 - \frac{1}{A(1+\gamma)} \ln \left( \cosh \left( \frac{1}{2} A(1+\gamma)(q-x)^2s \right) \right),
\]
where \(\cosh\) denotes the hyperbolic cosine.\(^6\)

In the absence of a hedging program (i.e., \(x = 0\)), the firm maximizes its certainty-equivalent profit when production satisfies the following first-order condition:
\[
\mu = c'(z^0) + Aq^0 \left( \sigma^2 + s \cdot \tanh(z^0) \right),
\]
where \(z^0\) is shorthand for \(\frac{1}{2}(Aq^0)^2(1+\gamma)s\). Under our assumptions, \(z^0\) is positive and so is \(\tanh(z^0)\). As in the previous example, the firm’s risk and ambiguity aversion reduce the level of production below the level that sets the marginal cost equal to the expected spot price. The firm’s production is decreasing in ambiguity aversion, increasing in optimism, decreasing in pessimism, and decreasing in the level of uncertainty.\(^7\) Therefore, the comparative statics have the same qualitative behavior as in the example of uncertainty about the expected spot price.

\(^6\) The hyperbolic functions are analogs of the ordinary trigonometric functions. The hyperbolic sine is the odd part of the exponential function, \(\sinh(z) = (e^z - e^{-z})/2\), the hyperbolic cosine the even part of the exponential function, \(\cosh(z) = (e^z + e^{-z})/2\), and the hyperbolic tangent is derived from the basic hyperbolic functions according to \(\tanh(z) = \sinh(z)/\cosh(z)\) for \(z \in \mathbb{R}\).

\(^7\) The second-order condition is given by
\[
\Pi^{CE}_{q\gamma}(q^0, 0) = -c''(z^0) - A \left( \sigma^2 + s \cdot \tanh(z^0) \right) - A^3(1+\gamma)(q^0 s)^2/\cosh^2(z^0) < 0.
\]
From the implicit function rule we derive
\[
\frac{\partial q^0}{\partial \gamma} = \frac{1}{2}(Aq^0)^3 s^2/\cosh^2(z^0) \frac{\Pi^{CE}_{q\gamma}(q^0, 0)}{\Pi^{CE}_{q\gamma}(q^0, 0)} < 0, \quad \frac{\partial q^0}{\partial \sigma^2} = \frac{Aq^0}{\Pi^{CE}_{q\gamma}(q^0, 0)} < 0.
\]
If the firm hedges a fixed level of production $q > 0$, we obtain its demand for hedging as the implicit solution to the following first-order condition:

$$
\mu = p^f + A(q - x^*) \left( \sigma^2 + s \cdot \tanh(z^*) \right),
$$

where $z^* = \frac{1}{2} A^2 (1 + \gamma) (q - x^*)^2 s$. The value of $z^*$ is always nonnegative and strictly positive unless the firm employs a full hedge. As before, the firm’s market view determines whether an underhedge, full hedge or overhedge is optimal because the large round bracket is strictly positive so that the sign of $(\mu - p^f)$ coincides with that of $(q - x^*)$. The firm’s demand for hedging is increasing (resp. constant, decreasing) in ambiguity aversion and the level of uncertainty for firms that underhedge (resp. fully hedge, overhedge). When it comes to optimism and pessimism, the effect also depends on whether the firm holds a favorable or an unfavorable market view. Under a favorable market view, the firm underhedges and optimism lowers the demand for hedging. If the firm holds an unfavorable market view, it overhedges and optimism increases the demand for hedging. These effects are consistent with the example of uncertainty over the expected spot price.

When production and hedging decisions are integrated, the first-order condition for optimal production is

$$
\mu = c'(q^*) + A(q^* - x^*) \left( \sigma^2 + s \cdot \tanh(z^*) \right). \tag{5}
$$

As before, this together with the first-order condition for hedging implies the separation theorem, $c'(q^*) = \mu$, and the optimal level of production is independent of the firm’s beliefs, risk and ambiguity preferences.

and

$$
\frac{\partial q^0}{\partial s} = \frac{Aq^0 \tanh(z^0) + \frac{1}{2} (Aq^0)^3 (1 + \gamma) s / \cosh^2(z^0)}{\Pi^{CE}_{qq}(q^0, 0)} < 0.
$$

$^8$ We denote the second-order condition by

$$
\Pi^{CE}_{xx}(q, x^*) = -A \left( \sigma^2 + s \cdot \tanh(z^*) \right) - A^2 (1 + \gamma) (q - x^*)^2 s^2 / \cosh^2(z^*) < 0.
$$

From the implicit function rule, we obtain

$$
\frac{\partial x^*}{\partial \gamma} = -\frac{1}{2} A^3 (q - x^*)^3 s^2 / \cosh^2(z^*) \cdot \frac{\Pi^{CE}_{xx}(q, x^*)}{\Pi^{CE}_{xx}(q, x^*)}, \quad \frac{\partial x^*}{\partial \sigma^2} = -\frac{A(q - x^*)}{\Pi^{CE}_{xx}(q, x^*)}
$$

and

$$
\frac{\partial x^*}{\partial s} = -\frac{A(q - x^*) \tanh(z^*) + \frac{1}{2} A^3 (1 + \gamma) (q - x^*)^3 s / \cosh^2(z)}{\Pi^{CE}_{xx}(q, x^*)}.
$$

In each case, the sign coincides with that of $(q - x^*)$. mortar.
3 Unhedged production decisions

We now return to the general model as outlined in Section 2.1. We first consider the role of the firm’s belief for production decisions in the absence of a hedging program. In terms of our model, we therefore set \( x = 0 \) in (2) and suppress \( x \) in our notation. Using subscripts to denote partial derivatives, we obtain the following first-order condition associated with the firm’s production objective:

\[
V_q(q^0|G) = \int_{\theta} \varphi' (U(q^0|\theta)U_q(q^0|\theta) \, dG(\theta) = 0, \tag{6}
\]

where \( q^0 \) denotes the optimal level of production. Given that it is costless at the margin to initiate production, some production is always optimal \( (q^0 > 0) \), and the strict concavity of the objective function warrants a unique interior solution.

Under assumption A1, we can distinguish between two scenarios. Let \( q \) and \( \bar{q} \) be the optimal production decisions under price distribution \( F(p) \) and \( \bar{F}(p) \), respectively, that is,

\[
q = \arg \max_q U(q|\theta) \quad \text{and} \quad \bar{q} = \arg \max_q U(q|\bar{\theta}).
\]

If \( q = \bar{q} \), uncertainty over the price distribution is inconsequential because any possible distribution implies the same optimal level of production. Therefore, a prerequisite for beliefs to matter is that \( q \neq \bar{q} \). In this case, two scenarios are possible.

A2. We speak of **concordant uncertainty** if \( \bar{q} > q \).

A2’. We speak of **discordant uncertainty** if \( \bar{q} < q \).

In the concordant case, the better distribution is associated with a higher level of production making the behavioral effect of uncertainty concordant with its effect on profitability. In the discordant case, matters are reversed and the better distribution implies a reduced level of production. Intuitively, one might think that only the concordant case is relevant. However, a change in the price distribution from \( F(p) \) to \( \bar{F}(p) \) induces an associated change in the distribution of profits at a given level of production. Under this new profit distribution, the firm might be more sensitive to risk and prefer to reduce its exposure via lowering \( q \). If this income effect dominates the substitution effect, optimal production decreases as the price distribution improves. In Appendix B.1, we discuss conditions under which uncertainty is concordant or discordant.

In a first step, we will revisit some existing results on comparative ambiguity aversion. This will allow us to determine the effect of ambiguity at the extensive margin. We state the following definition

**Definition 3** (Klibanoff et al., 2005). A firm is **more ambiguity-averse** than another one if they share the same vNM utility function, the same belief, and if the ambiguity function of the former is more concave than the ambiguity function of the latter.
We can then operationalize an increase in ambiguity aversion according to Pratt's (1964) approach to comparative risk aversion. If \( \psi \) and \( \varphi \) represent the ambiguity preferences of two different firms with the same utility function and the same belief, \( \psi \) represents greater ambiguity aversion than \( \varphi \) if and only if \( \psi(U) = k(\varphi(U)) \) for an increasing and concave transformation function \( k \) for any \( U \) in the relevant domain. When applied to the production decision of the competitive firm under price uncertainty, we obtain the following result from the literature.

**Proposition 1 (Wong 2015b)**. Greater ambiguity aversion reduces production if and only if the covariance between \( k'(\varphi(U(q^0|\theta))) \) and \( \varphi'(U(q^0|\theta))U_q(q^0|\theta) \) is negative.

This is Wong’s (2015b) Proposition 4. When applied to the distinction between concordant and discordant uncertainty, we obtain the following corollary.

**Corollary 1.** Under assumption A1, greater ambiguity aversion:

(i) Lowers production for concordant uncertainty.

(ii) Raises production for discordant uncertainty.

We provide a proof in Appendix A.2. The intuition derives from Gollier’s (2011) interpretation in terms of observational equivalence. We can rewrite the first-order condition (6) as follows:

\[
V_q(q^0|G) = \int_\theta^\theta \xi(\theta)U_q(q^0|\theta) \, dG(\theta) = 0, \quad \text{with} \quad \xi(\theta) = \frac{\varphi'(U_q(q^0|\theta))}{\int_\theta^\theta \varphi'(U_q(q^0|\theta)) \, dG(\theta)}.
\]

Now \( \xi(\theta) \) is a Radon-Nikodym derivative describing the distortion in the firm’s belief introduced by ambiguity aversion. As such the behavior of the ambiguity-averse firm coincides with that of a subjective expected-utility maximizing firm with a distorted belief and is therefore observationally equivalent. The firm’s distorted belief is more pessimistic than its undistorted belief, and the degree of pessimism is positively associated with the firm’s degree of ambiguity aversion. But worse price distributions are associated with lower (resp. higher) production levels in the concordant (resp. discordant) case, thus explaining Corollary 1. Wong’s (2015b) Propositions 5 and 6 both illustrate the concordant case and are thus consistent with our Corollary 1(i).

We can also use Corollary 1 to derive the effect of ambiguity on production at the extensive margin. A special case of greater ambiguity aversion is the comparison between an ambiguity-averse firm and an ambiguity-neutral firm. Under the smooth ambiguity model, the behavior of an ambiguity-neutral firm in the presence of ambiguity coincides with the behavior of an ambiguity-averse firm in the absence of ambiguity. Hence, the effect of ambiguity at the extensive margin coincides with the effect of greater ambiguity aversion at the intensive margin, which yields the following result.
Corollary 2. Under assumption A1, ambiguity:

(i) Lowers production for concordant uncertainty.

(ii) Raises production for discordant uncertainty.

Wong’s (2015b) Propositions 2 and 3 both illustrate the concordant case and are therefore in line with Corollary 2(i). We emphasize that no assumption other than ambiguity aversion is required to obtain this result. While the extensive margin addresses the comparison of a situation without ambiguity and a situation with ambiguity, it does not inform about the more realistic case where ambiguity is an inevitable part of the firm’s decision-making environment but some firm’s might perceive a greater level of uncertainty than others. Likewise, Corollary 2 does not cover different levels of optimism and pessimism because the expected commodity price under the expected price distribution (i.e., under \( \int_\theta F(p(\theta)) dG(\theta) \)) coincides with \( p_{\text{exp}} \) due to Fubini’s Theorem. To formulate our results on changes in ambiguity, we first define two measures of the firm’s ambiguity preferences.

Definition 4. Consider a firm with ambiguity preferences represented by \( \varphi \).

a) The firm’s relative ambiguity aversion is given by \( R(U) = -U \varphi''(U)/\varphi'(U) \).

b) The firm’s relative ambiguity prudence is given by \( P(U) = -U \varphi'''(U)/\varphi''(U) \).

Relative ambiguity aversion is the utility-elasticity of the marginal ambiguity function \( \varphi'(U) \) whereas relative ambiguity prudence is the utility-elasticity of the ambiguity function’s second derivative \( \varphi''(U) \). These measures are reminiscent of their counterparts in risk theory and are useful to determine the effects of changes in ambiguity on optimal behavior. For example, Huang and Tzeng (2018) apply them to portfolio choice, Peter and Ying (2019) to insurance demand under nonperformance risk and Peter (2019) to precautionary saving. We then obtain the following results.

Proposition 2. Consider the competitive firm under price uncertainty and assume that A1 holds. Then:

(i) A more optimistic firm raises (resp. lowers) production under concordant (resp. discordant) uncertainty if relative ambiguity aversion is bounded by unity.

(ii) A more pessimistic firm lowers (resp. raises) production under concordant (resp. discordant) uncertainty if relative ambiguity aversion is bounded by unity.

(iii) Greater ambiguity lowers (resp. raises) production under concordant (resp. discordant) uncertainty if relative ambiguity prudence is bounded by 2.

We give a formal proof in Appendix A.3. Comparing Proposition 2(iii) and Corollary C2, we see that an additional assumption is needed to sign the effect of ambiguity at the intensive rather than the extensive margin. To provide intuition for this result, we point out
the presence of two underlying effects as ambiguity increases. Consider the case of concordant uncertainty; if the level of ambiguity increases, there is a negative substitution effect, which incentivizes the firm to produce less in order to save on costs. This is the obvious effect of greater ambiguity because we would expect the heightened uncertainty about the future spot price to compromise production. However, there is also a potentially conflicting precautionary effect. Greater ambiguity increases the dispersion in the level of expected utility. For reasons of precaution, the firm will then aspire to take action that leads to an increase in expected utility on average. In the concordant case we have $q > q^0 > q$ so that $q^0$ is too high for low values of $\theta$ and too low for high values of $\theta$. Hence, the firm experiences an incentive to lower production when $\theta$ is low, consistent with the negative substitution effect, and an incentive to raise production when $\theta$ is high, which countervails the substitution effect. The restriction on relative ambiguity prudence ensures for the substitution effect to dominate, resulting in a negative net effect.

A similar intuition applies to changes in optimism and pessimism. Take again the case of concordant uncertainty; if the firm becomes more optimistic, there is a positive substitution effect because a more promising outlook on spot market prices encourages production. On the other hand, there is an ambiguity aversion effect because under the more optimistic belief the anticipated levels of expected utility are better in an FSD sense. Under ambiguity aversion, the marginal ambiguity function is diminishing. Therefore, the marginal value of additional profit and hence increased expected utility is high when expected utility is low and low when it is high. So the FSD shift encourages production for low values of $\theta$ and discourages production for high values of $\theta$. The upper bound on relative ambiguity aversion ensures that the substitution effect dominates.

The preferences conditions stated in Proposition 2 are sufficient but not necessary. Take the case of greater ambiguity; if relative ambiguity prudence does not exceed 2, greater ambiguity has the intuitive effect which is to lower production in the concordant case and to increase production in the discordant case. If a firm’s relative ambiguity prudence exceeds 2, the effect of greater ambiguity may or may not be as intuitively expected. Said differently, some firms with relative ambiguity prudence above 2 will reduce production in the concordant case while others will increase it. So at the firm level higher production may not indicate optimism and lower production may not indicate more uncertainty unless the restrictions on preference stated in Proposition 2 are satisfied. Berger and Bosetti (2019) conduct an experiment with students and a panel of policymakers and find relative ambiguity aversion of 0.53, which is below unity. They also find evidence of constant relative ambiguity aversion, in which case the estimate of relative ambiguity aversion implies relative ambiguity prudence of 1.53. So the existing

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Jouini et al. (2013) identify the necessary and sufficient conditions for changes in risk to have definitive comparative static effects on behavior. In their analysis of payoff risk, a similar situation arises that those consumers whose preferences violate the restriction on relative risk aversion will exhibit the reverse behavior for some initial endowment levels.
evidence supports our assumptions on ambiguity preference, although we recognize that field evidence about the size of relative ambiguity attitudes as well as evidence on ambiguity attitudes by firms is needed.

4 The demand for hedging

4.1 A fixed output level

In this section, we consider the firm’s hedging decisions. We start with a fixed level of production $q > 0$ and suppress $q$ in our notation. The following first-order condition characterizes the firm’s hedging behavior,

$$V_x(x^*|G) = \int_\theta ^\bar{\theta} \varphi'(U(x^*|\theta))U_x(x^*|\theta)\,dG(\theta) = 0,$$

(7)

with $x^*$ being shorthand for the optimal demand for hedging. The effect of hedging on expected utility is given by

$$U_x(x^*|\theta) = \int_p ^\bar{p} u'(\Pi(x^*|p))(p^f - p)\,dF(p|\theta).$$

Hedging increases profit and raises expected utility when spot prices are below the forward price, which we can think of as the marginal utility benefit of hedging. On the other hand, if spot prices exceed the forward price, hedging decreases profit and lowers expected utility, corresponding to the marginal utility cost of hedging. This trade-off is resolved differently depending on the spot price distribution $F(p|\theta)$, and Eq. (7) states that the firm strikes a balance across all the possible spot price distributions while taking into account its sensitivity to ambiguity.

If the firm adopts a full hedge ($x = q$), its profit simplifies to $p^f q - c(q)$ and no longer depends on the realized spot price because none of the commodity is sold or purchased on the spot market at date 1. Consequently, the firm’s profit and its expected utility do not depend on the distribution of spot prices either. The first-order expression at $x = q$ then simplifies to

$$\varphi'(u(p^f q - c(q)))u'(p^f q - c(q)) \cdot \left[ p^f - \int_\theta ^\bar{\theta} \int_p ^\bar{p} p\,dF(p|\theta)\,dG(\theta) \right],$$

where the square bracket simplifies to $p^f - \bar{p}_G^{exp}$. Together with the concavity of $V$ in $x$, this implies the following result.

\[\text{Baillon and Placido (2019) find evidence of decreasing relative ambiguity aversion, in which case relative prudence exceeds relative ambiguity aversion by more than one. Then, Berger and Bosetti’s point estimate no longer implies relative ambiguity prudence below 2 but does not contradict with it either. It depends on how fast relative ambiguity aversion decreases.}\]
Proposition 3. The competitive firm under price uncertainty:

(i) underhedges \((x^* < q)\) if it holds a favorable market view;

(ii) employs a full hedge \((x^* = q)\) if it holds a neutral market view;

(iii) overhedges \((x^* > q)\) if it holds an unfavorable market view.

Proposition 3 is similar to Iwaki and Osaki’s (2012) Theorem 2. They assume that the firm’s market view is neutral and distinguish between futures markets exhibiting normal backwardation, unbiasedness and contango, respectively. We, in turn, allow for different market views but keep the operation of the futures market fixed. Against the background of decomposition (1), it is clear that different factors can drive a wedge between the firm’s expected market price \(p_{exp}^G\) and the prevailing futures price \(p^f\). Therefore, our Proposition 3 corresponds to Iwaki and Osaki’s Theorem 2.

In each of the three cases, underhedging, full hedging and overhedging, we can now investigate the effect of greater ambiguity aversion, increased optimism and pessimism, and greater ambiguity. As in Section 3, we distinguish between concordant and discordant uncertainty. More formally, let

\[
\begin{align*}
x &= \arg\max_x U(x|\theta) \quad \text{and} \quad \bar{x} = \arg\max_x U(x|\bar{\theta})
\end{align*}
\]

be the demand for hedging under the best and the worst possible CDF. We can then make the following distinction.

A3. We speak of concordant uncertainty if \(\bar{x} > x\).

A3’. We speak of discordant uncertainty if \(\bar{x} < x\).

Similar to our study of optimal production decisions, the exposure to price uncertainty is endogenous and depends on the firm’s demand for hedging. However, unlike in case of production, also the direction of the exposure is endogenous. Assume, for example, that the family \(\{F(p|\theta)\}_{\theta}\) is ordered according to FSD with some distributions implying an expected spot price below \(p^f\) and others implying an expected spot price above \(p^f\). If the firm’s belief results in a favorable market view, it underhedges per Proposition 3(i) and it prefers higher prices over lower prices. Then, \(\theta\) orders the CDFs in ascending order if \(\mathcal{F}(p)\) dominates \(\mathcal{F}(p)\) in the sense of FSD. If, however, the firm’s belief results in an unfavorable market view, it overhedges according to Proposition 3(iii) and it prefers lower prices over higher prices. In order to satisfy the futures contracts the firm will have to buy some of the commodity on the spot market at date 1 so that a lower rather than higher price increases profit. In this case, \(\theta\) orders the CDFs in ascending order if \(\mathcal{F}(p)\) has FSD over \(\mathcal{F}(p)\). In Appendix 3.2, we identify conditions for concordant or discordant uncertainty. Here is our next result.

Proposition 4. Consider the competitive firm under price uncertainty who hedges a fixed level of production. Under assumption A1, greater ambiguity aversion:
(i) Raises (resp. lowers) the demand for hedging under concordant (resp. discordant) uncertainty when the firm’s market view is favorable.

(ii) Does not affect the demand for hedging when the firm’s market view is neutral.

(iii) Lowers (resp. raises) the demand for hedging under concordant (resp. discordant) uncertainty when the firm’s market view is unfavorable.

A proof is given in Appendix [A.4]. Applying the rule of the covariance to $V_x(q,x^*|G)$ allows us to establish a relationship between the amount that is hedged by the firm, depending on the firms expected output price.

$$V_x(q,x^*|G) = \int_\theta \varphi'(U(q,x^*|\theta)) \left( \text{Cov}[u'(\Pi), (p_f - p)] + \int_p u'(\Pi) dF(p|\theta) \int_p (p_f - p) dF(p|\theta) \right) dG(\theta),$$

$$= \int_\theta \varphi'(U(q,x^*|\theta)) \int_p u'(\Pi) dF(p|\theta) \int_p (p_f - p) dF(p|\theta) dG(\theta)$$

$$+ \int_\theta \varphi'(U(q,x^*|\theta)) \text{Cov}[u'(\Pi), (p_f - p)] dG(\theta).$$ (8)

Equation 8 allows us to analysis the relationship between the produced output, the amount hedge and the expected spot and forward price. This leads to the following proposition

We can directly infer from proposition [3] the sign of the $V_x(q,x^*|G)$ in each case. The above proposition allows us to examine the effect of an increase in ambiguity aversion on the demand for hedging. Let $x^0$ denote the optimal demand for hedging for the less ambiguity averse firm. The first-order condition of the more ambiguity averse firm, evaluated at the optimal level of hedging ($x^*$) is then given by

$$V_x(q,x^*|G) = \int_\theta k'(\varphi(U(q,x^*|\theta))) \varphi'(U(q,x^*|\theta))U_x(q,x^*|\theta) dG(\theta) = 0.$$ (9)

Applying the covariance rule and evaluating the first-order condition of the more ambiguous firm at the optimal level of hedge of the less ambiguous firm yields

$$V_x(q,x^*|G) = \text{Cov} \left[ k'(\varphi(U(q,x^0|\theta))), \varphi'(U(q,x^0|\theta)U_x(q,x^0|\theta)) \right]$$

$$+ \int_\theta k'(\varphi(U(q,x^0|\theta))) dG(\theta) \int_\theta \varphi'(U(q,x^0|\theta)U_x(q,x^0|\theta)) dG(\theta),$$ (10)

and by making use of the fact that $V_{x^0} = 0$, we denote

$$V_x(q,x^*|G) = \text{Cov} \left[ k'(\varphi(U(q,x^0|\theta))), \varphi'(U(q,x^0|\theta)U_x(q,x^0|\theta)) \right],$$ (11)
and hence, whether and increase in ambiguity reduces demand for hedging depends on the sign of the covariance.

\[
\text{sign} \{ \text{Cov}[k'(\varphi(U(q,x^0|\theta))), \varphi'(U(q,x^0|\theta)U_x(q,x^0|\theta))] \} \leftrightarrow \text{sign} \{ x^* - x^0 \} \tag{12}
\]

The first term in the covariance \( k'(\varphi(U(q,x^0|\theta))) \) is decreasing in \( \theta \) due to ambiguity aversion. Moreover, \( \varphi'(U(q,x^0|\theta)) \) is also decreasing in \( \theta \) also due to ambiguity aversion. Hence the sign of the covariance depends on the reaction of the marginal utility of hedging to an increase in ambiguity, \( U_x(q,x^0|\theta) \). As we differentiate uncertainty into a concordant and a discordant case, the sign of \( U_{x\theta} \) will depend on the scenario and hence differs from the analysis in Iwaki and Osaki (2012). There are three possible cases depending on the firm’s expectations about the expected spot price. Considering the first case, when the firm has realistic beliefs it chooses always to full hedge, hence any change or increase in ambiguity does not change the result that the firm will choose a full hedge.

We start the analysis with the scenario when the firm is optimistic, \( p^{exp} > p^f \) and the firm under-hedges. From proposition 3 we know that \( U_x < 0 \). Hence, in the concordant case \( U_{x\theta} < 0 \) and the covariance is negative. Therefore, an increase in ambiguity decreases the demand for hedging. In the discordant case, signs change and the demand for hedging increases. Considering now the second case, when the firm is pessimistic, \( p^{exp} < p^f \) and the firm over-hedges. From proposition 3 we infer that \( U_x > 0 \). In the concordant case \( U_{x\theta} > 0 \) and the covariance is positive. Hence, an increase in ambiguity aversion increases the demand for hedging. For the discordant case, signs change and the demand for decreases for hedging.

Table 1 summarizes the previous results for \( U_x \) and \( U_{x\theta} \) and the implication for the demand for hedging.

<table>
<thead>
<tr>
<th>Expectation</th>
<th>Decision</th>
<th>( U_x ) concordant</th>
<th>( U_x ) discordant</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p^{exp} \geq p^f )</td>
<td>( x^* = q )</td>
<td>( = 0 )</td>
<td>( = 0 ) const.</td>
</tr>
<tr>
<td>( p^{exp} &gt; p^f )</td>
<td>( x^* &lt; q )</td>
<td>( &lt; 0 )</td>
<td>( &lt; 0 )</td>
</tr>
<tr>
<td>( p^{exp} &lt; p^f )</td>
<td>( x^* &gt; q )</td>
<td>( &gt; 0 )</td>
<td>( &gt; 0 ) +</td>
</tr>
</tbody>
</table>

Note: (+) indicates an increase in the demand for hedging and (−) indicates a decrease for the demand of hedging.

5 Conclusion

In this paper, we study optimal production and the demand for hedging of a risk- and ambiguity-averse firm. Specifically, we analyze how the firm incorporates its market view into those decisions. We identify different economic variables that matter in this regard. First, the
distinction between concordant and discordant uncertainty determines the intuitive direction of optimism and ambiguity on production and hedging decisions. At the extensive margin, this intuition prevails without any additional presuppositions. At the intensive margin instead, preferences need to be restricted to sign the comparative statics. Thus, our results provide testable hypothesis on firm-level determinants of the demand for hedging.
References


Appendix A Mathematical proofs

A.1 Concavity of objective function \([2]\)

Using subscripts to denote partial derivatives, we obtain \(\Pi_q = p - c'(q), \Pi_{qq} = -c''(q), \Pi_x = p^f - p, \Pi_{xx} = 0\) and \(\Pi_{qy} = 0\). Therefore, for any \(p \in [\underline{p}, \overline{p}]\), \(\Pi\) is strictly concave in \(q\), linear in \(x\) and concave in \((q, x)\). Hence, \(U\) is concave in \((q, x)\) for any \(\theta \in [\underline{\theta}, \overline{\theta}]\) because \(u\) is concave and the expectation respects monotonicity. Likewise, \(V\) is concave in \((q, x)\) for any belief \(G\) due to the concavity of \(\varphi\) and the monotonicity of the expectation operator. If either \(u\) or \(\varphi\) is strictly concave, so is \(V\).

A.2 Proof of Corollary \([1]\)

From Proposition \([1]\) we know that the sign of the covariance between \(k'(\varphi(U(q^*|\theta)))\) and \(\varphi'(U(q^*|\theta))U_q(q^*|\theta)\) determines whether greater ambiguity aversion increases or decreases production. Due to the first-order condition \([\theta]\), we arrive at

\[
\text{Cov} [k'(\varphi(U(q^*|\theta))), \varphi'(U(q^*|\theta))U_q(q^*|\theta)] = \int_{\theta} \varphi'(U(q^*|\theta))U_q(q^*|\theta) \, dG(\theta),
\]

which is the first-order condition of the more ambiguity-averse firm evaluated at the optimal level of production of the less ambiguity-averse firm. We denote the integrand above by \(\zeta(\theta)\), that is, \(\zeta(\theta) = k'(\varphi(U(q^*|\theta)))\varphi'(U(q^*|\theta))U_q(q^*|\theta)\).

Under assumption A1, we obtain

\[
U_q(q^*|\theta) = \frac{\overline{\theta} - \theta}{\overline{\theta} - \underline{\theta}} \cdot U_q(q^*|\overline{\theta}) + \frac{\theta - \underline{\theta}}{\overline{\theta} - \underline{\theta}} \cdot U_q(q^*|\underline{\theta})
\]

and

\[
U_{q\theta}(q^*|\theta) = U_q(q^*|\overline{\theta}) - U_q(q^*|\underline{\theta}).
\]

Optimal production \(q^*\) under belief \(G\) lies between \(\underline{q}\) and \(\overline{q}\), and strictly so unless \(G\) is the CDF of the Dirac measure \(\delta_{\underline{q}}\) or \(\delta_{\overline{q}}\) (i.e., the firm’s belief is maximally pessimistic or optimistic)\([11]\).

Therefore, we have \(U_{q\theta}(q^*|\theta) > 0\) for all \(\theta\) in the concordant case and \(U_{q\theta}(q^*|\theta) < 0\) for all \(\theta\) in the discordant case. But the strict monotonicity of \(U_q(q^*|\theta)\) in \(\theta\) together with first-order condition \([\theta]\) imply the existence of a \(\hat{\theta} \in (\underline{\theta}, \overline{\theta})\) where \(U_q(q^*|\theta)\) switches sign. We can use \(\hat{\theta}\) to decompose \(\zeta(\theta)\) according to

\[
\zeta(\theta) = 1_{[\underline{\theta}, \hat{\theta})}(\theta) \cdot \zeta(\theta) + 1_{[\hat{\theta}, \overline{\theta})}(\theta) \cdot \zeta(\theta),
\]

\([11]\) Under concordant uncertainty \((\overline{q} > \underline{q})\), we obtain \(V_q(q|G) \geq 0\) and \(V_q(\overline{q}|G) \leq 0\) with both inequalities strict unless the firm is maximally pessimistic or optimistic. Therefore, \(q \leq q^* \leq \overline{q}\) with strict inequalities for firms who do not have extreme beliefs. The argument for discordant uncertainty is analogous.
where \(1_{\Theta}(\theta)\) abbreviates the indicator function for \(\Theta \subseteq [\theta, \theta]\). In the concordant case, the concavity of \(k\) implies

\[
1_{[\theta, \hat{\theta}]}(\theta) \cdot \zeta(\theta) < k'(\varphi(U(q^*|\hat{\theta}))) \cdot 1_{[\theta, \hat{\theta}]}(\theta) \varphi'(U(q^*|\theta))U_q(q^*|\theta)
\]

and

\[
1_{[\hat{\theta}, \theta]}(\theta) \cdot \zeta(\theta) \leq k'(\varphi(U(q^*|\hat{\theta}))) \cdot 1_{[\hat{\theta}, \theta]}(\theta) \varphi'(U(q^*|\theta))U_q(q^*|\theta).
\]

So that

\[
\int_{\theta}^{\hat{\theta}} \zeta(\theta) \, dG(\theta) \leq k'(\varphi(U(q^*|\hat{\theta}))) \int_{\theta}^{\hat{\theta}} \varphi'(U(q^*|\theta))U_q(q^*|\theta) \, dG(\theta) = 0,
\]

with a strict inequality unless \(G\) is the CDF of the Dirac measure \(\delta_{\hat{\theta}}\). Therefore, optimal production decreases. In the discordant case, the inequalities are reversed and optimal production increases.

### A.3 Proof of Proposition 2

We introduce auxiliary function \(g(\theta) = \varphi'(U(q^*|\theta))U_q(q^*|\theta)\), which allows us to rewrite the firm’s first-order condition (6) as \(\int_{\theta}^{\hat{\theta}} g(\theta) \, dG(\theta) = 0\). As shown by Rothschild and Stiglitz (1970) and more generally by Ekern (1980), the signs of subsequent derivatives of \(g(\theta)\) inform about how changes in the firm’s belief \(G\) affect the first-order condition. The first and second derivatives of \(g(\theta)\) are given by

\[
g'(\theta) = \varphi''(U(q^*|\theta))U_\theta(q^*|\theta)U_q(q^*|\theta) + \varphi'(U(q^*|\theta))U_\theta q(\theta)\]

and

\[
g''(\theta) = \varphi'''(U(q^*|\theta))U_\theta^2(q^*|\theta)U_q(q^*|\theta) + 2\varphi''(U(q^*|\theta))U_\theta q(\theta)U_\theta q(\theta)
\]

because \(U_\theta q(\theta) = U_\theta q(\theta) = 0\) for all \(\theta \in [\theta, \theta]\) due to the linearity assumption A1.

To sign those derivatives, we also introduce the following auxiliary function:

\[
h(\theta) = \frac{U_\theta(q^*|\theta)U_q(q^*|\theta)}{U(q^*|\theta)U_\theta q(q^*|\theta)}.
\]

Given that utility function \(u(\Pi)\) is unique up to increasing affine transformations (see Neilson 2010 Theorem 1), we can assume \(U(q^*|\theta)\) to be positive without loss of generality. Furthermore, \(U_\theta(q^*|\theta) > 0\) for all \(\theta \in [\theta, \theta]\) because \(\mathcal{F}\) is preferred over \(\hat{\mathcal{F}}\). In the concordant case, we have \(U_\theta q(\theta) > 0\) for all \(\theta \in [\theta, \theta]\) so that \(U_q(q^*|\theta)\) switches from positive to negative at \(\theta = \hat{\theta}\). In the discordant case, we have \(U_\theta q(\theta) < 0\) for all \(\theta \in [\theta, \theta]\) so that \(U_q(q^*|\theta)\) switches from negative to positive at \(\theta = \hat{\theta}\). In either case, \(h(\theta)\) is negative for \(\theta < \hat{\theta}\), zero for \(\theta = \hat{\theta}\), and positive for \(\theta > \hat{\theta}\). We can then show that \(h(\theta)\) does not exceed unity whenever it is positive. Assume it did so that we can find \(\theta' > \hat{\theta}\) with \(h(\theta') > 1\). Due to the continuity of \(h(\theta)\), there must then be a \(\theta'' > \hat{\theta}\) with \(h(\theta'') = 1\), \(h(\theta) \geq 1\) for all \(\theta \in [\theta'', \theta']\) and \(h(\theta) > 1\) for some
\[ \theta \in [\theta'', \theta'] \] with positive Lebesgue measure. From the fundamental theorem of calculus, we obtain \( h(\theta') - h(\theta'') = \int_{\theta''}^{\theta'} h'(\theta) \, d\theta \). The derivative of \( h(\theta) \) is given by

\[
h'(\theta) = \frac{U_\theta(q^*|\theta) U_{q^\theta}(q^*|\theta) \cdot [U_{q^\theta}(q^*|\theta) U(q^*|\theta) - U_\theta(q^*|\theta) U_q(q^*|\theta) ]}{U(q^*|\theta)^2 U_{q^\theta}(q^*|\theta)^2},
\]

with the square bracket being nonpositive in the concordant case and nonnegative in the discordant case for those \( \theta \) with \( h(\theta) \geq 1 \). Consequently, \( h'(\theta) \) is nonpositive on \([\theta'', \theta']\) and strictly positive for some \( \theta \in [\theta'', \theta'] \) with positive Lebesgue measure. Hence, \( \int_{\theta''}^{\theta'} h'(\theta) \, d\theta < 0 \) whereas \( h(\theta') - h(\theta'') > 0 \) because \( h(\theta') > 1 \) and \( h(\theta'') = 1 \). This yields a contradiction so that \( h(\theta) \leq 1 \) holds uniformly on \([\theta, \theta]\).

Returning to auxiliary function \( g(\theta) \), we can now rewrite \( g'(\theta) \) and \( g''(\theta) \) as

\[
g'(\theta) = -\varphi'(U(q^*|\theta)) U_{q^\theta}(q^*|\theta) \left\{ \frac{\mathcal{R}(U(q^*|\theta)) \cdot h(\theta) - 1}{\leq 1} \right\}_{<0}
\]

and

\[
g''(\theta) = -\varphi''(U(q^*|\theta)) U_{q^\theta}(q^*|\theta) U_{q^\theta}(q^*|\theta) \left\{ \frac{\mathcal{P}(U(q^*|\theta)) \cdot h(\theta) - 2}{\leq 2} \right\}_{\leq 1}
\]

respectively. So if relative ambiguity aversion is bounded by unity and the uncertainty is concordant (i.e., if \( U_{q^\theta}(q^*|\theta) > 0 \)), then \( g'(\theta) \) is nonnegative for all \( \theta \in [\theta, \theta] \) and strictly positive for some \( \theta \in [\theta, \theta] \) with positive Lebesgue measure. In this case, an FSD improvement of \( G \) raises \( \int_{\theta}^{\theta} g(\theta) \, dG(\theta) \), which indicates that a higher level of production is optimal, while an FSD deterioration of \( G \) lowers \( \int_{\theta}^{\theta} g(\theta) \, dG(\theta) \), so that a lower level of production is optimal. The reverse is obtained in case of discordant uncertainty (i.e., if \( U_{q^\theta}(q^*|\theta) < 0 \)). This proves Proposition 2 (i) and (ii). Similar arguments apply to the sign of \( g''(\theta) \), which then allows us to conclude whether \( \int_{\theta}^{\theta} g(\theta) \, dG(\theta) \) increases or decreases following an increase in risk of \( G \). This demonstrates Proposition 2 (iii) and completes the proof.

### A.4 Proof of Proposition 4

The proof is by and large analogous to the proof of Corollary 1 in Appendix A.2. We evaluate the first-order expression of the more ambiguity-averse firm with ambiguity function \( \psi = \varphi(U) \) at the optimal demand for hedging of the less ambiguity-averse firm and determine its sign. We obtain

\[
\int_{\theta}^{\theta} k'(\varphi(U(x^*|\theta))) \varphi'(U(x^*|\theta)) U_{x^*}(x^*|\theta) \, dG(\theta),
\]

and abbreviate the integrand to \( \zeta(\theta) \).

Assumption A1 implies that \( U_{x^\theta}(x^*|\theta) = U_{x^\theta}(x^*|\theta) - U_{x^\theta}(x^*|\theta) \). From the concavity of \( V \) in \( x \) it follows that the demand for hedging \( x^* \) under belief \( G \) is between \( \underline{x} \) and \( \overline{x} \), and strictly so unless \( G \) is the CDF of the Dirac measure \( \delta_{\underline{x}} \) or \( \delta_{\overline{x}} \).
The first-order condition of the firm at the optimal level of hedging and denoting the integrand by
\[ g(\theta) = \varphi'(U(q, x^*|\theta))U_x(q, x^*|\theta). \] (13)
Analysing the signs of the first and second derivatives of \( g(\theta) \) allows us to analyse how risk changes in the distribution of \( \theta \). There are two cases where the firm’s expectations matter. In the first scenario the firm is optimistic \( p^{exp} > p^f \) and the second scenario when the firm is pessimistic \( p^{exp} < p^f \). As in the prior analysis for each case, the effects of concordant and discordant uncertainty on \( U_x \) and \( U_{x\theta} \) are different. We will limit out of parsimonious reason the analysis to the first case when the firm is optimistic. However, we provide in Tabel XY an overview of each scenario and the obtained sign for \( g'(\theta) \) and \( g''(\theta) \). Denoting the first and second derivative of \( g(\theta) \) as
\[ g'(\theta) = \varphi''(U(q, x^*|\theta))U_\theta(q, x^*|\theta)U_x(q, x^*|\theta) + \varphi'(U(q, x^*|\theta))U_{x\theta}(q, x^*|\theta), \]
\[ g''(\theta) = \varphi''(U(q, x^*|\theta))U_\theta^2(q, x^*|\theta)U_x(q, x^*|\theta) + 2\varphi''(U(q, x^*|\theta))U_\theta(q, x^*|\theta)U_{x\theta}(q, x^*|\theta), \] (14)
as \( U_{\theta\theta} = U_{x\theta\theta} = 0 \), due to the linearity assumption in XX.

We also define the following function \( h(\theta) \)
\[ h(\theta) = \frac{U_\theta(q, x^*|\theta)U_q(q, x^*|\theta)}{U(q, x^*|\theta)U_{x\theta}(q, x^*|\theta)}. \] (15)

Function \( h(\theta) \) allows us to rewrite the first derivatives of \( g(\theta) \) later. Due to the monotonicity of \( U_x(q, x^*|\theta) \), evaluating \( U_x(q, x^*|\theta) \) at \( \theta = \hat{\theta} \) yields \( h(\theta) = 0 \). Moreover, \( h(\theta) \) is positive for \( \theta > \hat{\theta} \) and negative for \( \theta < \hat{\theta} \).

\[ h'(\theta) = \frac{U_\theta(q, x^*|\theta)U_{x\theta}(q, x^*|\theta)[U_{x\theta}(q, x^*|\theta)U_x(q, x^*|\theta) - U_\theta(q, x^*|\theta)U_x(q, x^*|\theta)]}{U(q, x^*|\theta)^2U_{x\theta}(q, x^*|\theta)^2} \] (16)

\( h(\theta) \) allows now to rewrite \( g'(\theta) \) and \( g''(\theta) \).

\[ g'(\theta) = -\varphi'(U(q, x^*|\theta)) \frac{U_{x\theta}(q, x^*|\theta)}{U(q, x^*|\theta)^2} \left\{ \mathcal{R}(U(q, x^*|\theta)) h(\theta) - 1 \right\} \] (17)

where \( \mathcal{R} \) defines the firm’s relative ambiguity aversion as defined in Definition 4. The second derivative of \( g(\theta) \) can be obtained as

\[ g''(\theta) = -\varphi''(U(q, x^*|\theta)) \frac{U_\theta(q, x^*|\theta)U_{x\theta}(q, x^*|\theta)}{U(q, x^*|\theta)^2} \left\{ \mathcal{P}(U(q, x^*|\theta)) h(\theta) - 2 \right\} \] (18)

The signs of \( g'(\theta) \) and \( g''(\theta) \) are determined by the signs of the curly brackets in Equations (17) and (18). If \( h(\theta) \) is smaller than unity and the restrictions on relative ambiguity aversion and relative prudence in ambiguity preferences stated in XX hold, then \( g'(\theta) \) is negative and
$g''(\theta)$ is positive. Assume there is a $\theta'$ with $h(\theta) > 1$. Given that we are in the concordant scenario, $\theta'$ has to be below $\hat{\theta}$ as $h(\theta) > 0$ for $\theta < \hat{\theta}$. $h(\theta)$ is continuous, and for it to connect continuously from $h(\theta) > 1$ to $h(\hat{\theta}) = 0$ there needs to be a $\theta'' \in (\theta', \hat{\theta})$ with $h(\theta'' ) = 1, h(\theta) \geq 1$ for $\theta \in [\theta', \theta'']$ with positive Lesbesgue measure. On $[\theta', \theta'']$ the square bracket in $h'(\theta)$ is non-negative and strictly positive for some $\theta$ in the interval so that $h'(\theta)$ is also non-negative on $[\theta', \theta'']$ and strictly positive for some $\theta$ in the interval. The fundamental theorem of calculus yields

$$h(\theta'') - h(\theta') = \int_{\theta'}^{\theta''} h'(\theta) > 0,$$

which is a contradiction, because $h(\theta'') = 1$ and $h(\theta') > 1$. Hence, $h(\theta) \leq 1$ for all $\theta$. $h(\theta)$ is also bounded by unity with discordant uncertainty and follows the concordant case. However, in the discordant case $U_\theta(q|\theta) < 0$ and $U_\varphi(q^*|\theta) > 0$ in the discordant case, both $g'(\theta) > 0$ and $g''(\theta) > 0$. The following table summarizes the results for optimistic and pessimistic expectations in the concordant and discordant scenario.

### Appendix B  Technical background and further illustrations

#### B.1  Concordant and discordant uncertainty for production decisions

In this section, we discuss assumptions on $u$, $E$ and $\bar{F}$ that allow us to rank the firm’s associated profits at a given level of production $q > 0$ as well as the associated optimal levels of production $q$ and $\bar{q}$. To this end we introduce stochastic orders from risk theory and apply them to the firm’s production decision. We set $E^{(1)}(p) = E(p)$, define $E^{(k)}(p) = \int_{\rho}^{p} E^{(k-1)}(\rho) \, d\rho$ for $k \geq 2$, and likewise for $\bar{F}$.

**Definition 5** (Liu 2014). Let $N \geq 1$ and $n \in \{1, \ldots, N-1\}$ be two integers. Then $\bar{F}$ dominates $E$ by $n$-moments preserving $N$th-order stochastic dominance if

(i) $F^{(N)}(p) \leq \bar{F}^{(N)}(p)$ for all $p \in [\underline{p}, \bar{p}]$, with a strict inequality for some $p$,

(ii) $F^{(k)}(\underline{p}) \leq \bar{F}^{(k)}(\underline{p})$ for $k \in \{1, \ldots, N-1\}$ with an equality for $i \in \{1, \ldots, n\}$.

We refer to this ordering assumptions as the $(n/N)$-order for brevity. For $n = 0$ none of the moments of $E$ and $\bar{F}$ are identical and we obtain first-order stochastic dominance for $N = 1$, second-order stochastic dominance for $N = 2$, third-order stochastic dominance for $N = 3$, etc. If $n = N-1$, all lower-order moments are preserved and $E$ and $\bar{F}$ only differ by the $N$th moment. So the $(n/N)$-order covers a Rothschild and Stiglitz (1970) increase in risk ($n = 1, N = 2$), a Menezes et al. (1980) increase in downside risk ($n = 2, N = 3$) and a Menezes and Wang (2005) increase in outer risk ($n = 3, N = 4$), etc. What’s more, it also covers intermediate cases, for example, if $E$ and $\bar{F}$ differ by third-order stochastic dominance but have equal means ($n = 1, N = 3$). The following result establishes the link between the $(n/N)$-order and risk preferences.
Theorem 1 (Liu 2014). The following are equivalent:

(i) $\tilde{F}$ dominates $F$ by the $(n/N)$-order,

(ii) $\int_{p}^{\tilde{p}} u(p) \, dF(p) > \int_{p}^{\tilde{p}} u(p) \, dF(p)$ for all $u$ that are mixed risk-averse from order $(n + 1)$ to $N$, that is, $\text{sgn}\, u^{(k)} = (-1)^{k+1}$ for $k \in \{n + 1, \ldots, N\}$.

We can now apply this result to the firm’s uncertain profit. Assume that $\tilde{p}$ is distributed according to $\tilde{F}$ and $\tilde{p}'$ according to $F'$; if $\tilde{F}$ dominates $F'$ by the $(n/N)$-order, then any positive affine transformation applied to $\tilde{p}$ and $\tilde{p}'$ preserves this ordering, which can be derived easily from Theorem 1. Hence, $\Pi(q, 0|\tilde{q})$ dominates $\Pi(q, 0|\tilde{q})$ by the $(n/N)$-order, and we obtain $U(q, 0|\tilde{q}) > U(q, 0|\tilde{q})$ from Theorem 1 when $u$ is mixed risk-averse from order $(n + 1)$ to $N$. For example, if we assume monotonicity and risk aversion, $u' > 0$ and $u'' < 0$, then any ordering of $\tilde{F}$ and $F'$ by FSD, SSD or IR allows us to rank $U(q, 0|\tilde{q})$ and $U(q, 0|\tilde{q})$ for any given level of production $q > 0$.

To evaluate how the ranking in terms of the $(n/N)$-order affects associated production decisions, we need additional restrictions on preferences. The following result summarizes.

Proposition 5. Assume that $\tilde{F}$ dominates $F$ by the $(n/N)$-order and that $u$ is mixed risk-averse from order $(n + 1)$ to $(N + 1)$. If relative $k$th degree risk aversion is bounded by $k$ for $k \in \{n + 1, \ldots, N\}$, that is, $-\Pi u^{(k+1)}(\Pi)/u^{(k)}(\Pi) \leq k$ for the relevant $\Pi$, then $\bar{q} > q$.

Proof. The first-order condition associated with $q$ is given by

$$\int_{p}^{\tilde{p}} (p - c'(q)) \cdot u'(pq - c(q)) \, dF(p) = 0.$$ 

Denoting the integrand by $f(p)$ and using $\Pi$ as shorthand for $pq - c(q)$, we obtain the subsequent derivatives of $f(p)$ as follows:

$$ f^{(k)}(p) = q^{k-1} \left[ ku^{(k)}(\Pi) + \Pi u^{(k+1)}(\Pi) + (c(q) - qc'(q)) \cdot u^{(k+1)}(\Pi) \right]. $$

Under the assumptions made, it holds that $\text{sgn}\, f^{(k)} = (-1)^{k+1}$ for $k \in \{n + 1, \ldots, N\}$. Per Theorem 1, the change from $F$ to $\tilde{F}$ then raises the first-order expression above zero, indicating that a higher level of production is now optimal.

Rothschild and Stiglitz (1971) were the first to consider an increase in risk in the output price ($n = 1, N = 2$) while Cheng et al. (1987) consider an FSD shift ($n = 0, N = 1$). Chiu et al. (2012) consider the effect of $n$th-degree risk increases on production (their Proposition 3(i)), $n = N - 1$ and that of $n$th-degree stochastic dominance (their Proposition 3(ii), $n = 0$). The intuition for the restriction on the relative risk aversion measures is that the firm balances

\[12\] The convexity of $c$ implies that $(c(q) - qc'(q)) > 0$ because $q > 0$. 

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two effects. The first one is a positive substitution effect because a better price distribution incentivizes higher production. However, the change in output price risk induces a change in the profit distribution. Under the new profit distribution the firm may be more sensitive to risk and prefer to produce less.

Alternatively, we may require that \( F \) dominates \( F' \) by the \((n/N)\)-order and by central dominance (CD, see Gollier, 1995) but lift the restrictions on the relative risk aversion measures. \( \text{Gollier (2011)} \) lists several stochastic orders that belong to the set of CD. Furthermore, if the price risk is binary, it is easy to show that a higher probability of the high price raises production. In short, there are many scenarios where uncertainty is concordant.

To obtain discordant uncertainty, some prerequisites are necessary. Consider the case where \( F \) dominates \( F' \) by the \((n/N)\)-order; for the dominating distribution to induce less production than the dominated one, we know from Proposition 5 that some relative risk aversion measures need to be larger than their corresponding order.

**B.2 Concordant and discordant uncertainty for hedging decisions**

To be completed.