Valuation and agency implications of performance-vesting stock grants with path-dependent (price- and earnings-based) vesting schedules

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Abstract

Relying on option pricing techniques, we quantify analytically the incentive properties of performance-vesting (p-v) stock grants, i.e. stock that vests with performance targets. Our paper’s contribution is twofold. First, we derive closed-form expressions for the value and incentive measures (ownership and risk-taking) of p-v stock grants with highly sophisticated vesting provisions. Second, we advance the understanding of incentives conveyed by these grants from a corporate finance perspective by studying the agency implications of different vesting provisions.

While both ownership incentives and risk-taking incentives can be substantial by t-v grants standards, incentives conveyed by these grants vary according to the degree of moneyness of the vesting schedule’s long and short components, implying that the agency implications of replacing stock options with p-v stock grants are substantial.

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We find that the introduction of p-v stock grants is likely to hurt both debtholders and shareholders' interests. We also focus on two key sets of vesting provisions, namely path-dependent vesting provisions and accounting-based vesting provisions. While we find that path dependence unambiguously improves debtholder interests, its effect on shareholder interests is mitigated. In addition, since p-v stock grants that vest with accounting metrics only are contingent on market-based performance and accounting-based performance, the alignment they convey combines a direct effect and an indirect, where the latter captures the fact that an increase in the stock price leads to grant value appreciation through the vesting of additional shares due to the correlation between market-based and accounting performance.

1 Introduction

Growing interest has been devoted of late to studying the incentive characteristics of performance-vesting (p-v) stock grants. Unlike stock options, commonly known as time-vesting grants (t-v), p-v stock grants vest according to an accounting-based (i.e. earnings) and/or market-based (i.e. stock price) vesting schedule, which defines the grants' contingent payout in terms of shares.

Recent studies show that firms in which managers are compensated with p-v stock grants, as opposed to t-v grants, are characterized by subsequent overperformance over the five consecutive years (Bettis et al., 2010) and are associated with an increase in pay-performance-sensitivity (Kuang and Qin (2006), Kuang and Qin (2009), and Bettis et al. (2016)). This suggests that the use of p-v stock grants is likely to improve agency conflicts traditionally associated with the separation of ownership and control (i.e. shareholder-manager agency issues).

We show, however, that while p-v stock grants can indeed provide a much stronger alignment between manager and shareholder interests, the agency implications of substituting t-v grants by p-v stock grants are significant for both shareholders and debtholders. First, the cost of achieving a high alignment is substantial for shareholders as they have to relinquish a very large number of shares to ensure that managers’ interests are highly aligned with theirs. Indeed, we show that the reason why p-v stock grants convey

1We borrow the terminology from Bettis et al., 2016.
very high ownership incentives is because they provide an “abnormally large number of shares” in comparison with t-v grants (Bettis et al., 2010). For instance, Amgen CEO Kevin Sharer’s 2008 p-v compensation plan specifies an award of 36’000, 73’000 and 146’000 shares at, respectively, the threshold, target, and ceiling performance levels of three-year annualized total stock return (Bettis et al., 2016).

Second, p-v stock grants’ risk-taking incentives can be considerable by t-v grants standards, even if the number of shares they potentially convert into is low. This feature is likely to be of concern. Convexity in t-v grants’ payoff structure, which was initially sought to mitigate managers’ risk aversion resulting from their poorly-diversified wealth, has been blamed as one of the many culprits for inducing too much risk taking from managers (under risk neutrality). Since p-v stock grants’ payoff structure, by contrast, exhibits both convex and concave regions due to the conversion into, respectively, a minimum and a maximum number of shares, do p-v stock grants address t-v grants’ shortcoming?

By construction, the effect of the vesting schedule is to concavify the p-v stock grant’s payoff function around the long position embedded in the schedule and to convexify it around the short position. Interestingly, the effect induced by the convexification of the payoff structure on risk-taking incentives is far greater in magnitude than the effect induced by the concavication. By t-v grants standards, the increase in risk-taking incentives in the convex portion of the grant’s payoff structure is indeed considerable. By contrast, the concavification of the grant’s payoff structure has the effect of lowering risk-taking incentives, and even providing negative values, close to the concave portion of the grant’s payoff structure.

The fact that p-v stock grants can convey high risk-taking incentives is likely to be of concern to policymakers, regulators and shareholders alike. This inevitably raises some issues. One of the purposes of granting p-v stock grants in the aftermath of the last financial crisis was precisely to address t-v grants’ incentives to take on excessive risk and to attach more stringent performance conditions to managers’ compensation (see Bettis et al. 2010). Yet, our findings (and prior literature) seem to suggest that, if anything, switching from t-v grants to p-v stock grants may in fact exacerbate rather than mitigate risk-taking behavior. Hence, because the higher reward to risk taking may affect the (ex post) risk of asset substitution, p-v stock grants unequivocally worsen debtholder interests – implying that the cost of debt has to increase when managers are compensated with p-v stock grants to
reflect the potential for greater risk shifting. Shareholder interests are also likely to be hurt since very high risk-taking incentives might induce managers to select a degree of riskiness that exceeds the (shareholder) value-maximizing objective.

To advance the understanding of incentives provided by p-v stock grants, we explore two novel vesting provisions. First, we study the implications of introducing a path-dependent feature in p-v stock grants’ vesting schedules. This entails substituting standard European call options by path-dependent call options. Second, we explore the extent to which price-based provisions differ from accounting-based vesting provisions. Our paper’s contribution is twofold. Besides deriving closed-form expressions for the value, ownership incentive (i.e. delta) and risk-taking incentive (i.e. vega) measures of p-v stock grants with these vesting provisions, we also study the implications of these provisions for both shareholders and debtholders.

Despite their interest from a corporate finance perspective, both vesting features have been relatively unexplored. Whereas the concept of path-dependent vesting provisions is novel to the literature and practically nonexistent in compensation policies (one exception being Credit Suisse’s PIPs and ISUs), accounting performance metrics have only been analyzed numerically (Bettis et al. 2010), despite the fact that they “have displaced to some extent stock price metrics” (Bettis et al. 2016) in compensation packages.

The attractiveness of path-dependent vesting provisions resides in the fact that the exercise decision associated with the conversion of grants into shares is contingent on the entire path of the schedule’s underlying asset, and not solely on its value at maturity. A closer alignment between shareholders and managers is thus expected because path dependence requires sustained overperformance over the entire vesting period. A closer alignment between debtholders and managers is also expected as managers are refrained from engaging in opportunistic risk taking close to vesting. The attractiveness of accounting-based vesting provisions stems from the fact that since it is more efficient to incentivize managers for actions that are more under their

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2These are the two standard incentive measures commonly used in the compensation literature. In accordance with prior literature, we define delta as the sensitivity of grant value to a one percent change in the underlying asset and vega as the sensitivity of grant value to a 0.01 change in underlying volatility.

3Closed-form expressions present several advantages. First, they can account for the nonlinearities in p-v stock grants’ payoffs. Second, they provide a consistent approach to estimating incentives stemming from p-v stock grants and t-v grants. Third, they allow for comparative statics of the grants’ incentives for varying degrees of moneyness and increasing difficulty of targets. Finally, they allow for the estimation of incentives of path-dependent p-v grants after the grant’s issuance date.

4There is, however, one exception.
control, accounting-based vesting provisions tend to be preferred from a corporate governance viewpoint. Whereas price values can be a noisy reflection of managerial performance (if subject to manipulation), accounting performance is more likely to better reflect managerial effort than stock prices.

Relying on option pricing techniques allows us to readily compare incentives conveyed by accounting-based, path-dependent and standard price-only vesting provisions. In order to obtain closed-form expressions for grants contingent on accounting-based vesting provisions, the joint underlying dynamics of the stock price and the accounting metric have to be modeled explicitly. Assuming a semi-strong form of the efficient market hypothesis, we rely on Bakshi and Chen’s (2005) extended version of the Gordon model in continuous time to model the contemporaneous relation between the stock price and the accounting metric. For grants whose value is contingent on the entire path of the underlying asset(s), we have to make explicit both the averaging process and the joint law of an asset and its average. We assume a lognormal distribution of the underlying asset(s) and a geometric (as opposed to arithmetic) averaging process of the underlying asset, which imply that their joint distribution is lognormal.

The valuation of p-v stock grants with price-only vesting provisions can be done under the risk-neutral measure since the stock price is a traded asset. By contrast, because the payoff of p-v stock grants with accounting-based provisions depends on the performance of both traded (i.e. stock price) and nontraded (i.e. earnings per share) assets, risk-neutral valuation is not applicable since the risk associated with the nontraded asset cannot be hedged. However, we show that under a number of assumptions, valuation of these p-v stock grants under the risk-adjusted probability measure is equivalent to valuation under the risk-neutral probability measure.

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5 In its simplest form, which we require for tractability, earnings per share and the stochastic discount factor follow a geometric Brownian motion with constant drift, constant volatility, and a constant correlation between earnings shocks and systematic shocks; the dividend per share represents a constant fraction of earnings; the dividend equals earnings per share times the payout ratio; and a constant plowback ratio.

6 The valuation of path-dependent claims can be technically challenging because the sum of correlated lognormal random variables is not lognormally distributed. Even though, in practice, averages are more likely to involve discrete values, the assumptions of a geometric averaging process and a continuous sampling of the logarithm of the grant’s underlying asset enable us to obtain closed-form expressions. The question of how good of an approximation the geometric average is to the arithmetic average can be answered, at least partially, by the following. Given that p-v stock grants are not traded, the assumption of geometric averaging with continuous sampling can be deemed reasonable as the intended purpose of their valuation provided in this paper is purely theoretical.

7 Assuming no arbitrage, complete markets and replicability of the grant’s payoff, the grant-date value is given by the expectation under the risk-neutral measure of the grant’s boundary condition.

8 See above.
Before exploring path-dependent and accounting-based provisions, our paper first details the valuation of the most standard p-v stock grant, i.e. with price-only vesting provisions absent accounting and path-dependent features. Since this grant converts at maturity into a number of shares according to performance conditions, its boundary condition is given by the product of the stock price at maturity and the value of the vesting schedule, which defines the contingency of the number of shares the grant converts into on the stock price. This number being constrained between a minimum (i.e. a long position) and a maximum (i.e. a short position), we model the value of the vesting schedule at maturity as the product of a bull spread of standard European call options written on the underlying (dividend-paying) stock price and the scaling coefficient. The latter is shown to have a significant impact on the grant’s incentives since it reflects the extent to which a marginal increase in underlying performance translates into an increase in the number of shares the grant converts into.

Since the schedule’s final payoff structure is characterized by convexity around the long strike price and concavity around the short strike price, it decomposes into three regions. Grant value is a convex function of the stock price only for stock price values between the long and short strike prices (i.e. the high-incentive region) because a marginal increase in the stock price leads to an increase in grant value through both the value of the stock holders are entitled to at maturity and the vesting of additional shares. Below the long strike price, i.e. in the no-incentive region, the grant’s final payoff is zero because the grant does not vest into shares. Above the short strike price, i.e. in the low-incentive region, grant value is increasing linearly in the stock price because price increases do not trigger the vesting of additional shares. Since the convex region (i.e. the high-incentive region) of the payoff structure combines incentives through two correlated sources, this region is characterized by the highest values of risk-taking incentives (displaying a bell-shaped property in the stock price) and the highest slope of ownership incentives (displaying a monotonic increase in the underlying stock price).

As p-v stock grants are progressively substituting t-v grants in compen-

\[9\] It is given by the difference in the maximum and minimum of shares scaled by the difference between the short and long strike prices. Its definition relies on the assumption of a continuous and linear payout schedule between the lowest and highest number of shares.

\[10\] In addition, we show that these grants’ incentives increase as the difference between the long and short strike prices decrease. This is due to the fact that a small difference results in a steeper slope of the vesting schedule, which in turn entails a greater reward for a given increase in the grant’s underlying asset. The effect is thus to reduce the high-incentive region.
sation packages, it is instructive to compare how incentives conveyed by the former contrast with those conveyed by the latter. Which grant features are conducive to high incentives? Are p-v stock grants’ high incentives due to (i) the slope of the vesting schedule (i.e. the fact that these grants potentially convert into a higher number of shares than t-v grants), (ii) the vesting schedule’s underlying appreciation mechanism (i.e. the spread of European call options on a dividend-paying asset), or (iii) a combination of both?

To provide a like-for-like comparison of incentives between t-v grants and p-v stock grants, we disentangle in the latter the slope effect from the effect embedded in the spread of call options. We first transform a p-v stock grant into its scaled equivalent, which we define as a p-v stock grant that converts into at most one share. Comparison between the delta (vega, resp.) of the p-v stock grant and the delta (vega, resp.) of its scaled equivalent allows us to isolate the component of the ownership (risk-taking, resp.) incentive conveyed by the number of shares the grant converts into. We then compare the delta (vega, resp.) of the scaled p-v stock grant with the delta (vega, resp.) of a European stock option on a dividend-paying asset with identical parameters. This comparison allows us to isolate the component of the grant’s ownership (risk-taking, resp.) incentive conveyed by the spread.

We find that the bulk of ownership incentives conveyed by p-v stock grants is provided by the scaling coefficient (i.e. the slope of the vesting schedule) and not by the mechanism underlying the spread of call options nested in the vesting schedule. In other words, the strength of p-v stock grants’ ownership incentives stems from the fact that these grants potentially convert into a large number of shares rather than from the long and short positions embedded in the vesting schedule. By contrast, both the scaling coefficient and the spread of call options convey substantial risk-taking incentives compared to stock options. That is, even after accounting for the scaling factor, the spread provides substantial risk-taking incentives.

These findings indicate that, if anything, the agency implications of replacing stock options with p-v stock grants are not negligible. While we find in line with prior literature that shifting away from t-v grants to p-v stock grants (with a price-contingent vesting schedule) may convey a higher degree of alignment between managerial and shareholder interests, we show that the cost of achieving this higher alignment is substantial for shareholders who have to relinquish a relatively large number of shares. Failing to account for this cost results in an inaccurate estimation of the agency cost associated with shifting from t-v grants to p-v stock grants.
In addition, we find that even p-v stock grants that convert into a low number of shares convey risk-taking incentives that are substantial compared to those conveyed by t-v grants. This finding implies that p-v stock grants unequivocally worsen debtholders’ interests because they increase the reward to risk taking. The introduction of p-v stock grants is also likely to hurt shareholders’ interests by causing some distortion between managerial incentives and shareholders’ interests. Indeed, although p-v stock grants’ high risk-taking incentives may directly serve shareholder interests by inducing potentially risk-averse managers to overcome their risk aversion and invest in risky value-creating projects, they may harm them indirectly as high risk-taking incentives may induce managers away from the shareholder-value maximizing objective.

Hence, by and large, our results tend to suggest that introducing p-v stock grants as managerial compensation tools may provide mixed results. To further our understanding of incentives conveyed by p-v stock grants, we ask what are the agency implications for both shareholders and debtholders of introducing a path-dependent feature in vesting schedules. From an option pricing approach, we thus contrast the incentive properties of p-v stock grants with vesting schedules that embed underlying path-dependent call options instead of standard European call options.

We find that debtholder interests unambiguously improve because managers holding p-v stock grants with a path-dependent vesting schedule have very little to virtually no incentive to shift to riskier assets as vega is substantially lower. Path dependence, by making \textit{ex-post} shifting to riskier assets less rewarding, contributes to weakening the asset substitution problem.

By contrast, the effect of path dependence on shareholder interests is mitigated. Because they lower the return to risk taking, path-dependent vesting provisions render risk taking less attractive than standard vesting provisions. Whether this feature ultimately benefits shareholders is debatable. One can contend that shareholders are better off when managers have less incentive to manipulate stock prices because (opportunistic) price manipulation is conducive to a lower alignment between managers and shareholders, as it induces the former to taking actions that allow them to reap short-term profits whereas the latter instead favor actions that create long-term shareholder value. By contrast, p-v stock grants’ high risk-taking incentives may directly serve shareholder interests by inducing potentially risk-averse managers to
overcome their risk aversion and invest in risky value-creating projects.\footnote{In comparison with p-v stock grants that vest according to a standard vesting schedule, ownership incentives are stronger under path dependence in the second half of the high-incentive region and the beginning of the low-incentive region, but lower in the remainder of the low-incentive region, the no-incentive region and the first half of the high-incentive region.}

Our findings have important implications. From a regulatory viewpoint, introducing path dependence in p-v stock grants’ vesting schedules can provide a means of mitigating these grants’ very strong risk-taking incentives without significantly affecting their ownership incentives. From an agency perspective, firms awarding p-v stock grants with path-dependent vesting schedules can ensure that shareholder and manager interests are more aligned and can improve shareholder-debtholder agency issues by lessening \textit{ex ante} the severity of the asset substitution problem.

Lastly, we turn to a type of p-v stock grants that is increasingly being used in managers’ compensation packages, i.e. p-v stock grants that vest with accounting metrics only. Although relatively uncommon in the traditional compensation literature, the contingency on accounting metrics is becoming a dominant feature of compensation grants, as “accounting performance metrics have displaced to some extent stock price metrics” (Bettis et al. 2016).

How do managerial incentives differ when vesting provisions are based on accounting performance as opposed to market-based performance? By construction, the vesting into shares according to an accounting-based schedule establishes that the number of shares is solely contingent on accounting performance. Yet, even if stock price increases do not \textit{per se} lead to the vesting of additional shares, the grant’s final value is ultimately impacted by price increases explicitly (as grant value is function of the stock price since value converts into shares) and implicitly through the vesting schedule (due to the correlation between stock price performance and accounting performance).

Specifically, Bakshi and Chen’s (2005) equity valuation framework allows us to isolate the alignment effect conveyed by the accounting metric by disentangling the component of the grant’s delta that is due to stock value appreciation (i.e. \textit{direct} effect that holds the accounting metric constant) from the component arising through the accounting-based vesting schedule (i.e. \textit{indirect} effect). The latter \textit{indirect} effect captures the fact that an increase in the stock price likely results in an increase in accounting performance due to the correlation between variables, which in turn leads to grant
value appreciation through the vesting of additional shares.\footnote{In the Bakshi and Chen framework we rely on, the correlation between the two arises from the fact that the stock price equals the infinite sum of future dividends, which are assumed to be a constant function of the accounting metric.}

Our analytical expression for the delta of the p-v stock grant is closely related to Bettis et al.'s (2016) numerical characterization of the grant’s aggregate delta, except for the fact that we explicitly derive the partial derivative of the stock price with respect to earnings per share from Bakshi and Chen’s equity valuation model. As in Bettis et al. (2016), we also explore whether the contribution provided by the accounting channel to total ownership incentives is economically meaningful. Our findings indicate that the alignment effect can be significant (in line with Bettis et al., 2016) – but need not be. Whereas they find that “[f]or p-v stock grants based on a single accounting metric, the accounting channel supplements marginal stock price delta to enhance executive delta incentives by approximately 72%,” our results indicate a more nuanced picture. We show that the contribution of the accounting metric to the aggregate delta decreases exponentially in the stock price. This results from the fact that the marginal accounting delta is a bell-shaped function of the stock price, due to the vesting schedule: For low values of the stock price, the marginal accounting delta exceeds the marginal stock delta, whereas the opposite holds for higher values of the stock price, implying that the aggregate delta becomes almost entirely driven by the marginal stock delta as the stock price increases.

Our paper belongs to an emerging literature that analyzes compensation grants for which the payoffs and vesting are subject to performance requirements. The literature so far has been mostly empirical (see for example, Kuang and Qin (2006), Gerakos et al. (2007), Kuang (2008), Kuang and Qin (2009), Bettis et al. (2010), Li and Wang (2016)), with the exception, however, of Bettis et al. (2016) who investigate incentive effects stemming from performance-vesting grants using numerical approaches, and of Holden and Kim (2013) as well as Bizjak et al. (2016). Both Johnson and Tian (2000a and 2000b) provide comparative analyses of different types of grants.

In addition, greater attention has been devoted to studying stock options with performance-vesting conditions (Johnson and Tian (2000), Gerakos et al. (2007), Kuang (2008), Kuang and Qin (2009)), and much less to the case involving the vesting of shares that is contingent on performance conditions (see Holden and Kim, 2013 and Bettis et al. 2016).

Bettis et al. (2010) provide an extensive overview and description of
performance-vesting grants. They find that these grants are characterized by significant incentives for executives and lead to better subsequent operating performance than control firms. Kuang and Qin (2006) show that although performance-vested stock options provide higher pay-performance sensitivity compared to unconditional stock options, too difficult targets negatively affect managers’ choice of effort and lead to a divergence of interests between managers and shareholders. In addition, Kuang and Qin (2009) show that performance-vested stock options in executive compensation contracts are associated with a greater alignment of interests between managers and shareholders as they provide better incentives for managerial effort than traditional stock options.

Bizjak et al. (2016) present a model to analyze the expected payoff from equity compensation with accounting-based payout conditions, incorporating characteristics such as the coupling of stock price and accounting targets, multiple accounting targets, sliding payout schedules, and the adoption of both “and” and “or” conditions that trigger payout. Gerakos et al. (2007) find that performance-vested stock option grants to U.S. CEOs represent a smaller share of option grants for firms with larger market-to-book ratios and those with larger holdings by pension funds.

In a very recent paper that relies on numerical approaches, Bettis et al. (2016) characterize performance-vesting awards and their determinants. Their contribution is to develop new methods to measure p-v stock grants’ value and incentive properties stemming from both the stock price and the accounting channel. Their approach consists in first defining the grant’s ex post value (that ultimately converts into the back-end instrument which may consist of shares or stock options), and then obtaining its ex ante grant-date expected value. Our approach is equivalent to theirs as we start off with defining the grant’s boundary condition and then compute the expected discounted grant-date value. In our setting which relies on option pricing techniques, the two are analytically related since the expected discounted grant-date value is obtained from the grant’s boundary condition.13

The remainder of this paper is organized as follows. The valuation of p-v stock grants with a price-based vesting schedule is provided for non-path-dependent schedules in Section 2.1 and for path-dependent schedules in Section 2.2. Section 3 deals with the valuation of accounting-based p-v

13What they define as the ex post value is equivalent to what we define as the grant’s boundary condition, and what they term ex ante grant-date expected value corresponds to what we define as the expected discounted grant-date value.
stock grants with both non-path-dependent schedules and path-dependent schedules. Finally, Section 4 provides concluding remarks.

2 Valuation of performance-vesting (p-v) stock grants with a price-contingent vesting schedule

2.1 Standard price-contingent vesting schedule

We begin by exploring the incentive properties of standard p-v stock grants with (stock) price-only vesting provisions, i.e. stock that vests according to a price-contingent vesting schedule. Our focus on p-v stock grants, as opposed to p-v stock option grants, is motivated by the fact that “stock is displacing options as the back-end security for both t-v and p-v grants, and that, likewise, so are p-v awards displacing t-v awards” (Bettis et al. 2016). In its most basic form, the price-contingent p-v stock grant vests at the end of the vesting period into a minimum number of shares if stock price performance equals a threshold value and into a maximum number of shares if stock price performance equals a ceiling value.

Before studying the incentive properties of these p-v stock grants, we first proceed with their valuation using option pricing techniques. In order to derive an analytical expression for a p-v stock grant, we first characterize the grant’s boundary condition, i.e. when the grant’s value eventually converts into its final payoff. The boundary condition of a p-v grant differs from that of a stock option in that vesting does not accrue with the passage of time (as do so-called time-vesting grants such as stock options), but requires performance conditions to be met. As each grant is expected to convert at maturity into a number of shares, the grant’s value at vesting date $T$ (equivalently, its boundary condition) is given by the product of the stock price at $T$ and the value of the performance-vesting schedule at $T$$S(T)g(S(T))$, \(^{(1)}\)

where $g(S(T))$ denotes the value of the performance-vesting schedule.\(^{14}\)

\(^{14}\)Hence, in comparison with t-v grants whose payoff is positive only if the stock price at maturity exceeds the option strike price, the final payoff of a p-v stock grant is given by the product of the stock price at maturity and the vesting schedule contingent on the performance metric. It thus reflects the
Modeling the performance-vesting schedule involves defining the contingency of the number of shares as a function of underlying performance. The simplest p-v stock grant converts at \( T \) into \( n \) shares if the underlying stock price performance at \( T \) is below a threshold value (which we denote by \( K_l \)) and into at most \( \pi \) shares if stock price performance reaches a ceiling value (which we denote by \( K_h \)).

Because the vesting schedule defines the contingency of the number of shares on performance of the underlying metric, it can be modeled as the product of two components, i.e. a bull spread of standard European call options written on the underlying (non-dividend-paying) asset and the scaling coefficient. The spread reflects the fact that the number of shares the grant converts into is constrained between a minimum (i.e. a long position) and a maximum (i.e. a short position), with the latter exceeding the former. The scaling coefficient, which represents the slope of the vesting schedule, reflects the extent to which a marginal increase in underlying performance translates into an increase in the number of shares the grant converts into.\(^{15}\)

Assuming a continuous and linear payout between \( n \) and \( \pi \), we can thus express the value of the vesting schedule at date \( T \) as follows

\[
g(S(T)) = n + \alpha \left[(S(T) - K_l)^+ - (S(T) - K_h)^+\right], \tag{2}
\]

where \( \alpha = (\pi - n)/(K_h - K_l) \) for \( \pi > n \geq 0 \) and \( K_h > K_l > 0 \). The spread consists of a long call option and a short call option with identical maturities \( T \), same underlying asset \( S \), where the short strike price \( K_h \) exceeds the long strike price \( K_l \). The slope \( \alpha \) is determined by the ratio of the difference in the highest (\( \pi \)) and lowest (\( n \)) payouts to the difference in the spread’s short (\( K_h \)) and long (\( K_l \)) strike prices. Since, empirically, the minimum number of share(s) a p-v stock grant vests into is usually set to zero, vesting schedules are typically characterized by \( n = 0 \) (see Bizjak et al., 2016).

Because holders of p-v stock grants are long performance in excess of the threshold value \( K_l \) and short performance in excess of the ceiling value \( K_h \), the schedule’s payoff structure exhibits convexity around \( K_l \) and concavity around \( K_h \). This implies that the grant’s final payoff structure is a convex two (correlated) sources of randomness, i.e. the stock price and the performance metric. Given that the vesting schedule determines the number of shares per each grant, the grant’s final payoff (i.e. the number of shares) is obtained by dividing its value at \( T \) by the stock price at \( T \).

\(^{15}\)It is given by the difference in the maximum and minimum of shares scaled by the difference between the short and long strike prices. Its definition relies on the assumption of a continuous and linear payout schedule between the lowest and highest number of shares.
function of the stock price between $K_l$ and $K_h$. Accordingly, the fact that the payoff structure exhibits regions of different degrees of convexity necessarily implies that the grant’s ownership and risk-taking incentives vary with the degree of moneyness of the long and short components of the vesting schedule.

For illustration, Figure 1a depicts the value at maturity of the performance-vesting schedule of a p-v stock grant that vests into $n = 0$ share if the stock price at $T$ is below $K_l = 20$ and into $n = 5$ shares if the stock price at $T$ is greater than $K_h = 80$. In between $K_l$ and $K_h$, a continuous and linear payout schedule is assumed. As shown in Figure 1b, the final payoff of the p-v stock grant decomposes into three regions. In the no-incentive region, where $S(T) \leq K_l$, the grant’s payoff at $T$ is zero because the stock price is below the threshold value, implying that the number of shares the grant vests into is zero. In the low-incentive region, where $S(T) \geq K_h$, grant value at $T$, which equals $\pi S(T)$, is increasing linearly in the stock price. Since the stock price is in excess of the ceiling value, increases in the stock price do not trigger the vesting of additional shares. Hence, grant value appreciation arises only through the stock price. By contrast, in the high-incentive region, where $K_l < S(T) < K_h$, grant value at $T$, which equals $\pi S(T)(S(T) - K_l)/(K_h - K_l)$, is a convex function of the price at maturity because a marginal increase in the stock price leads to an increase in grant value through both the value of the stock holders are entitled to at maturity and the vesting of additional shares. Hence, grant value is a convex function of the stock price only for stock price values between $K_l$ and $K_h$. 

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We now proceed with the derivation of the grant-date value of the p-v stock grant. Given that the stock price is a traded asset, we can value the grant under the risk-neutral measure. Under the assumptions of no arbitrage, complete markets and replicability of the grant’s payoff, risk-neutral pricing entails that the grant-date value is given by the expectation under the risk-neutral measure of the grant’s boundary condition

\[ V_0^s (T) = \alpha e^{-rT} \mathbb{E}^Q \left[ (S(T) - K_t)^+ - (S(T) - K_h)^+ \right] , \]

where \( r \) denotes the constant risk-free rate and \( \mathbb{E}^Q[.] \) the expectation operator under the risk-neutral measure.

Stock price dynamics of a dividend-paying asset under the risk-neutral measure are given by

\[
\frac{dS(t)}{S(t)} = (r - q) \, dt + \sigma \, dZ(t),
\]

where the process \( \{Z(t)\}_{0 \leq t \leq T} \) represents a \( \mathbb{Q} \)-standard Brownian motion with respect to the filtration \( \{\mathcal{F}_t\}_{0 \leq t \leq T} \), and \( q \) and \( \sigma \) denote, respectively, the constant dividend yield and the constant stock-return volatility.

Proposition 1 provides the closed-form formula for the value of a p-v stock grant with a standard price-based vesting schedule for \( n = 0 \).

**Proposition 1** Under the assumptions of no arbitrage, complete markets, and replicability of the grant’s payoff, the risk-neutral grant-date value of the p-v stock grant with a standard price-based vesting schedule is given by

\[
v_0^s (S(T)) = v_{0,l}^s (S(T)) - v_{0,h}^s (S(T)),
\]

where

\[
v_{0,j}^s (S(T)) = s_0 \alpha \left\{ s_0 e^{-(q - \frac{3}{2} \sigma^2)T} \Phi \left( -d_j^s + 2 \sigma \sqrt{T} \right) \\ - K_j e^{-\frac{1}{2} \left( r + q - \frac{\sigma^2}{2} \right)T} \Phi \left( -d_j^s + \sigma \sqrt{T} \right) \right\}, \quad S(0) = s_0,
\]

and

\[
d_j^s := \frac{\log \left( \frac{K_j}{s_0} \right) - \frac{1}{2} \left( r - q - \frac{\sigma^2}{2} \right) T}{\sigma \sqrt{T}},
\]
with \( \alpha = (\bar{n} - n)/(K_h - K_l) \), for \( j \in \{l, h\} \), and \( \Phi(.) \) indicating the cumulative distribution function of the Gaussian variable.

**Proof.** See Appendix available upon request.

It is instructive to contrast our approach with the existing literature. Bettis et al. (2016) develop new numerical methods to measure the value and incentive properties of p-v stock grants. Their approach consists in first defining the grant’s *ex post* value and then obtaining its *ex ante* grant-date expected value. Our approach is equivalent to theirs as we start off with defining the grant’s boundary condition and then compute the expected discounted grant-date value. In our setting which relies on option pricing techniques, the two are analytically related since the expected discounted grant-date value is obtained from the grant’s boundary condition.

### 2.1.1 Incentives of p-v stock grants with a price-based vesting schedule

We now proceed with the formal analysis of incentives conveyed by p-v stock grants with price-only vesting provisions. The two standard measures of incentives used in the compensation literature are the *ownership* incentive and the *risk-taking* incentive.

**Ownership incentive.** The delta at date 0 of a p-v stock grant with price-only vesting provisions is given by the partial derivative of the grant value with respect to a one percent increase in the stock price, holding everything else constant, as stated in the following proposition.

**Proposition 2** The expression at date 0 for the delta of the p-v stock grant with a standard price-based vesting schedule is given by

\[
\delta^s := \frac{s_0}{100} \times \left[ \frac{\partial v_{0,l}^s (S(T))}{\partial s_0} - \frac{\partial v_{0,h}^s (S(T))}{\partial s_0} \right],
\]

where

\[
\frac{\partial v_{0,j}^s (S(T))}{\partial s_0} = \alpha \left\{ s_0 e^{-\left(q + \frac{3}{2}\sigma^2\right)T} \left[ 2\Phi \left(-d_j^s + 2\sigma\sqrt{T}\right) - s_0 \phi \left(-d_j^s + 2\sigma\sqrt{T}\right) \frac{\partial (d_j^s)}{\partial s_0} \right] 
- K_j e^{-\frac{t}{2} \left(r + q - \frac{\sigma^2}{2}\right)T} \left[ \Phi \left(-d_j^s + \sigma\sqrt{T}\right) - s_0 \phi \left(-d_j^s + \sigma\sqrt{T}\right) \frac{\partial (d_j^s)}{\partial s_0} \right] \right\},
\]

(9)
for \( j \in \{l, h\} \) and \( \phi(.) \) indicating the probability density function of the Gaussian variable.

**Risk-taking incentives.** The risk-taking incentive conveyed by a grant is measured by the grant’s vega, which represents the sensitivity of the grant value to the grant’s underlying volatility, generally measured with respect to a 0.01 change in the underlying volatility.

**Proposition 3** The expression for the vega at date 0 of the p-v stock grant with a standard price-based vesting schedule is given by

\[
\nu^s := 0.01 \times \left[ \frac{\partial \nu_{0,l}^s (S(T))}{\partial \sigma} - \frac{\partial \nu_{0,h}^s (S(T))}{\partial \sigma} \right],
\]

where

\[
\frac{\partial \nu_{0,j}^s (S(T))}{\partial \sigma} = s_0 \alpha \left\{ s_0 e^{-\left(\frac{3}{2} \sigma^2\right)T} \left[ 3 \sigma T \Phi \left( -d_j^s + 2 \sigma \sqrt{T} \right) + \phi \left( -d_j^s + 2 \sigma \sqrt{T} \right) \left( \frac{\partial (-d_j^s)}{\partial \sigma} + 2 \sqrt{T} \right) \right] - K_j e^{-\frac{1}{2}(r+\frac{\sigma^2}{2})T} \left[ 0.5 \sigma T \Phi \left( -d_j^s + \sigma \sqrt{T} \right) + \phi \left( -d_j^s + \sigma \sqrt{T} \right) \left( \frac{\partial (-d_j^s)}{\partial \sigma} + \sqrt{T} \right) \right] \right\},
\]

for \( j \in \{l, h\} \).

Figure 2 plots the delta of the p-v stock grant and Figure 4 plots the vega of the p-v stock grant against the stock price at 0 for parameter values of the grant defined above. The solid vertical lines represent the value of the long \((K_l = 20)\) and short \((K_h = 80)\) strike prices of the vesting schedule. Other parameter values are defined as follows: stock-return volatility \( \sigma = 0.17 \), risk-free rate \( r = 2.5\% \), maturity \( T = 4 \), and expected dividend yield \( q = 0.02 \).

Figure 2 indicates that while the grant’s delta is monotonically increasing in the stock price – a direct consequence of the fact that the grant converts into shares – its sensitivity to a marginal increase in the underlying stock price is highly dependent on the degree of moneyness of the vesting schedule’s long and short components. In other words, the slope of the delta function (i.e. as a function of the underlying asset) depends on how close the stock price \( s_0 \)
is to the threshold value $K_l$ or to the ceiling value $K_h$. The high slope in the high-incentive region (i.e. $K_l < s_0 < K_h$) is due to the fact that incentives arise not only from the price of the share(s) which the grant holder is entitled to at vesting, but also from the performance-vesting schedule where marginal increases in the stock price result into the vesting of shares. By contrast, in both the no-incentive region (i.e. $s_0 \leq K_l$) and the low-incentive region (i.e. $s_0 \geq K_h$), the number of shares vested is independent of stock price performance. The slope of the delta function in the low-incentive region reflects the fact that grant value is increasing in the stock price.

Figure 4 indicates that in contrast to the delta’s increasing monotone pattern, the grant’s vega displays a sinusoidal pattern, with increasing positive values alternating with decreasing (and even negative) values. Peak values occur in the high-incentive region between the vesting schedule’s long $K_l$ and short $K_h$ strike prices, with vega initially increasing due to convexity of the schedule’s payoff around $K_l$ and subsequently decreasing due to concavity of the schedule’s payoff around $K_h$. The increase (decrease, respectively) in risk-taking incentives reflects the fact that holders of p-v stock grants are long

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16 This region is characterized initially by a convex segment (due to convexity of the vesting schedule’s payoff at $K_l$) and subsequently by a concave segment (due to concavity of the payoff at $K_h$).

17 In our numerical example illustrated in Figure 2, the grant’s delta is close to zero in the no-incentive region (i.e. $s_0 \leq 20$), and its sensitivity to the underlying asset is very small, reflecting the region’s low-incentive characteristic. In this region, there is no incentive at all for grant holders to increase the underlying stock price because marginal price increases do not translate into the vesting of shares. Conversely, in the low-incentive region (i.e. $s_0 \geq 80$), marginal stock price increases only affect grant value through the value of the share grant holders are entitled to, since marginal price increases do not trigger the vesting of additional shares. The increase of delta in the underlying asset is accordingly slightly higher than in the no-incentive region. By contrast, in the high-incentive region (i.e. $20 < s_0 < 80$), the sensitivity of grant value to the underlying stock price compounds twice the same source of appreciation, i.e. the stock price. Ownership incentives are the strongest in this region, as they compound the alignment effects through both the stock price and the vesting schedule. Accordingly, the increasingness of delta in the underlying asset is highest because a price increase leads to a grant value appreciation through a higher value of the stock price and through the vesting of additional shares.
(short, respectively) performance above $K_l$ ($K_h$, respectively). Risk-taking incentives in this region are particularly high because a marginal increase in stock-return volatility has two effects on the expected grant value. First, it increases the likelihood of the long component being more in-the-money at maturity, which in turn leads to the vesting of additional shares. Second, it leads to a higher expected stock price at maturity.

By comparison, risk-taking incentives in the low-incentive region are increasing, albeit in smaller magnitude, because an increase in stock-return volatility can only lead to a price increase at maturity, and not to the vesting of additional shares. Finally, risk-taking incentives in the no-incentive region are very low; they increase as the long component of the vesting schedule becomes more in-the-money.

### 2.1.2 Comparison between incentives conveyed by p-v stock grants and t-v grants

As firms are progressively substituting t-v grants by p-v stock grants in managerial compensation packages, it is of interest to study how incentives conveyed by p-v stock grants compare with those conveyed by t-v grants such as stock options. Figure 3a shows that the delta and vega of p-v stock grants with standard price-contingent vesting schedules are considerably higher than those of t-v grants with comparable parameter values (see below).

While determining which grant features drive p-v stock grants’ high delta and vega compared to stock options is of interest from a corporate finance perspective, it is a priori not obvious. On the one hand, p-v stock grants potentially convert into a larger number of shares than t-v grants – yet, the latter provide unlimited upside potential. On the other hand, p-v stock grants bear the risk of vesting into zero share (assuming $n = 0$) whereas t-v grants always vest into one share. We thus investigate whether p-v stock grants’ high incentives are due to (i) the slope of the vesting schedule (i.e. the fact that these grants potentially convert into a higher number of shares than t-v grants), (ii) the vesting schedule’s underlying appreciation mechanism (i.e. the spread of European call options) or (iii) a combination of both.

To provide a like-for-like comparison of incentives between t-v grants and p-v stock grants, we need to disentangle in the latter the slope effect from the effect embedded in the spread of call options. We proceed in two steps. We first transform a p-v stock grant into its scaled equivalent, which we define as a p-v stock grant that converts into a maximum of one share (i.e. $\bar{n} = 1$).
We then compare the delta (vega, resp.) of the p-v stock grant with the delta (vega, resp.) of its scaled equivalent in order to isolate the component of the ownership (risk-taking, resp.) incentive that is conveyed by the slope of the delta (vega, resp.) function (i.e. \( \overline{\pi} \)). In a second step, we compare the delta (vega, resp.) of the scaled p-v stock grant with the delta (vega, resp.) of a European stock option on a dividend-paying asset with identical parameter values. This allows us to isolate the component of the p-v stock grant’s ownership (risk-taking, resp.) incentive that is conveyed by the spread of call options embedded in the vesting schedule.

For completeness, we recall the (well-known) expression for the grant-date value of a European stock option with strike price \( K \) and maturity \( T \) on a dividend-paying asset \( S \) under the risk-neutral probability measure\(^{18}\)

\[
v_0(S(T)) = s_0 e^{-qT} \Phi \left( \tilde{d} \right) - K e^{-rT} \Phi \left( \tilde{d} - \sigma \sqrt{T} \right), \quad S(0) = s_0 \quad (12)
\]

as well as the expressions for the stock option’s delta

\[
\delta := \frac{s_0}{100} e^{-qT} \Phi \left( \tilde{d} \right), \quad (13)
\]

and vega

\[
\nu := 0.01 e^{-qT} \phi \left( \tilde{d} \right) s_0 \sqrt{T}, \quad (14)
\]

where

\[
\tilde{d} := \frac{\log \left( \frac{s_0}{K} \right) + T \left( r - q + \frac{\sigma^2}{2} \right)}{\sigma \sqrt{T}}, \quad (15)
\]

with \( \Phi(\cdot) \) indicating the cumulative distribution function of the Gaussian variable, \( r \) the constant riskfree interest rate and \( q \) the expected dividend yield.

Figure 3a compares the delta of the p-v stock grant (blue line), the delta of the scaled-equivalent p-v stock grant (grey line), and the delta of the European stock option on a dividend-paying asset (yellow line) against the stock price at date 0. For comparative purposes, we assume that the option strike price equals the strike price of the long component of the p-v stock grant vesting schedule’s, i.e. \( K = K_l = 20 \). We further assume same parameter values for both grants, i.e. \( \sigma = 17\% \), \( r = 2.5\% \), \( T = 4 \), and \( q = 2\% \).

\(^{18}\)Stock price dynamics of a dividend-paying asset under the risk-neutral measure are given by (4).
Much to our surprise, we find that the bulk of ownership incentives conveyed by p-v stock grants is provided by the scaling factor $\eta$ (i.e. the slope of the vesting schedule) and not by the mechanism underlying the spread of call options nested in the vesting schedule. In other words, the strength of p-v stock grants’ ownership incentives stems from the fact that these grants potentially convert into a large number of shares rather than from the long and short positions on the schedule’s underlying asset.

By contrast, both the scaling factor in the slope of the vesting schedule (Figure 5a) and the spread of long and short call options (Figure 5b) convey substantial risk-taking incentives compared to stock options. That is, even after accounting for the scaling factor, the vesting schedule embedded in p-v stock grants provides substantial risk-taking incentives. Differences in risk-taking incentives between t-v grants and scaled p-v stock grants are sizeable (see Figure 5b). While the vega of the stock option exhibits an asymmetric bell-shaped property with no risk-taking incentive as the option becomes deep in-the-money, the vega of the scaled p-v stock grant displays a sinusoidal pattern with extremely high values arising between the long and short strike prices of the vesting schedule. In addition, the peak value, even on a scaled basis, is much higher than that of the stock option.

\[\text{Figure 3a. Price-based vesting schedule: delta of p-v stock grant, of scaled p-v stock grant, and of stock option}\]

\[\text{Figure 3b. Price-based vesting schedule: delta of scaled p-v stock grant vs. delta of stock option}\]

\[\text{In addition, as shown in Figure 3b, controlling for the scaling factor (i.e. assuming that } \eta = 1)\text{, we find that incentives provided by the scaled p-v stock grant (grey line) largely exceed those of the t-v grant (yellow line) for stock price values between the schedule’s long } K_l \text{ and short } K_h \text{ strike prices, where they combine the two appreciation sources in a multiplicative setting. Above the short strike price, the difference in incentives between the two grants diminishes. Interestingly, the t-v grant conveys stronger ownership incentives than the scaled p-v stock grant for low values of the grant’s underlying asset. This is due to the fact that the t-v grant’s potential upside is unlimited, giving rise to stronger incentives when the grant is, ceteris paribus, more out-of-the-money. In fact, as the latter comparison contrasts ownership incentives conveyed by a (long) call option versus an embedded spread of long and short call options, it essentially amounts to quantifying the implication of limiting potential upside gain on managers’ ownership incentives.}\]

21
2.1.3 What are the agency implications for both shareholders and debtholders of substituting t-v grants for p-v stock grants in managerial compensation packages?

Our findings indicate that, if anything, the agency implications of replacing stock options with p-v stock grants are non trivial. With regards to shareholder-manager agency issues, prior literature finds that firms in which managers are compensated with p-v grants, as opposed to t-v grants, are characterized by subsequent overperformance over the five consecutive years (Bettis et al., 2010) and are associated with an increase in pay-performance-sensitivity (Kuang and Qin (2006), Kuang and Qin (2009), and Bettis et al. (2016)) – thus suggesting that the use of p-v grants is likely to improve agency conflicts traditionally associated with the separation of ownership and control (i.e. shareholder-manager agency issues).

While our findings also indicate that shifting away from t-v grants to p-v stock grants (with a price-contingent vesting schedule) may convey a higher degree of alignment between managerial and shareholder interests, our paper is the first to show that the cost of achieving this higher alignment is substantial on at least two counts. First, in order for managers’ equity-based wealth to increase significantly with the underlying stock price, shareholders have to relinquish a relatively large number of shares, as p-v stock grants that convert into a small expected number of shares do not really achieve a higher
alignment of shareholder-manager interest than t-v grants. Empirically, this is indeed the case as p-v stock grants generally provide an “abnormally large number of shares” in comparison with t-v grants (Bettis et al., 2010). Failing to account for this cost results in an inaccurate estimation of the agency cost associated with shifting from t-v grants to p-v stock grants.

Second, we find that even p-v stock grants that convert into a lower number of shares convey risk-taking incentives that are substantial compared to those conveyed by t-v grants. This feature is likely to hurt both debtholders and shareholders’ interests. Indeed, the use of p-v stock grants unequivocally worsens debtholders’ interests by increasing the reward to risk taking. If this higher incentive is not accurately priced in risky debt, debtholders are worse off if managers are compensated with p-v stock grants instead of t-v grants.

The introduction of p-v stock grants is also likely to cause some distortion between managerial incentives and shareholders’ interests in the sense that rewarding managers for the taking of risk potentially shifts their objective away from the shareholder-(equity) value maximization. This is particularly the case for stock price values up to half-way in the high-incentive region, where p-v stock grant holders’ wealth is found to increase more following a marginal increase in underlying stock-return volatility than following a marginal increase in the underlying stock price, with the opposite holding for higher values of the stock price (see Figure 5d). This implies that p-v stock grants can only achieve an alignment of managerial and shareholder interests for stock price values in excess of a threshold.

By and large, our results suggest that introducing p-v stock grants as managerial compensation tools may provide mixed results. While they ameliorate manager-shareholder agency issues by providing significant ownership incentives, the agency cost for shareholders to ensure that managers’ interests align more with their own is substantial. In addition, because these grants convey a higher reward to risk taking, they may deteriorate both

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20 For example, Amgen CEO Kevin Sharer’s 2008 p-v compensation plan specifies an award of 36’000, 73’000 and 146’000 shares at, respectively, the threshold, target, and ceiling performance levels of three-year annualized total stock return (Bettis et al., 2016).

21 Indeed, comparing both incentive measures, Figure 5c contrasts the value gain per grant induced by a marginal increase in the underlying stock price and the value gain per grant induced by a marginal increase in underlying volatility. We find that ownership incentives are stronger than risk-taking incentives for stock prices that exceed the middle value of the high-incentive region. By contrast, risk-taking incentives are stronger than ownership incentives in the no-incentive region and in the first half of the high-incentive region. In other words, risk-taking incentives are higher (lower, respectively) than ownership incentives when the vesting schedule’s long component is, ceteris paribus, more (less, respectively) out-of-the-money and for stock price values lower than half-way between the vesting schedule’s long and short strike prices.
manager-shareholder agency issues (particularly for low stock price values) and manager-debtholder agency issues.

2.2 Path-dependent price-contingent vesting schedule

In this section, we explore a feature that so far has been unexplored in the compensation literature. We study the incentive properties of p-v stock grants that vest into shares according to vesting provisions measured with respect to the average of the performance metric, as opposed to the end-of-period value of the performance metric. Our focus is thus the study of p-v stock grants with a path-dependent vesting schedule rather than a standard vesting schedule. Whereas the concept of path dependence in p-v stock grants’ vesting schedule is novel to the literature on p-v stock grants, its usage in compensation policies dates back to at least a decade. For illustration, in 2004-2006, Credit Suisse granted as part of its executive compensation policy complex compensation grants with path-dependent vesting provisions labeled Performance Incentive Plans (PIPs) and Incentive Share Units (ISUs).

Our goal in studying the incentive properties of p-v stock grants with path-dependent vesting provisions is to investigate the impact of path dependence on shareholder-manager and debtholder-manager agency problems. Given that the value of path-dependent call options increases less with risk than the value of standard call options (as the exercise decision embedded in path-dependent vesting provisions is contingent on the entire path of the vesting schedule’s underlying asset), the path-dependent feature is expected to favor debtholder interests through a mitigation of the asset substitution problem. Moreover, because managers’ reward in shares depends on all values of the performance metric during the vesting period, path dependence can be expected to mitigate shareholder-manager agency conflicts by aligning managers’ short-termism with shareholders’ long-term horizon.
From an option pricing approach, path-dependent vesting provisions entail that the vesting schedule embeds underlying path-dependent call options, as opposed to standard call options. Vesting into shares depends on the entire path of the grant’s underlying asset, and the exercise decision is measured with respect to the average value over the vesting period. At the end of the \([0, T]\) period, the grant converts into \(n\) share(s) if the average underlying stock price performance between 0 and \(T\) is below a threshold value (which we denote by \(K_l\)) and into at most \(\pi\) share(s) if average stock price performance reaches a ceiling value (which we denote by \(K_h\)). In between the lowest and highest payoffs, a continuous and linear appreciation scheme is assumed. This latter assumption enables us to model the vesting schedule as the product of a spread of path-dependent European call options and the schedule’s slope, in addition to the minimum payoff \(n\).

Formally, the boundary condition of a p-v stock grant with a path-dependent vesting schedule is given by the product of the value of the stock price at \(T\) and the vesting schedule defined over \([0, T]\)

\[
S(T)g\left(G_S^{[0,T]}\right),
\]

where the expression for the value of the vesting schedule at \(T\) is given by

\[
g\left(G_S^{[0,T]}\right) = n + \alpha \left[\left(G_S^{[0,T]} - K_i\right)^+ - \left(G_S^{[0,T]} - K_h\right)^+\right],
\]

where \(\alpha = (\bar{n} - n)/(K_h - K_l)\) denotes the slope of the vesting schedule, with \(n\) and \(\bar{n}\) denoting, respectively, the minimum and maximum number of share(s) each grant can convert into at \(T\) if the average stock price over the entire period is respectively below \(K_i\), and equal or greater than \(K_h\), for \(\bar{n} > n \geq 0\) and \(K_h > K_i > 0\).

Under risk-neutral valuation, the grant-date value of a p-v stock grant with path-dependent vesting provisions is given by the following expectation

\[
v_0^s\left(S(T), G_S^{[0,T]}\right) = e^{-rT} \mathbb{E}^Q\left[S(T)g\left(G_S^{[0,T]}\right) | \mathcal{F}_0\right],
\]

with \(r\) denoting the constant risk-free rate. Stock price dynamics of a dividend-paying asset under the risk-neutral measure are given in the preceding section.

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\(22\) The spread therefore consists of a long call option and a short call option with identical maturities and identical (averaged) underlying asset but with the short strike price exceeding the long strike price. The slope is determined by the ratio of the difference in the highest (\(\pi\)) and lowest (\(n\)) payoffs to the difference in the spread’s short (\(K_h\)) and long (\(K_i\)) strike prices.
Computation of the expectation in (18) requires that we model the joint law of the stock price and its average, which entails specifying the averaging process of the stock price. Given that the arithmetic average of the stock price is not lognormally distributed, we assume a geometric averaging process in continuous time. This allows us to obtain an analytical expression for equation (18) since the stock price (which is assumed to follow a geometric Brownian motion) and its geometric average are jointly lognormally distributed. We thus define $G^{[0,T]}_S$ as the continuously-sampled geometric average of the logarithm of the stock price $S$ between 0 and $T$, i.e.

$$G^{[0,T]}_S := e^{\frac{T}{2} \int_0^T \log S(u) \, du}.$$ (19)

Since $\left( \int_0^T W(u) \, du, W(T) - W(0) \right)$ is equal in distribution to $\left( \frac{T^{3/2}}{\sqrt{3}} \xi, \sqrt{T} z \right)$, where $\xi$ and $z$ are standard Gaussian variables with correlation $\rho = \sqrt{3}/2$, we can – as far as their distribution is concerned – replace $\int_0^T W(u) \, du$ and $W(T) - W(0)$ by, respectively, $\frac{T^{3/2}}{\sqrt{3}} \xi$ and $\sqrt{T} z$.

Let $S(0) = s_0$ and define $G^{[0,T]}_S$ by $\tilde{h}(s)$. We can then rewrite (19)

$$G^{[0,T]}_S = \tilde{h}^{T \, s_0} \left( \frac{\sqrt{3}}{T^{3/2}} \int_0^T W(u) \, du \right) \overset{\text{dist.}}{=} \tilde{h}^{T \, s_0} (\bar{\xi}),$$ (20)

where

$$\tilde{h}^{s}(y) = se^{\frac{1}{2} \left( r - q - \frac{\sigma^2}{2} \right) t + \sigma \sqrt{T} y},$$ (21)

with the superscript in $\tilde{h}$ indicating the time interval over which the averaging process takes place, and the subscript the starting value of the averaging process.

Given (20) and (21), we can express (18) as follows

$$v^\ddot{a}_0 \left( s_0, \tilde{h}^{T \, s_0} \right) = s_0 \alpha e^{-\left( q + \frac{\sigma^2}{2} \right) T} \mathbb{E}^Q_0 \left[ e^{\sigma \sqrt{T} z} \left\{ \left( \tilde{h}^{T \, s_0} (\bar{\xi}) - K_l \right)^+ - \left( \tilde{h}^{T \, s_0} (\bar{\xi}) - K_h \right)^+ \right\} \right]$$ (22)

following an application of Itô’s lemma to the logarithm of the stock price and assuming $\underline{a} = 0$.

Solving the expectation in the preceding yields Proposition 4 which provides the closed-form formula for the value of a p-v stock grant with a path-dependent price-based vesting schedule.
Proposition 4 Under the assumptions of no arbitrage, complete markets, and replicability of the grant’s payoff, the expression for the grant-date value of the p-v stock grant with a path-dependent price-based vesting schedule under the risk-neutral probability measure is given by

\[ v_s(0, h_{s0}) = v_{0,l}(s_0, h_{s0}) - v_{0,h}(s_0, h_{s0}), \]  

(23)

where

\[ v_{0,j}(s_0, h_{s0}) = s_0 \alpha e^{-qT} \left( e^{\sigma \sqrt{T} \tilde{d}^j} + \sigma \rho \sqrt{T} \right) \]

\[ -K_j e^{-qT} \Phi \left( \tilde{d}^j + \sigma \nu' \sqrt{T} \right), \]

(24)

with

\[ \tilde{d}^j := \frac{\log \left( \frac{s_0}{K_j} \right) + \frac{1}{2} \left( r - q + \sigma^2 \right) T}{\sigma \sqrt{T}}, \]

(25)

for \( j \in \{l, h\} \) where \( \alpha = \pi/(K_h - K_l) \), \( \nu' := \rho - 1/\sqrt{3} \), and \( \rho = \sqrt{3}/2 \) is the coefficient of correlation of the joint probability distribution function of the standard bivariate Gaussian distribution and \( \Phi(.) \) denotes the cumulative distribution function of the standard Gaussian law.

Proof. See Appendix available upon request.\[ \]

2.2.1 Ownership incentives

In line with standard practice in the compensation literature, we define the ownership incentive associated with the p-v stock grant with a path-dependent price-based vesting schedule at date 0 by the grant’s delta, which

\[ v_s(0, h_{s0}) = s_0 \alpha e^{-qT} \phi \left( e^{\sigma \sqrt{T} \delta^j} + \sigma \rho \sqrt{T} \right) \]

\[ -K_j e^{-qT} \Phi \left( \delta^j + \sigma \nu' \sqrt{T} \right), \]

(24)

with

\[ \delta^j := \frac{\log \left( \frac{s_0}{K_j} \right) + \frac{1}{2} \left( r - q + \sigma^2 \right) T}{\sigma \sqrt{T}}, \]

(25)

for \( j \in \{l, h\} \) where \( \alpha = \pi/(K_h - K_l) \), \( \nu' := \rho - 1/\sqrt{3} \), and \( \rho = \sqrt{3}/2 \) is the coefficient of correlation of the joint probability distribution function of the standard bivariate Gaussian distribution and \( \Phi(.) \) denotes the cumulative distribution function of the standard Gaussian law.

Proof. See Appendix available upon request.\[ \]

2.2.1 Ownership incentives

In line with standard practice in the compensation literature, we define the ownership incentive associated with the p-v stock grant with a path-dependent price-based vesting schedule at date 0 by the grant’s delta, which

\[ v_s(0, h_{s0}) = s_0 \alpha e^{-qT} \phi \left( e^{\sigma \sqrt{T} \delta^j} + \sigma \rho \sqrt{T} \right) \]

\[ -K_j e^{-qT} \Phi \left( \delta^j + \sigma \nu' \sqrt{T} \right), \]

(24)

with

\[ \delta^j := \frac{\log \left( \frac{s_0}{K_j} \right) + \frac{1}{2} \left( r - q + \sigma^2 \right) T}{\sigma \sqrt{T}}, \]

(25)

for \( j \in \{l, h\} \) where \( \alpha = \pi/(K_h - K_l) \), \( \nu' := \rho - 1/\sqrt{3} \), and \( \rho = \sqrt{3}/2 \) is the coefficient of correlation of the joint probability distribution function of the standard bivariate Gaussian distribution and \( \Phi(.) \) denotes the cumulative distribution function of the standard Gaussian law.

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Proof. See Appendix available upon request.\[ \]
is given by the partial derivative of the grant’s value with respect to a one percent change in the underlying stock price, ceteris paribus.

**Proposition 5** The expression for the delta of the p-v stock grant with a path-dependent price-based vesting schedule at date 0 is given by

\[
\delta_{pd}^s := \frac{s_0}{100} \times \left[ \frac{\partial v_{0,j}^s \left( s_0, \tilde{h}_s^T \right)}{\partial s_0} - \frac{\partial v_{0,b}^s \left( s_0, \tilde{h}_s^T \right)}{\partial s_0} \right],
\]

(26)

where

\[
\frac{\partial v_{0,j}^s \left( s_0, \tilde{h}_s^T \right)}{\partial s_0} = \alpha \left\{ s_0 e^{\frac{1}{2} \left( r - 3q + 2 \sigma^2 \right) T} \left[ 2 \Phi \left( \tilde{d}_j^s + \sigma \rho \sqrt{T} \right) + \phi \left( \tilde{d}_j^s + \sigma \rho \sqrt{T} \right) \frac{\sqrt{3}}{\sigma \sqrt{T}} \right] 
- K_j e^{-qT} \left[ \Phi \left( \tilde{d}_j^s + \sigma \nu' \sqrt{T} \right) + \phi \left( \tilde{d}_j^s + \sigma \nu' \sqrt{T} \right) \frac{\sqrt{3}}{\sigma \sqrt{T}} \right] \right\},
\]

(27)

for \( j \in \{l, h\} \).

2.2.2 Risk-taking incentives

In line with the compensation literature, we define the risk-taking incentive of a p-v stock grant with a price-based path-dependent vesting schedule at date 0 by its vega, defined as the partial derivative of the grant’s value with respect to a 0.01 change in underlying volatility, holding everything else constant.

**Proposition 6** The expression for the vega of the p-v stock grant with a price-based path-dependent vesting schedule at date 0 is given by

\[
\nu_{pd}^s := 0.01 \times \left[ \frac{\partial v_{0,l}^s \left( s_0, \tilde{h}_s^T \right)}{\partial \sigma} - \frac{\partial v_{0,h}^s \left( s_0, \tilde{h}_s^T \right)}{\partial \sigma} \right],
\]

(28)
where

\[
\frac{\partial v_{0,j}^{s}}{\partial \sigma} (s_{0}, \tilde{h}_{s_{0}}^{T}) = s_{0} \alpha \left\{ s_{0} e^{\frac{1}{2}(r-3q+\frac{s}{6} \sigma^{2})T} \left[ \left( \frac{5}{6} \sigma T \right) \Phi \left( -\tilde{d}_{j}^{s} + \sigma \rho \sqrt{T} \right) 
\right.
\right.
\]

\[
\left. + \phi \left( -\tilde{d}_{j}^{s} + \sigma \rho \sqrt{T} \right) \left( \frac{\partial \left( -\tilde{d}_{j}^{s} \right)}{\partial \sigma} + \rho \sqrt{T} \right) \right] \left. \right\} - K_{j} e^{-qT} \phi \left( -\tilde{d}_{j}^{s} + \sigma \nu \sqrt{T} \right) \left( \frac{\partial \left( -\tilde{d}_{j}^{s} \right)}{\partial \sigma} + \nu \sqrt{T} \right) \left. \right\} ,
\]

for \( j \in \{ l, h \} \), \( \alpha = \pi / (K_{h} - K_{l}) \), and \( n = 0 \).

### 2.2.3 Agency implications of p-v stock grants with path-dependent vesting schedules

We now explore how path-dependent vesting provisions affect incentives conveyed by p-v stock grants, by contrasting these grants’ incentives with those of p-v stock grants that do not nest a path-dependent feature, i.e. p-v stock grants with standard vesting schedules.

For comparative purposes, we define all parameter values as in the example of a p-v stock grant with a standard price-based vesting schedule (see preceding Section). We thus consider a p-v stock grant that vests into \( n = 0 \) share if the average stock price between dates 0 and \( T \) is below \( K_{l} = 20 \) and into \( n = 5 \) shares if the average stock price between dates 0 and \( T \) is greater than \( K_{h} = 80 \), assuming a continuous and linear payout schedule between \( K_{l} \) and \( K_{h} \). We further assume same parameter values, namely \( \sigma = 17\% \), \( r = 2.5\% \), \( T = 4 \), and \( q = 2\% \).

We first study the impact of path dependence on ownership incentives. Figure 6 compares the delta of a p-v stock grant with a path-dependent vesting schedule with the delta of a p-v stock grant with a standard vesting schedule against the stock price at 0. The comparison suggests that the impact of path-dependent vesting provisions on shareholder-manager agency issues is mitigated, as path dependence conveys a lower alignment (i.e. a lower delta) of shareholder-manager interest for a subset of stock prices and a higher alignment (i.e. a higher delta) of shareholder-manager interest for
other subsets of stock prices. In comparison with p-v stock grants that vest according to a standard vesting schedule (grey line), ownership incentives are stronger under path dependence in the second half of the high-incentive region and the beginning of the low-incentive region, but lower in the remainder of the low-incentive region, the no-incentive region and the first half of the high-incentive region. This feature contrasts with the monotone increase of the delta of a standard vesting schedule in the underlying asset (grey line).

What are these differences attributed to? Since path dependence entails that a price increase at any date affects the degree of moneyness of the vesting schedule’s long and short components, the number of shares vested at maturity depends on the value of the stock price on each single day of the vesting period. In the no-incentive region (where the long and short strike prices are both out-of-the-money) and first half of the high-incentive region (where the long strike price is in-the-money and the short strike price out-of-the-money), the sensitivity of ownership incentives is lower under path dependence. This is the case because the stock price at date 0 being so low, a marginal price increase has no effect on the vesting of shares since it is very unlikely that the average stock price will exceed the threshold value $K_l$ at maturity. By comparison, in the first half of the high-incentive region (where the long strike price is in-the-money and the short strike price out-of-the-money), the stock price at date 0 is so low that a marginal increase in the stock price results in a proportionally smaller increase in the payout in shares under path dependence than under non-path dependence. In other words, the reward to a marginal stock price increase in terms of additional alignment is lower under path dependence.

By contrast, in the second half of the high-incentive region and the first half of the low-incentive (where the long and short strike prices are both in-the-money), the stock price at date 0 is sufficiently high such that a marginal
price increase has a proportionally higher impact under path dependence on the number of shares the grant converts into at maturity. The stronger effect under path dependence is due to the fact that since all price increases carry over until maturity, grant holders are incentivized to over-perform on each day of the vesting period because the reward in terms of shares depends on all stock prices during the vesting period.

In comparison, in the second part of the low-incentive region, the lower sensitivity of ownership incentives arises because for sufficiently high price values, a marginal price increase has no effect on the vesting of shares since the average stock price exceeds the ceiling value $K_h$. Hence, path dependence contributes to weakening incentives to increase the underlying stock price.

We next study the impact of path dependence on risk-taking incentives. Figure 7 contrasts the vega of a p-v stock grant with a path-dependent vesting schedule (blue line) with the vega of a p-v stock grant with a standard vesting schedule (grey line) against the stock price at 0. We find that path dependence substantially reduces risk-taking incentives, as vega is much lower in path-dependent vesting schedules for all values of the grant’s underlying asset.

This finding has important agency implications because it entails that path dependence contributes to making risk taking less attractive vis-a-vis standard vesting schedules. Indeed, while managers holding p-v stock grants with standard vesting schedules (grey line) may have an incentive to manipulate the stock price at date 0 (when it is located halfway between the long and short strike prices of the vesting schedule and as the stock price increases above the short strike price) in order to increase grant value, this incentive is considerably weaker if they hold p-v stock grants with a path-dependent vesting schedules (blue line). It is nonexistent for stock price values around the short strike price where path-dependent vesting schedules clearly disincentivize price manipulation.

By lowering the return to risk taking, path dependence necessarily affects both shareholder-manager and manager-debtholder agency conflicts. The latter obviously improve because managers holding p-v stock grants with path-dependent vesting schedules have very little to virtually no incentive to shift to riskier assets because the vega is considerably lower under path dependence. This is particularly the case for stock price values greater than

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24 Differences are particularly striking in both the midst of the high-incentive region (where peak values are considerably lower in the case of path-dependent vesting schedules) and for high values of the underlying asset (which are associated with a substantially lower vega under path dependence).
the short strike price where grant value barely increases with stock-return volatility. Path dependence is thus likely to improve manager-debtholder agency conflicts by making ex-post shifting to riskier assets less rewarding, thereby making the asset substitution problem less severe.

Manager-shareholder agency issues are also likely to improve. Shareholders are better off when managers have less incentive to manipulate stock prices because price manipulation is arguably conducive to a lower alignment between managers and shareholders, as it induces the former to taking actions that allow them to reap short-term profits whereas the latter instead favor actions that create long-term shareholder value[25]. Path dependence can thus be seen as inducing a greater alignment of shareholder-manager interest.

However, the net impact of path-dependent vesting provisions on shareholder-manager agency issues is ambiguous. Whereas shareholders also stand to gain from reduced opportunistic risk-taking behavior (as it reduces managers’ incentive to depart from the shareholder value-maximizing objective), the fact that path dependence has been shown to lead to lower ownership incentives for certain values of the stock price indicates that path dependence also affects shareholder interests through agency issues associated with the separation of ownership and control. For those stock price values, the net effect of path-dependent vesting provisions on manager-shareholder agency conflicts is thus à priori indeterminate since managers’ behavior is affected in opposite ways: While on the one hand, they are being induced into much less reckless risk-taking behavior, on the other hand, they can either share a higher or a lower degree of alignment with shareholders.

Indeed, a paradox of rewarding managers with compensation grants with convex payoff structures (such as stock options and p-v stock grants) is that by providing them with a means of overcoming the risk aversion implied by their poorly diversified wealth, it leads them instead to taking actions that maximize the payoff of the grants they hold rather than the value of shareholders’ equity.
If a comparison between a grant’s delta and its vega is of any interest, it can be argued that what matters to shareholders, debtholders and regulators is whether, under path dependence, the change in grant value due to a marginal increase in the stock price exceeds the change in grant value due to a marginal increase in stock-return volatility. Comparison of Figure 5c and Figure 8 indicates that path-dependent vesting schedules achieve a higher degree of alignment than standard vesting schedules because the interval of price values over which managers and shareholders’ interests are at odds is much smaller. In particular, compared to standard vesting schedules, path-dependent vesting schedules convey a stronger alignment for average stock prices close to the vesting schedule’s long strike price.

Our findings have important implications. From a regulatory viewpoint, introducing path dependence in p-v stock grants’ vesting schedules can provide a means of mitigating these grants’ very strong risk-taking incentives without significantly affecting their ownership incentives. From an agency perspective, firms awarding p-v stock grants with path-dependent vesting schedules can ensure that shareholder and manager interests are more aligned for low stock price values, and can improve shareholder-debtholder agency issues by lessening *ex ante* the severity of the asset substitution problem.
3 P-v stock grants with accounting-based vesting schedules

3.1 Valuation

Having explored the incentive properties of p-v stock grants that vest according to (stock) price-only performance conditions, we now turn to a type of p-v stock grants that is increasingly being used in managers’ compensation packages, i.e. p-v stock grants that vest with accounting metrics. Although relatively uncommon in the traditional compensation literature, the contingency on accounting metrics is becoming a dominant feature of compensation grants, as “accounting performance metrics have displaced to some extent stock price metrics” (Bettis et al. 2016).

By construction, such grants vest into shares according to an accounting-based schedule which establishes that the number of shares is solely contingent on accounting performance. Yet, even if stock price increases do not per se lead to the vesting of additional shares, grant value is ultimately impacted by price increases in an explicit fashion (as grant value is function of the stock price since value converts into a number of shares) and in an implicit fashion through the vesting schedule (due to the correlation between stock price performance and accounting performance).

From a valuation standpoint, the contingency of the grant’s payoff on the performance of both the stock price and earnings per share entails at least two implications. First, because the grant’s payoff depends on the performance of traded (i.e. stock price) and nontraded (i.e. earnings per share) assets, risk-neutral valuation is not applicable since the risk associated with the nontraded asset cannot be hedged.\(^{26}\) Valuation of these p-v grants is thus computed on a risk-adjusted basis under the physical measure. Second, the valuation of these grants is highly dependent on the assumptions made regarding the underlying dynamics of the stock price and the accounting measure. Indeed, as the p-v grant’s boundary condition involves two correlated random payoffs, assumptions must be formulated with respect to the joint law of the stock price and the accounting metric.

Given that earnings per share represent the most widespread accounting metric used in grants’ vesting schedules (Bettis et al., 2010, Bizjak et al. 2016), we proceed with the valuation of p-v stock grants with vesting

\(^{26}\)See Holden and Kim (2013) for an alternative approach.
provisions contingent on earnings performance. We use an earnings-based (as opposed to a dividend-based) stock valuation model to account for the instantaneous correlation between stock price and earnings per share. We rely on Bakshi and Chen’s (2005) extended version of the Gordon model in continuous time. In its simplest form – which we require for tractability – both earnings per share and the stochastic discount factor follow a geometric Brownian motion with constant drift and volatility, and dividends per share represent a constant fraction of earnings.

Under the complete markets and no-arbitrage assumptions (which we impose in order to guarantee the uniqueness of the stochastic discount factor), the stochastic discount factor and earnings are assumed to have the following dynamics under the physical measure

\[
\frac{dM(t)}{M(t)} = -\mu_m \, dt - \sigma_m \, dW_m(t),
\]

\[
\frac{dX(t)}{X(t)} = g \, dt + \sigma \, dW_x(t),
\]

where \(\mu_m\) and \(g\) denote the constant drift rate of, respectively, the pricing kernel and earnings per share, and \(\sigma_m\) and \(\sigma\) their respective constant volatilities. The processes \(\{W_m(t)\}_{0 \leq t \leq T}\) and \(\{W_x(t)\}_{0 \leq t \leq T}\) represent \(\mathbb{P}\)-standard Brownian motions with respect to the filtration \(\{\mathcal{F}_t\}_{0 \leq t \leq T}\). We assume

\[
\mathbb{E}^\mathbb{P}[W_x(s)W_m(t)] = \rho_x \min\{s, t\}, \forall s, t \geq 0,
\]

which implies that the correlation between earnings shocks and systematic shocks is given by \(\langle W_x, W_m \rangle_t = \rho_x t\) (See Bakshi and Chen). In addition, we assume that the dividend equals earnings per share times the payout ratio, i.e. \(D(t) = (1 - p)X(t)\), where \(p\) denotes the constant plowback ratio, for \(p < 1\). We also assume a constant dividend yield, i.e. \(q = D(t)/S(t)\).

According to the assumptions of Bakshi and Chen’s extended version of the Gordon model in a continuous-time setting, equity per share \(S\) at any given date is defined as the infinite sum of future expected dividends discounted by the stochastic discount factor

\[
S(t) = (1 - p) \int_t^\infty \mathbb{E}^\mathbb{P}\left[\frac{M(u)}{M(t)}X(u) \mid \mathcal{F}_t\right] \, du,
\]

\[27\] The assumption that earnings per share follow a geometric Brownian motion implies that earnings cannot be negative.

\[28\] Our assumptions imply the following dividend stream \(\{D(t)\}_{0 \leq t \leq T}\), i.e. \(D(t) = (1 - p)X(t)\).
where $\mathcal{F}_t$ denotes the smallest sigma-algebra containing $\{W_m(s)\}_{s \leq t}$ and $\{W_x(s)\}_{s \leq t}$ and $\mathbb{E}^\mathbb{P}[.|\mathcal{F}_t]$ the $\mathcal{F}_t$-conditional expectation operator under $\mathbb{P}$.

Solving the expectation in (32) yields the following expression for the risk-adjusted value of equity per share $S$ at date $t > 0$

$$S(t) = \left(\frac{1 - p}{r + \rho_x \sigma_m - g}\right) X(t),$$

(33)

for $r + \rho_x \sigma_m > g$ (Appendix available upon request).

Equation (33) characterizes the contemporaneous relation between earnings per share and the stock price. It states that earnings per share and the stock price have identical dynamics, and the stock price equals the fraction of earnings per share paid out as dividend discounted at the risk-free rate minus the risk-adjusted drift of earnings, $g - \rho_x \sigma_m$. The latter equals the difference between the physical drift and the risk premium.

Because of the contingency of the vesting schedule on a nontraded asset (earnings per share), risk-neutral valuation is not applicable. We instead compute the risk-adjusted grant-date value, which is given by the expectation under the physical measure of the grant’s discounted final payoff

$$v_0^\mathbb{P} \left( S(T), G_{X}^{[0,T]} \right) = \mathbb{E}^\mathbb{P}_0 \left[ \frac{M(T)}{M(0)} S(T) f(X) \right],$$

(34)

where $S(T)f(X)$ represents the grant’s boundary condition (to be defined below), with $M$ and $X$ dynamics given by, respectively, (30) and (31).

In order to compute the expectation in the preceding expression, we first operate a change of measure. To this end, we define

$$\zeta_u = -\int_0^u \sigma_m \, dW_m(v),$$

(35)

and the Doléans-Dade exponential

$$\mathcal{E} (\zeta_u) = e^{-\int_0^u \sigma_m \, dW_m(s) - \frac{1}{2} \int_0^u \sigma_m^2 \, ds}.$$  

(36)

By Girsanov’s theorem, there exists a probability measure $\mathbb{P}^\ast$ such that its Radon-Nikodým density equals the Doléans-Dade exponential

$$\frac{d\mathbb{P}^\ast}{d\mathbb{P}} \bigg|_{\mathcal{F}_u} = \mathcal{E} (\zeta_u),$$

(37)
∀u ≥ 0 \[29\]

It can be shown that if, in addition, we assume a constant dividend yield, i.e. \( q = D(t)/S(t) \), valuation of the p-v stock grant with an earnings-based vesting schedule under the risk-adjusted probability measure is equivalent to its valuation under the risk-neutral probability measure, i.e. \( dX(t)/X(t) = (g - \rho_x \sigma_m) \, dt + \sigma \, dW(t) = (r - q) \, dt + \sigma \, dW(t) \), and the contemporaneous relation between the stock price and earnings per share under the latter is given by

\[
S(t) = \left( \frac{1 - p}{q} \right) X(t), \quad (38)
\]

∀q > 0 \[30\]

Hence, under the assumptions listed above, we can equivalently value (34) under the risk-neutral measure, i.e.

\[
v^x_0 \left( S(T) \right) = e^{-rT} \mathbb{E}^Q \left[ S(T) \left\{ (X(T) - X_d)^+ - (X(T) - X_u)^+ \right\} \right]. \quad (39)
\]

We first explore p-v stock grants with a standard earnings-based vesting schedule, i.e. a schedule composed of standard European call options. We then explore the extent to which path dependence in the vesting schedule has an effect on the grants’ incentives.

For grants that embed a standard earnings-based vesting schedule, the value of the vesting schedule is given by

\[
f(X) = \frac{\pi}{X_u - X_d} \left[ (X(T) - X_d)^+ - (X(T) - X_u)^+ \right]. \quad (40)
\]

Valuation of the grant under the risk-neutral probability measure involves discounting the grant’s final payoff at the risk-free rate, i.e.

\[
v^x_0 \left( X(T) \right) = e^{-rT} \frac{\pi}{X_u - X_d} \mathbb{E}^Q \left\{ S(T) \left\{ (X(T) - X_d)^+ - (X(T) - X_u)^+ \right\} \right\}. \quad (41)
\]

\[29\] Since \( W_z := \{W_z(t)\}_{0 \leq t \leq T} \) is a \( P \)-standard Brownian motion, \( W := \{W(t)\}_{0 \leq t \leq T} \) is a standard Brownian motion under the risk-adjusted measure \( P^* \) defined as \( W(t) = W_z(t) - \langle W_z, \xi \rangle_t = W_z(t) + \rho_x \sigma_m t \).

\[30\] Indeed, if we define \( q \) as the expected dividend yield, where \( q = D(t)/S(t) \), we can rewrite equation (34) as follows \( g - \rho_x \sigma_m = r - q \). Hence, the risk-adjusted drift under the probability measure \( P^* \) is equal to the risk-neutral drift of a dividend-paying asset. Since the right-hand side of the preceding expression represents the risk-neutral drift of a dividend-paying asset, the expression provides the constraint on the diffusion coefficient of the stochastic discount factor required in order to be under the risk-neutral measure, i.e. \( \sigma_m = \frac{g - (r - q)}{\rho_x} \). The equality of drifts arises because the pricing condition can equivalently be rewritten as the condition to obtain the equivalent martingale measure (e.m.m.), i.e. that the cum-dividend discounted price process be a martingale (see also Cochrane 2005). The equivalence between the e.m.m and the pricing condition of the extended version of the Gordon model assumed in this paper therefore results in the risk-adjusted drift and the risk-neutral drift being equal if the preceding is satisfied.
where risk-neutral dynamics of earnings per share are given above.

Solving the preceding yields Proposition 7.

**Proposition 7** Under the assumptions of no arbitrage, complete markets, and replicability of the grant’s payoff, the expression for the value at date 0 under the risk-neutral probability measure of the p-v stock grant with a standard earnings-based vesting schedule is given by

\[
v^*_0 (X(T)) = v^*_0,d (X(T)) - v^*_0,u (X(T)),
\]

where

\[
v^*_{0,k} (X(T)) = s_0 \beta \left\{ x_0 e^{-\frac{1}{2} q^2 T} \Phi \left( -d^*_k + 2 \sigma \sqrt{T} \right) - X_k e^{-\frac{1}{2} \left( r + q - \frac{q^2}{2} \right) T} \Phi \left( -d^*_k + \sigma \sqrt{T} \right) \right\},
\]

with \( \beta = \overline{\pi}/(X_u - X_d) \), and

\[
d^*_k := \frac{\log \left( \frac{X_k}{x_0} \right) - \frac{1}{2} \left( r - q - \frac{q^2}{2} \right) T}{\sigma \sqrt{T}},
\]

for \( k \in \{d, u\} \). Appendix available upon request.

We now explore the incentive properties of p-v stock grants that vest according to accounting-based metrics.

**Ownership incentives.** In this section, we explore how the contingency of p-v stock grant value on the accounting metric affects ownership incentives conveyed by these grants. The alignment effect these grants convey is different from that of p-v stock grants with a price-only vesting schedule in that the underlying asset of the vesting schedule is an accounting metric; vesting into shares depends, therefore, only on the performance of the accounting metric. We show that p-v stock grants that vest with accounting metrics exhibit two additive sources of manager-shareholder alignment. Indeed, the total value gain associated with a marginal price increase decomposes into the sum of a **direct** gain associated with a change in the underlying stock price (holding earnings per share constant) and an **indirect** gain that arises through earnings per share within the grant’s vesting schedule. The indirect effect captures the fact that earnings per share likely increase following a
marginal increase in the stock price due to the correlation between the stock price and the accounting metric, which in turn results into the vesting of additional shares. Hence, an increase in the stock price leads to an increase in grant value through both a higher stock price and the vesting of additional shares.

Formally, the total derivative of $w := h(S, f(S))$ with respect to the stock price of a p-v stock grant with an accounting-based vesting schedule is composed of the sum of two components, i.e. a direct component and an indirect component through earnings per share

$$\frac{dw}{dx} \bigg|_{x=S} = \frac{\partial h}{\partial x} \bigg|_{x=S} + \frac{\partial h}{\partial y} \frac{dy}{dx} \bigg|_{x=S, y=X},$$

(45)

where $\partial h(,)/\partial u$ denotes the first-order partial derivative with respect to $u$ holding all other variables constant and $dX/dS = q/(1 - p)$ for $p < 1$.

It is instructive to compare the analytical expression in (45) with Bettis et al.’s (2016) definition of the delta of a grant with an accounting-based vesting schedule. Relying on numerical approaches, they define the grant’s aggregate delta as the total sensitivity of grant value to a marginal increase in the stock price arising through both the value of the stock price of the share(s) grant holders are entitled to at vesting and through the vesting schedule.

For comparative purposes, we define the delta of a p-v stock grant with an earnings-based vesting schedule in a similar fashion as in Bettis et al.

$$\delta^{agg} = \underbrace{\delta^s}_{\text{direct effect}} + \underbrace{\frac{q}{1 - p} \delta^x}_{\text{indirect effect through vesting schedule}},$$

(46)

where

$$\delta^s := \frac{\partial v^x_0 (X(T))}{\partial s_0} \bigg|_{x_0=\text{cst}} \times \frac{s_0}{100},$$

(47)

$$\delta^x := \frac{\partial v^x_0 (X(T))}{\partial x_0} \bigg|_{s_0=\text{cst}} \times \frac{x_0}{100}.$$

(48)

**Proposition 8** The expression for the delta at date 0 of the p-v stock
grant with a standard earnings-based vesting schedule is given by

\[
\delta_{\text{agg}} = \left[ \frac{\partial v_{0,d}^x(X(T))}{\partial s_0} \bigg|_{x_0 = \text{cst}} - \frac{\partial v_{0,u}^x(X(T))}{\partial s_0} \bigg|_{x_0 = \text{cst}} \right] \times \frac{s_0}{100} \\
+ \left[ \frac{\partial v_{0,d}^x(X(T))}{\partial x_0} \bigg|_{s_0 = \text{cst}} - \frac{\partial v_{0,u}^x(X(T))}{\partial x_0} \bigg|_{s_0 = \text{cst}} \right] \times \frac{s_0}{100 \left( 1 - p \right)},
\]

(49)

where

\[
\frac{\partial v_{0,k}^x(X(T))}{\partial s_0} \bigg|_{x_0 = \text{cst}} = \beta \left\{ x_0 e^{-(q-\frac{3}{2}\sigma^2)T} \Phi \left( -d_k^x + 2\sigma \sqrt{T} \right) \\
- X_k e^{-\frac{1}{2} \left( r+q-\frac{3}{2}\sigma^2 \right)T} \Phi \left( -d_k^x + \sigma \sqrt{T} \right) \right\}, \tag{50}
\]

\[
\frac{\partial v_{0,k}^x(X(T))}{\partial x_0} \bigg|_{s_0 = \text{cst}} = s_0 \beta \left\{ e^{-(q-\frac{3}{2}\sigma^2)T} \Phi \left( -d_k^x + 2\sigma \sqrt{T} \right) \\
+ e^{-\frac{1}{2} \left( r+q-\frac{3}{2}\sigma^2 \right)T} \phi \left( -d_k^x + 2\sigma \sqrt{T} \right) \frac{1}{\sigma \sqrt{T}} \\
- X_k e^{-\frac{1}{2} \left( r+q-\frac{3}{2}\sigma^2 \right)T} \phi \left( -d_k^x + \sigma \sqrt{T} \right) \frac{1}{\sigma \sqrt{T} x_0} \right\}. \tag{51}
\]

for \( k \in \{d, u\} \). Appendix available upon request.

Expression (46) is equivalent to what Bettis et al. (2016) term the grant’s aggregate delta which combines the two effects of the stock price on grant value. It is composed of the sum of the sensitivity of the grant value to a one percent increase in the stock price (which they define as the marginal stock delta \( \delta^s \)) and the product of the sensitivity of the grant value to a one percent increase in the accounting metric (which they define as the marginal accounting delta \( \delta^x \)) and the partial derivative of the stock price to the accounting metric. While the direct impact of the stock price arises through the value of the stock grant holders are entitled to at vesting, the indirect impact arises through the correlation between stock price and accounting performance. Hence, the impact of the accounting metric is only through
We now explore ownership incentives conveyed by p-v stock grants with an accounting-based vesting schedule. For illustration, we consider the numerical example of a p-v stock grant whose conversion into shares is determined by a vesting schedule which yields \( n = 0 \) share if earnings per share at \( T \) are below \( X_d = 2 \) and \( n = 5 \) shares if earnings per share at \( T \) are greater than \( X_u = 6 \), where \( X_d \) and \( X_u \) represent the vesting schedule’s long and short strike prices. Other parameter values are defined similarly as in previous sections, i.e. \( \sigma = 17\% \), \( r = 2.5\% \), \( T = 4 \), and \( q = 0.1 \). In addition we assume a plowback ratio of \( p = 0.7 \).

Figure 9a provides a comparison of the extent to which the grant’s aggregate ownership is driven by stock price performance (marginal stock delta) or by accounting performance (marginal accounting delta). Our finding indicates that the alignment effect conveyed by the accounting metric is significant – in line with Bettis et al. (2016) – but need not be. Indeed, we find that aggregate and marginal ownership incentives exhibit a wide divergence of values. This is due to the fact that because the marginal accounting delta arises through the vesting schedule, its value is affected by the degree of moneyness of the long and short strike prices of the schedule. Graphically, the marginal accounting delta (orange line) is characterized by an asymmetric bell-shaped function of the stock price at date 0, due precisely to the vesting schedule. For low values of the stock price, the value of the marginal

\[ \text{marginal stock delta} \]

\[ \text{marginal accounting delta} \]

\[ \text{aggDelta} \]

\[ \text{value of underlying asset at 0} \]

\[ \text{value of underlying asset at 0} \]

---

31 It should be noted that we cannot estimate or compute the additional incentive conveyed by accounting-based metrics since the expression for delta differs whether vesting occurs according to stock price or accounting metrics. Option pricing techniques provide a rationale for why ownership incentives are lower in the case of a price-dependent vesting schedule than in that of an earnings-dependent vesting schedule. Whereas the former is shown to compound the effects occurring through the stock price (of the share entitled to at vesting) and the vesting schedule in a multiplicative fashion, the latter disentangles into two effects in an additive fashion. Indeed, because value creation in the case of p-v stock grants with an accounting-dependent vesting schedule may arise from two correlated performance measures (i.e. the stock price and earnings per share), the delta of these stock grants decomposes into a direct effect arising from the stock price only (i.e. holding earnings per share constant) and an indirect effect arising from the stock price through earnings per share.
accounting delta is considerable. For higher values of the stock price, it is lower.

As in Bettis et al. (2016), we explore whether the contribution of the accounting channel to the grant’s ownership incentive is economically meaningful. In order to match Bettis et al. (2016), we define the contribution of the accounting channel in percentage of the contribution of the stock price channel as $100\frac{q\delta x}{(1-p)\delta s}$. Figure 10 plots the percentage represented by the accounting channel against the stock price at date 0. Whereas Bettis et al. (2016) find that, “[f]or p-v stock grants based on a single accounting metric, the accounting channel supplements marginal stock price delta to enhance executive delta incentives by approximately 72%,” our results indicate a more nuanced picture. While the contribution of the accounting channel is close to 180% that of the stock price when the latter equals 0, it is extremely low when the stock price equals 30 (in our example, the 72% relative contribution of the accounting metric arises only for the stock price value between $s_0 = 10$ and $s_0 = 11$). Hence the relative alignment effect conveyed by the accounting metric is initially very high and decreases exponentially towards zero for higher values of the stock price.

Consequently, our findings indicate that the alignment effect conveyed by the accounting metric depends on the degree of moneyness of the vesting schedule’s long and short components. Indeed, as shown in Figure 9b (which is identical to Figure 9a but for a smaller interval of stock price values), comparing marginal stock delta with marginal accounting delta, we find that for low values of the stock price, the latter exceeds the former, whereas the opposite holds for high values of the stock price.

**Risk-taking incentives.** Given that the assumptions we use for Bakshi and Chen’s extended version of the Gordon model in continuous time imply that the only source of uncertainty driving the stock price is that driving earnings per share, the vega of p-v stock grants with an earnings-based vest-
ing schedule measures the change in grant value that results from a marginal increase in stock-return (or, equivalently, earnings) volatility.

Therefore, in line with common practice in the compensation literature, we define the grant’s vega by the partial derivative of grant value to a 0.01 increase in stock-return volatility, holding everything else constant.

**Proposition 9** The expression for the vega at date 0 of the p-v stock grant with a standard earnings-based vesting schedule is given by

\[
\nu^x = 0.01 \times \left[ \frac{\partial v_{0,d}^x (X (T))}{\partial \sigma} \bigg|_{x_0,s_0=cst} - \frac{\partial v_{0,u}^x (X (T))}{\partial \sigma} \bigg|_{x_0,s_0=cst} \right],
\]

(52)

where

\[
\frac{\partial v_{0,k}^x (X (T))}{\partial \sigma} \bigg|_{s_0,x_0=cst} = s_0 x_0 e^{-(q-\frac{1}{2} \sigma^2)} T^\beta \left\{ 3 \sigma T \Phi \left( -d_k^x + 2 \sigma \sqrt{T} \right) 
\right.
\]

\[+ \phi \left( -d_k^x + 2 \sigma \sqrt{T} \right) \left( \frac{\partial (-d_k^x)}{\partial \sigma} + 2 \sqrt{T} \right) \} 
\]

\[-X_k s_0 e^{-(r+q-\frac{1}{2} \sigma^2)} T \frac{\sigma T}{2} \Phi \left( -d_k^x + \sigma \sqrt{T} \right) \]

\[+ \phi \left( -d_k^x + \sigma \sqrt{T} \right) \left( \frac{\partial (-d_k^x)}{\partial \sigma} + \sqrt{T} \right) \},
\]

(53)

for \( k \in \{d, u\} \).

As shown in Figure 11, vega is positive and increasing for higher values of the stock price. Compared to price-based vesting schedules, vega exhibits the same sinusoidal pattern, albeit lower in magnitude.

### 3.2 Path-dependent earnings-based vesting schedule

In this section, we study the incentive properties of p-v stock grants with path-dependent earnings-based vesting schedules. The study of such incentives is of interest. Whether such p-v stock grants provide less scope for earnings management and manipulation is unclear. The recent empirical literature provides little guidance. While Bettis et al. (2010) find no evidence of earnings manipulation, Bizjak, Hayes and Kalpathy (2015) report earnings
management or misreporting, and stock price manipulation close to targets. Yet, if anything, results reported in the preceding section indicate that the benefit from increasing stock-return volatility is not at the threshold or ceiling values, but at values of the stock price located between the long and short strike prices of the earnings-based vesting schedule.

From an option pricing approach, when the vesting into shares is determined according to average performance throughout the vesting period, the grant’s vesting schedule includes a spread of long and short path-dependent call options, where each option is characterized by a fixed strike price and an average price. To obtain a closed-form expression for the grant’s value, we assume a geometric averaging process in continuous time, and thus define \( G_{X}^{[0,T]} \) as the continuously-sampled geometric average of the logarithm of earnings per share \( X \) between 0 and \( T \), i.e.

\[
G_{X}^{[0,T]} := e^\frac{1}{T} \int_0^T \log X(u) \, du. \tag{54}
\]

A path-dependent earnings-based vesting schedule entails that the grant vests into shares according to average earnings per share performance. The grant converts at \( T \) into \( n \) shares if average earnings per share between 0 and \( T \) is below \( X_d \) and into \( \pi \) if average earnings per share exceed \( X_u \). Assuming a continuous and linear payout schedule, we model the value of the path-dependent earnings-contingent vesting schedule as the sum of \( n \) and the product of the schedule’s slope and a spread of European path-dependent call options. In this case, the grant’s vesting schedule is given by

\[
f(X) = f \left( G_{X}^{[0,T]} \right),
\]

where

\[
f \left( G_{X}^{[0,T]} \right) = n + \frac{\pi - n}{X_u - X_d} \left[ \left( G_{X}^{[0,T]} - X_d \right)^+ - \left( G_{X}^{[0,T]} - X_u \right)^+ \right]. \tag{55}
\]

with \( \pi \geq n > 0 \) and \( X_u > X_d > 0 \).

**Proposition 10** Under the assumptions of no arbitrage, complete markets, and replicability of the grant’s payoff, the expression for the risk-neutral value at date 0 of the \( p,v \) stock grant with a path-dependent earnings-based vesting schedule for \( n = 0 \) is given by

\[
v_0^x \left( x_0, \tilde{h}_{x_0}^T \right) = v_{0,d}^x \left( x_0, \tilde{h}_{x_0}^T \right) - v_{0,u}^x \left( x_0, \tilde{h}_{x_0}^T \right), \tag{56}
\]
where

\[
v_{0,k}^x(x_0, \tilde{h}_x^T) = s_0 \beta \left[ x_0 e^{\frac{1}{2} (r-q+\frac{\sigma^2}{3}) T} \Phi \left( \tilde{d}_k^x + \sigma \rho \sqrt{T} \right) - X_k e^{-q T} \Phi \left( \tilde{d}_k^x + \sigma \nu' \sqrt{T} \right) \right], \quad s_0 = S(0), x_0 = X(0),
\]

(57)

and

\[
\tilde{d}_k^x := \frac{\log \left( \frac{x_0}{X_k} \right) + \frac{1}{2} \left( r - q + \frac{\sigma^2}{3} \right) T}{\sigma \sqrt{\frac{T}{3}}},
\]

(58)

for \( k \in \{d, u\} \), \( \nu' := \rho - 1/\sqrt{3} \), \( \rho = \sqrt{3}/2 \) is the coefficient of correlation of the joint probability density function of the standard bivariate Gaussian distribution, and \( \Phi(.) \) denotes the cumulative distribution function of the standard Gaussian law.

Appendix available upon request.

Indeed, since \( \left( \int_0^T W_u du, W(T) - W(0) \right) \) is equal in distribution to \( \left( \frac{\zeta}{\sqrt{T}}, \sqrt{T} z \right) \), where \( \zeta \) and \( z \) are standard Gaussian variables with correlation \( \rho = \sqrt{3}/2 \), we can – as far as their distribution is concerned – replace \( \int_0^T W_u du \) and \( W(T) - W(0) \) by, respectively, \( \frac{\zeta}{\sqrt{T}} \) and \( \sqrt{T} z \) (Appendix available upon request). If we let \( S(0) = s_0 \) and \( X(0) = x_0 \), and define \( G_{X}^{[0,T]} \) by \( \tilde{h}(x) \), we can then rewrite (54) as

\[
G_{X}^{[0,T]} = h_{x_0}^T \left( \frac{\sqrt{3}}{T^{3/2}} \int_0^T W(u) du \right) \overset{\text{dist.}}{=} \tilde{h}_{x_0}^T(\zeta),
\]

where

\[
\tilde{h}_{x_0}^T(y) = x e^{\frac{1}{2} \left( r-q - \frac{\sigma^2}{3} \right) T + \sigma \sqrt{T} y},
\]

with the superscript in \( \tilde{h} \) indicating the time interval over which the averaging process takes place, and the subscript the starting value of the averaging process. Given the preceding, we can express (55) as follows

\[
v_{0}^x \left( x_{0}, \tilde{h}_{x_0}^T \right) = s_0 \frac{\pi}{X_u - X_d} e^{-r T E_0} e^{\left( r-q - \frac{\sigma^2}{3} \right) T + \sigma \sqrt{T} E_0} \left\{ \left( \tilde{h}_{x_0}^T(\zeta) - X_d \right)^+ - \left( \tilde{h}_{x_0}^T(\zeta) - X_u \right)^+ \right\},
\]

following an application of Itô’s lemma to the logarithm of earnings per share and assuming \( n = 0 \). As shown in equations (56) and (57), the grant-date value is given by the product of the stock price at date 0 and the difference between the date-0 Black-Scholes value of two path-dependent European call options written on average earnings per share with same time-to-maturity \( T \) and \( X_u > X_d \).
In unreported results, we find that the path-dependent feature achieves a lower delta than non path-dependence for all values of the underlying asset. In addition, the aggregate delta defined as in Bettis et al. (2016) is lower under path dependence. Moreover, we find that the path-dependent feature provides more efficient incentives: P-v stock grants with a path-dependent vesting schedule are shown to provide a better alternative to standard p-v grants by inducing less opportunistic risk-taking behavior from managers. In this regard, p-v grants with a path-dependent feature may contribute to improving both manager-shareholder conflicts and manager-debtholder conflicts.

4 Conclusion

Relying on option pricing techniques, we explore the incentive properties of performance-vesting (p-v) stock grants with different kinds of vesting provisions. Despite the fact that such grants are gaining both in usage and complexity, and are progressively replacing restricted stock and traditional time-vesting (t-v) grants such as stock options, relatively little is known about their incentive implications.

Our paper’s contribution to this emerging literature is twofold. First, we derive closed-form expressions for the value and incentive measures of both simple p-v stock grants contingent on a single performance metric and p-v stock grants with highly sophisticated vesting provisions. We provide closed-form expressions for the two standard incentive measures commonly used in the compensation literature, namely the ownership incentive and the risk-taking incentive. Second, we advance the understanding of incentives conveyed by these grants from a corporate finance perspective by studying the agency implications of different vesting provisions.

Our general finding is that both ownership incentives and risk-taking incentives can be substantial by t-v grants standards. However, since holders of p-v stock grants are long performance in excess of the threshold value and short performance in excess of the ceiling value through the vesting schedule, incentives conveyed by these grants vary according to the degree of moneyness of the vesting schedule’s long and short components. This implies that firms awarding p-v stock grants to managers might actually provide a different, i.e. possibly lower, alignment between shareholder and manager interests than expected. In addition, we find that grants that embed a sequential
conversion mechanism provide greater risk-taking incentives by making the kinked-shaped payoff structure much more pronounced, thereby amplifying the convexity of the grants’ payoff structure at maturity.

Our paper first contrasts incentives conveyed by the most standard p-v stock grant (i.e. with price-only vesting provisions) with those of t-v grants. Our findings indicate that, if anything, the agency implications of replacing stock options with p-v stock grants are not negligible. For instance, while we find in line with prior literature that shifting away from t-v grants to p-v stock grants (with a price-contingent vesting schedule) may convey a higher degree of alignment between managerial and shareholder interests, achieving this high alignment of interest is quite costly for shareholders because they have to relinquish a relatively large number of shares. In addition, the finding that even p-v stock grants that convert into a low number of shares convey risk-taking incentives that are substantial compared to those conveyed by t-v grants implies that p-v stock grants unequivocally worsen debtholders’ interests because they increase the reward to risk taking. The introduction of p-v stock grants is also likely to hurt shareholders’ interests by causing some distortion between managerial incentives and shareholders’ interests.

To further our understanding of incentives conveyed by p-v stock grants, we subsequently focus on two key sets of vesting provisions which have been relatively unexplored, namely path-dependent vesting provisions and accounting-based vesting provisions. In comparison with standard contingent claims, path-dependent claims require sustained over-performance over the period because the exercise decision is taken with respect to the average of the underlying asset rather than to the value of the underlying asset at maturity. We explore the agency implications for both shareholders and debtholders of introducing a path-dependent feature in vesting schedules. While we find that debtholder interests unambiguously improve because managers holding p-v stock grants with a path-dependent vesting schedule have very little to virtually no incentive to shift to riskier assets as vega is substantially lower, the effect of path dependence on shareholder interests is mitigated.

In the case of p-v stock grants that vest with accounting metrics only, we show that the alignment conveyed by these grants draws on two sources, i.e. market-based performance and accounting-based performance. We isolate the alignment effect conveyed by the accounting metric by disentangling the component of the grant’s delta that is due to stock value appreciation that holds the accounting metric constant from the component arising through the accounting-based vesting schedule. The latter effect captures the fact
that an increase in the stock price likely results in an increase in accounting performance due to the correlation between variables, which in turn leads to grant value appreciation through the vesting of additional shares.

5 Bibliography


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