Risk-Sharing and Investment According to Cournot and Arrow-Debreu∗

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Abstract

We provide a framework for analyzing how the presence of large traders with price impact affects risk-sharing, capital allocation, and asset prices in a complete-markets production economy. We find that imperfect competition hampers risk-sharing because agents strategically distort asset supply to capture price rents. This has aggregate effects consistent with recent macroeconomic trends: investment, the risk-free rate, and measured TFP all fall, Tobin’s $q$ rises for risky investments, and risk premia may be compressed. While both buyers and sellers are strategic, sellers exert more market power because their marginal utility function is endogenously flatter. This tilts prices up and returns down. Distortions are most severe when there is high cross-sectional dispersion in investment risk, productivity, and preferences.

We explore the role of market decentralization and limited commitment, and provide links to classical industrial organization theory.

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1 Introduction

The key roles of a financial system are to facilitate the sharing of risks and to allocate capital to productive uses. In the classical view, equilibrium asset prices efficiently coordinate much of this activity if markets are competitive and market participants are price-takers. Yet there is growing evidence that many markets (including centralized financial markets) are not perfectly competitive, and that market participants frequently take into account that their trading behavior affects asset prices. We are interested in understanding how such price impact affects the ability of markets to efficiently allocate capital and risk, and how this feeds back into investment decisions and asset prices. To this end, we study a complete-markets production economy with a number of large strategic traders who interact with a competitive fringe.

Our first key result is that price impact leads to inefficient risk-sharing. This is because agents strategically distort the supply of financial assets that are used for hedging by other agents. This has aggregate consequences: investment and the risk-free rate are low, but Tobin’s $q$ is high. Perhaps surprisingly, the market risk premium may also be compressed. The reason is that sellers find it particularly cheap to distort high-income states, thereby disproportionately lowering the returns earned on claims to good aggregate states. Our second main result is that the same distortions also hamper the reallocation of resources to the most efficient production technologies. This lowers measured productivity and output.

The strength of these effects is tightly linked to the dispersion of production risk, productivity, and risk preferences: there are strong incentives to exploit price impact only when there are large potential gains from trade. As a result, price impact is most harmful when trade is most valuable. This has several implications. First, the market power channel is particularly strong during recessions, where cross-sectional disper-
sion in idiosyncratic productivity is high. Second, it provides a new perspective on
the secular relation between market concentration and the rise (decline) of idiosyncratic
(aggregate) risk (Campbell, Lettau, Malkiel, and Xu (2001)). Third, price impact af-
facts the pass-through of monetary policy to asset prices and risk-sharing because the
risk-free rate determines the outside option of not trading in financial markets. Fourth,
risk premia, investment levels and productive efficiency are not invariant to changes in
the cross-sectional productivity distribution, even if the aggregate technology frontier is
unchanged.

In our framework, risk-averse strategic agents finance investment and share pro-
duction risks by trading a full set of state-contingent Arrow securities. Our baseline
specification assumes that markets are centralized, but we also consider an extension to
decentralized trading. Agents can be thought of as large firms or financial institutions,
and production technologies may represent direct investments in real assets or lending
opportunities to outside borrowers. The model differs from standard settings only in
that agents take into account that their portfolio choices affect asset prices. We model
this strategic behavior as a multilateral Cournot game in which agents choose which se-
curities to supply and demand, and then choose quantities knowing that their portfolio
affects asset prices. This gives rise to a strategic motive to distort quantities to tilt prices
for private gain.

The basic motive for asset trading is standard. Agents buy claims on states with
low private returns to hedge risk, and sell claims on states with high private returns
to fund investment. Strategic considerations, however, prevent the realization of all
potential gains from trade: sellers strategically lower supply to raise prices, and buyers
strategically lower demand to lower prices. Hence, sellers remain excessively exposed to
states in which their output is high, while buyers garner too little insurance against states
in which their output is low. This discourages risky investment and, if agents refrain
from investing or financing high-risk high-return projects when they cannot hedge the
resulting risks, also lowers measured TFP.

We then consider implications for asset pricing. A-priori, the effect of price impact

\[ \text{Allen and Gale (2000) argue that complete markets is a realistic approximation among financial insti-
tutions who not only have deposits with each other and own each other’s debt, but also trade derivatives}
\]
on asset prices is not obvious because there are strategic traders with opposing motives on both sides of every market. Hence, it depends on whether buyers or sellers have a stronger incentive to distort. When agents have symmetric preferences, we show that it must be sellers. The downside of distorting their portfolios is that, fixing prices, agents must trade away from their preferred asset position. This is particularly costly when marginal utility is high, as it is for buyers who demand assets precisely in states with low idiosyncratic returns. As such, the natural sorting of agents into buying and selling roles is always such that sellers are willing to distort their asset positions more than buyers. In the case of heterogeneous preferences, it follows that relatively risk-tolerant agents (or well-capitalized agents, in the case of decreasing absolute risk aversion) more aggressively exploit their price impact and earn higher rents. We also explore the effects of introducing limited commitment as in Kehoe and Levine (1993), which leads to endogenous constraints on sellers’ ability to supply assets. We show that these constraints reallocate equilibrium market power to buyers when they bind, and that sufficiently tight constraints reverse the comparative static of asset prices with respect to market power. This is because agents with small gross positions have weaker incentives to distort, and provides an analogue to capacity constraints in classical industrial organization theory.

We provide several additional characterization results. First, asset prices can be decomposed into three components: a standard risk premium, a term reflecting binding borrowing constraints similar to Alvarez and Jermann (2000), and a term linked to market power. The last term implies the existence of arbitrage profits, which are absent in a competitive setting. Second, we provide a sharp equivalence result for risk-sharing: the observed degree of risk-sharing in our model with complete markets and price impact is as if markets are competitive but incomplete. Specifically, a set of securities with a strictly smaller asset span is sufficient to replicate the observed degree of risk-sharing among agents for any investment policy. This asset span is intimately related to the extent of price impact, as the implied asset payoffs are orthogonal to the differences in the price impact of any two agents from their trading in state-contingent securities, which are the unrealized potential gains from trade.

Third, since investment is endogenous in our model, incomplete risk-sharing hampers productive efficiency by discouraging investment in high-return, high-idiosyncratic risk technologies. This has additional asset pricing implications. While the risk pre-
mium must rise if a fixed quantity of risk is shared less efficiently, agents in our model can avoid risk by investing less in risky production. As such, price impact can lead to a simultaneous compression of the risk premium and an increase in Tobin’s $q$ (as measured by the marginal return to risky investment relative to the user cost of capital). The same mechanism also generates spillovers from price impact across multiple securities: if agents respond by distorting investment, the under-provision of insurance in one market may lower the availability of funds in other states of the world, further hampering risk-sharing.

Fourth, we explore how the effects of price impact vary with market structure. Specifically, we consider an extension in which agents can only trade with a fixed set of counterparties, as in an incomplete network. This means that the same security can trade at different prices across the network, and that central agents exert more market power. We show that arbitrage profits are a necessary outcome for any such network. Conversely, when links must be formed at a cost, arbitrage profits can provide incentives for an agent to become central. We also show how one can use network centrality measures to measure the implied market power of individual agents given balance sheet information. We conclude by discussing the model’s empirical implications.

1.1 Related Literature


payoffs. A key difference in our setting is that we investigate how strategic trading interacts with production and consider asymmetric agents and equilibria. While Gupta (2019) argues that market concentration can lead to excess credit provision because large banks want to prop up house prices, our model suggests that market concentration can lead to the under-provision of credit through the strategic rationing of financial claims.

Since we include collateral constraints, our paper is related to the literature on the effects of imperfect risk-sharing because of limited commitment, including Alvarez and Jermann (2000), Kehoe and Levine (1993) and Kocherlakota (1996). We differ from these papers in two ways: we study strategic agents with market power, and we consider production rather than endowment economies. Rampini and Viswanathan (2010) and Rampini and Viswanathan (2019) study risk management by collateral-constrained agents, and show that poorly capitalized agents insure less. We instead consider distortions that arise from price impact, and show that they disproportionately distort the supply of insurance. Since we also include collateral constraints, we show that our mechanism distorts insurance above and beyond limited commitment alone. This leads to additional empirical predictions, and we also consider implications for asset prices.

not to obtain insurance against certain insurable risks. A common theme in most of these papers is that markets are incomplete, and production is either wholly absent or highly stylized. This means that the basic motives for trade are also exogenously fixed. We instead consider endogenous risk-sharing needs through production to explore the aggregate implications of strategic asset trading. Our asset pricing predictions also do not depend on a particular market structure, and our implications for real distortions arise from the ex ante misallocation of capital.

2 Model

**Demographics.** Time is discrete and there are two dates, \( t = \{1, 2\} \). The economy is populated by \( N \) strategic agents indexed by \( i \in \{1, \ldots, N\} \), and a mass of non-strategic agents that we refer to as the competitive fringe. Agents are risk-averse, endowed with initial equity capital \( e_i > 0 \), and obtain utility \( u^1_i(c_{i1}) + u^2_i(c_{i2}(z)) \) from consumption at dates 1 and 2. \( u^2_i(\cdot) \) is strictly increasing, continuously differentiable, satisfies the Inada condition, and \( u^2_i(\cdot) \) is homothetic of degree \( \gamma > 0 \) and convex. Unless specified otherwise, preferences are homogeneous across all agents and that \( u^1_i(\cdot) \) satisfies these properties as well. We think of strategic agents as representing financial institutions that trade in financial markets and provide funds to the real economy, such as banks, asset managers, hedge funds, or mutual funds. However, our framework may also have applications to strategic interaction between large firms.

**Technology.** Uncertainty is represented by set of states of the world \( \mathcal{Z} \). State \( z \in \mathcal{Z} \) is realized at date 2 with probability \( \pi(z) \). Agent \( i \) has access to a risky investment opportunity that transforms \( k_i \) units of capital into \( y_i(z)k_i \) units of consumption good in state \( z \). These investment opportunities can be thought of as portfolios of loans, and differences in productivity represent differences their risk profiles. \( y_i(z) \) is bounded \( a.s. \), and can be expressed as \( y_i(z) = Y(z) + \varepsilon_i(z) \geq 0 \), where \( Y \) is systematic risk and \( \varepsilon_i \) is risk specific to agent \( i \). All agents also have access to a storage technology that can transfer resources \( s_i \geq 0 \) elastically from date 1 to date 2 at a riskless rate of transformation \( R > 0 \). We assume that \( E[y_i(z)] > R \) for all \( i \). Storage cannot be negative to ensure that borrowing must occur in financial markets. The competitive fringe passively participates in financial markets but does not invest.
Since there are a finite number of strategic agents, the aggregate return to production \( y(z) = \sum_{i=1}^{N} y_i(z) \) is a random variable. In what follows, we use vector notation, in which case \( \bar{x} \geq \bar{y} \) implies \( x_i \geq y_i \) for all \( i \), while \( \bar{x} > \bar{y} \) implies \( x_i \geq y_i \) for all \( i \) with at least one inequality sharp. In addition, \( \bar{x}_{-i} \) is the vector \( \bar{x}_i \) omitting the \( i^{th} \) element.

**Financial Markets.** Markets are complete. There is a full set of Arrow-Debreu securities (one for each state) that can be traded in centralized markets at date 1. These can be thought of as financial claims or trade credit extended for future deliverables. Without loss, we assume that the claim on state \( z \) promises one unit of the numeraire in state \( z \) and zero in all other states. The claim to state \( z \) has price \( q(z) \), and the quantity of this contract held by agent \( i \) is \( a_i(z) \). We say that \( i \) is a seller if \( a_i(z) < 0 \), and a buyer otherwise. The aggregate net demand for claim \( z \) is \( A(z) = \sum_i a_i(z) \).

Similar to other models of strategic interaction, we assume that the competitive fringe is deep-pocketed relative to total wealth of strategic agents.\(^4\) Given that markets are complete, we assume that there is a separate competitive fringe for each \( z \). This makes transparent how strategic trading affects asset prices state by state. It is sufficient for our results that the demand of the fringe can be expressed as

\[
D(z) = f(q(z), z) \sum_{j=1}^{N} e_j, \tag{1}
\]

where \( f(\cdot, z) \) is a strictly decreasing \( C^2 \) function of its first argument, \( q(z) f(\cdot, z) \) is bounded from above for all \( z \), and \( f(\cdot, z) \). That is, we assume a well-behaved downward-sloping residual demand curve for each state-contingent claim. Specifying that demand scales with strategic agents’ equity ensures that their policies are homothetic in equity even with price impact. Appendix B provides several examples of demand functions derived from standard utility functions that satisfy these properties.

We can then model the strategic interaction between strategic agents as a multi-lateral Cournot game. The market-clearing condition for the claim on state \( z \) is

\[
D(z) + A(z) = 0. \tag{2}
\]

\(^4\)A competitive fringe is also present in Eisenberg and Noe (2001), Xiong (2001), Brunnermeier and Pedersen (2005), Pritsker (2005), and Koijen and Yogo (forthcoming).
Inverting the demand curve of the competitive fringe yields the pricing function

\[
q(z) = f^{-1}(-A(z), z) \quad \text{for all } z. \tag{3}
\]

Strategic agents take this pricing function as given. Hence price impact satisfies

\[
\frac{\partial q(z)}{\partial a_i(z)} = \frac{\partial q(z)}{\partial A(z)} > 0 \quad \text{for all } i \text{ and } z \tag{4}
\]

and it is symmetric because asset prices depend on aggregate demand only.

**Decision Problem.** Agents choose risky investment, storage, and an asset portfolio to maximize expected utility over consumption at dates 1 and 2. Agent \(i\)'s budget constraint at date 1 is

\[
c_{1i} + k_i + \sum_{z \in \mathcal{Z}} q(z) a_i(z) + s_i \leq e_i \tag{5}
\]

and date-2 consumption in state \(z\) is

\[
c_{i2}(z) = y_i(z) k_i + a_i(z) + Rs_i. \tag{6}
\]

To understand the role of quantity constraints, we assume that agents have limited commitment as in Alvarez and Jermann (2000). Specifically, agents can choose to default on their liabilities at date 2 at the cost of incurring a deadweight loss of a fraction \(\xi\) of wealth. Because markets are complete, it is without loss to assume that no default occurs on the path of play. However, the threat of default limits the quantity of a given claim that an agent can issue. Date 2 consumption after a default is

\[
c_{i2}^D(z) = (1 - \xi) \left( y_i(z) k_i + Rs_i \right). \tag{7}
\]

and the incentive-compatibility constraint that ensures repayment in \(z\) is

\[
c_{i2}(z) \geq c_{i2}^D. \tag{8}
\]

This constraint is trivially satisfied if \(i\) is a buyer of the state-\(z\) claim \((a_i(z) > 0)\), and it never binds if \(\xi = 1\). It is convenient to state this condition as the **leverage constraint**

\[
- a_i(z) \leq \xi (y_i(z) k_i + Rs_i) \quad \text{for all } z. \tag{9}
\]
This formulation highlights that limited commitment constrains sales but not purchases. The decision problem of agent $i$ is

$$U^i = \max_{\{c_{i1}, k_i, s_i, \bar{\alpha}_i\}} u_1 (c_{i1}) + \sum_{z \in \mathcal{Z}} \pi (z) u_2 (c_{i2} (z))$$

s.t. (5), (6), (7) for all $z$

where agents internalize that their portfolio $\bar{\alpha}_i = \{a_i (z)\}_{z \in \mathcal{Z}}$ affects equilibrium prices.

**Equilibrium Concept.** Because agents internalize price impact, we use as our equilibrium concept sequential equilibrium in demand functions. In such an equilibrium, large agents take the beliefs and investment, as well as the security demand, and supply schedules of other agents as given. This can be viewed as a generalization of the Nash-equilibrium–in–demand–schedules concept of Kyle (1989). Since there are strategic complementarities in investment by agents, there can potentially be multiple equilibria.

**Definition 1** A Nash Sequential Equilibrium in Demand Schedules is a list of strategy profiles $\{\sigma_i\}_{i = \{1, \ldots, N\}} \in \Sigma_i$, with policy tuples $(c_{i1}, k_i, s_i, \bar{\alpha}_i (z)) \in \sigma_i$, and prices $\{q (z)\}_{z \in \mathcal{Z}}$ such that:

1. Taking other agents’ strategy profiles $\sigma_{-i}$ and its own date-1 policy as given, agent $i$ finds it optimal to repay its debt in date 2 for all $z$.

2. Agent $i$ finds it optimal to pursue strategy $\sigma_i$ given $\sigma_{-i}$.

3. The market for each claim clears with zero excess demand: $D (z) + \sum_{i} a_i (z) \leq 0$ for all $z$.

4. All agents form rational expectations of their and others’ strategies.

**Isolating Price Impact.** Our main objective is to study comparative statics of asset prices and quantities with respect to price impact. This requires a structure that can isolate changes in price impact from other model primitives. To this end, we consider a sequence of economies indexed by $\mu \in (0, 1]$. Economy $\mu$ has a total of $\frac{N}{\mu}$ strategic agents that are evenly distributed across $N$ types. Types are indexed by $i$, and all agents of type $i$ inherit the preferences, endowments, and technology of strategic agent $i$ in the baseline model. Put differently, we split each strategic agent into $\frac{1}{\mu}$ smaller but otherwise identical agents, and choose their mass to ensure that the total mass of each type is constant. We will index an individual agent of type $i$ by subscript $ji$. By symmetry, it is without loss of generality to take as given that all agents of a given type choose the
same portfolios, \( a_{ji} = a_i(z) \) for all \( z \). Their aggregate demand for the state-\( z \) claim is
\[
\sum_{i=1}^{N} \sum_{j=1}^{1/\mu} \mu a_{i}(z) = \sum_{i}^{N} a_i(z),
\]
and is independent of \( \mu \). However, price impact scales with
\( \mu \) because it determines relevance for aggregate demand. That is,
\[
\frac{\partial q(z)}{\partial a_{ji}(z)} = \mu \frac{\partial q(z)}{\partial A(z)}.
\]
We recover the baseline economy if \( \mu = 1 \), and price impact vanishes, and agent’s act as
price-takers, as \( \mu \to 0 \). Hence we will use this limit as our benchmark of a competitive
equilibrium. We denote it by superscript \( CE \), and observe that the competitive equilib-
rium is constrained efficient. More generally, \( \mu \) alters price impact without affecting the
aggregate production frontier. Hence comparative statics with respect to \( \mu \) isolate the
aggregate consequences of price impact. We exploit this property in Section 4. To facil-
itate comparisons with competitive equilibria in the literature, we assume that markets
clear internally in the limit without price impact.

**Assumption 1** \( D(z) \) is such that \( \lim_{\mu \to 0} A(z) = 0 \) for all \( z \).

### 3 Equilibrium

We begin by characterizing the optimal portfolios of strategic agents. It is convenient to
define the marginal value of additional date-2 consumption in state \( z \) relative to addi-
tional consumption at date 1. Because of limited commitment, this marginal value will
depend on the tightness of the leverage constraint (7) in all states. Let \( \pi(z) \psi_i(z) \) denote
the associated Lagrange multiplier for each \( z \). The relevant marginal value is then given
by agent \( i \)'s state price for state \( z \):
\[
\Lambda_i(z) = \pi(z) \left[ \frac{u_2(c_{i2}(z)) + \xi \psi_i(z)}{u_1'(c_{i1})} \right]
\]
(8)
The next result shows that an equilibrium exists, and characterizes optimal portfolios.

**Proposition 1** There exists an equilibrium in which the optimal policies of agent \( i \) are homoge-
neous of degree 1 in $e_i$, and $c_{ii}$, $k_i$, $s_i$, and $a_i(z)$ satisfy the optimality conditions

$$k_i : \sum_{z \in Z} \Lambda_i(z) y_i(z) \leq 1 \ (\text{and} \ if \ k_i > 0),$$

$$s_i : \sum_{z \in Z} \Lambda_i(z) \leq 1, \ (\text{and} \ if \ s_i > 0),$$

$$a_i(z) : \Lambda_i(z) = q(z) + \frac{\partial q(z)}{\partial a_i(z)} a_i(z).$$

The optimality conditions for risky investment and storage are standard: at the margin, state-price weighted expected returns are set equal to the unit marginal cost of investment. The first-order conditions for financial claims are not: they are modified to incorporate price impact. Price impact scales with position size, and its sign depends on whether agent $i$ is a buyer or a seller. The scale effect arises because there are larger infra-marginal benefits to distorting prices when the agent takes a large position. Since state price $\Lambda_i(z)$ is decreasing in $a_i(z)$, sellers, who take negative positions, have an incentive to under-supply a claim, and buyers under-demand.

The next proposition establishes stringent conditions for the irrelevance of price impact: it does not affect equilibrium outcomes if and only there are no gains from trade in the competitive equilibrium. This can be measured the extent of trade in the competitive equilibrium.

**Proposition 2** The equilibrium with price impact is equivalent to the competitive equilibrium if and only if there are no gains from trade in the competitive equilibrium, $a_i^{CE}(z) = 0$ for all $i, z$.

The statement follows directly from the first-order condition for $a_i(z)$, since price impact is irrelevant when $a_i(z) = 0$. A natural example that generates no trade is the case of pure systematic risk and homogeneous preferences. More important is the converse implication: price impact distorts financial trading whenever there are gains from trade.

Since the distortion from price impact scales with position size, moreover, increasing gains from trade lead to larger distortions. It follows that idiosyncratic production risk introduces natural pricing power in financial markets that grows with cross-sectional dispersion. Similarly, differences in risk tolerance introduce pricing power when trading claims to aggregate risk. This relates our work to Athanasoulis and Shiller (2000) and Athanasoulis and Shiller (2001), who study the securities that are most valuable for risk-sharing when aggregate risk is traded across agents with different risk aversions, and
when agent-specific risk is traded between agents with similar risk aversions. We show that such gains from trade simultaneously lead to distortions from price impact.

There is also an instructive link between our model of risk-sharing and investment with price impact and classical industrial organization theory. Define $i$’s market share in the state-$z$ claim as $\alpha_i(z) = a_i(z)/A(z)$, and the inverse price elasticity of demand as

$$1/\eta(z) = -\left(\frac{\partial q(z)}{\partial A(z)}\right) A(z)/q(z)$$

(9)

Then we can restate the first-order condition for $a_i(z)$ as

$$\frac{q(z) - \Lambda_i(z)}{q(z)} = -\frac{\alpha_i(z)}{\eta(z)}.$$  

(10)

The right-hand side is the standard notion of market share scaled by price elasticity. The only difference is that market shares can be positive or negative. The left-hand side can be interpreted as a financial Lerner index that determines the optimal markup for sellers, where the state price is the appropriate notion of marginal cost for state-contingent claims. For buyers, the right-hand side is negative since $\alpha_i > 0$. From their perspective, the Lerner index thus measures the optimal utility loss from strategic demand distortions, where $q(z)$ is the cost of obtaining additional state-contingent consumption and $\Lambda_i(z) > q(z)$ is its marginal value.

We differ from classical industrial organization in two ways. Sellers’ marginal cost in state $z$ is an equilibrium object that depends both on investment and the efficiency of risk-sharing in all states $z' \neq z$. Since risk-sharing is the outcome of strategic behavior, this implies that there are spillovers from price impact across markets. Second, there is endogenous sorting into the buy-side and sell-side, and strategic agents on both sides of every market. For any investment policy, moreover, we can recover the sorting of agents in a given asset market by their ranking of state prices under the presumption of zero trade in financial markets.

Our model can also be viewed as Cournot competition in a production network in which firms produce differentiated intermediate goods that are all inputs to each other’s final production. To see this, consider the special case in which the large agents have CRRA preferences with CRRA index $\gamma$ and do not value initial consumption, $u^i_1(c_{i1}) \equiv 0$. Then, we can rewrite the certainty equivalent expected utility of date two consumption
\[ CE \left( E \left[ u_2^i (c_{i2}) \right] \right) = \left( \sum_{z \in Z} \pi (z) c_{i2}^{1-\gamma} \right)^{\frac{1}{1-\gamma}}. \]  

(11)

Notice that this resembles a CES production function with final output the certainty equivalent consumption, share weights \( \pi (z) \) that sum to unity, and elasticity of substitution \( \frac{1}{\gamma} \). Although there is one consumption good in the economy, consumption in each state represent the differentiated intermediate goods in this joint production. The shading of supply and demand by the large agents distorts their joint production by introducing a wedge in the marginal products of each good. An additional novel aspect of our analysis is that this competition effectively occurs in the futures markets referencing this production. In addition, the trading of these futures contracts feeds back into (the efficiency of) each agent’s capital allocation decisions since the proceeds from the sales of state-contingent securities help finance their investment.

### 3.1 Risk-sharing and Endogenous Market Incompleteness

Next we characterize how price impact hampers risk-sharing. The basic mechanism is that agents voluntarily distort quantities to capture rents. As a result, state prices (i.e. marginal valuations of state-contingent consumption) are not aligned across agents. This indicates that there are gains from trade not consummated for purely strategic reasons.

**Proposition 3** If agent \( j \) is a seller and agent \( i \) is a buyer of claim \( z \), then

\[ \Lambda_i (z) > \Lambda_j (z), \]

and price impact hinders risk-sharing above and beyond the impact of limited commitment. In addition, leverage constraints from limited commitment are less likely to bind.

There is no dispersion of state prices in competitive complete-markets economies, but such dispersion is the hallmark of incomplete-markets models with heterogeneous agents. It arises in our complete-markets model because of strategic behavior. Similar to Constantinides and Duffie (1996), financial markets would therefore appear incomplete to an outside econometrician under the maintained hypothesis that markets are competitive. Specifically, dispersion in marginal utilities across states would indicate that agents do not have access to the financial assets required to further share risk. We formalize
this claim by constructing an equivalence between consumption allocations in our model and a counterfactual with perfectly competitive incomplete markets.

**Proposition 4** Let the implied state price deflator of agent $i$ in state $z$ be

$$\Lambda_i(z) = q(z) + \frac{\partial q(z)}{\partial a_i(z)} a_i(z) \forall (i, z).$$

Define $\frac{\partial \bar{q}}{\partial \bar{a}_i}$ to be the $K \times 1$ vector with entries $\frac{\partial q(z)}{\partial a_i(z)}$, $\bar{a}_i$ to be the $K \times 1$ vector with entries $a_i(z)$, and $\Lambda_i$ to be the $K \times 1$ vector with entries $\Lambda_i(z)$ of agent $i$. Then, there exists a market structure with $M < |Z|$ assets, indexed by the $M \times K$ dividend yield matrix of the traded assets $\frac{D'}{p}$, that replicates the asset span in Proposition 4. This counterfactual dividend yield satisfies

$$\frac{D'}{p} \Lambda_i = \iota_M \quad \text{and} \quad \frac{D'}{p} \left( \frac{\partial \bar{q}}{\partial \bar{a}_j} \odot (\bar{a}_j - \bar{a}_j) \right) = 0_{M \times 1} \forall j.$$

and the implied market incompleteness can be characterized by:

$$\text{Cov} \left( \Lambda^*(z), \Lambda_i(z) - \Lambda_j(z) \right) = 0,$$

where $\Lambda^*(z)$ is the market-implied state price deflator.

The first part of Proposition 4 reveals that the intentional mispricing arising from imperfect competition is isomorphic to forced disagreements about state prices stemming from missing markets with perfect competition. As this is reflected in prices, one infer the strength of market power from the implied incompleteness. In Section 4 we use this proposition to construct two examples in which the implied incomplete market structure consists of either a risk-free asset or a market index only.

Similar to incomplete markets, the dispersion of agent state prices also implies that they can have different valuations for non-traded assets, despite markets being complete. In contrast to Alvarez and Jermann (2000), thus, the recovered state-contingent claims prices cannot be used to recover the price of any non-traded but redundant, security. The second part of the claim reveals that the incomplete market span is such that any potential gains from trade must be un-priced.
3.2 Investment

We now show that quantity distortions due to price impact affect investment through two related channels: inefficient risk-sharing deters risky investments, and low trading volumes hamper the reallocation of funds to agents with the most efficient production technologies. This lowers aggregate investment volumes and expected productivity through a misallocation of resources. The next proposition formally states these results. To identify misallocation, we compare hypothetical outcomes under perfect competition and when all financial trading is prohibited (autarky). The competitive equilibrium is constrained efficient. Hence $i$ chooses a larger scale of production under autarky only if $i$’s production technology is jointly dominated by the production technologies available to other agents.

**Proposition 5** Aggregate risky investment and expected productivity under price impact are lower than in the competitive equilibrium, and aggregate investment in storage is higher. The cross-section of investment has the following properties:

(i) If $k_i^{CE} \geq k_i^{Aut}$, then $k_i < k_i^{CE}$ and $s_i \geq s_i^{CE}$ due to price impact.

(ii) If $k_i^{CE} < k_i^{Aut}$, then $k_i$ may be larger than $k_i^{CE}$ due to price impact.

The aggregate results follow from Proposition 3, which established the inefficiency of risk-sharing under price impact, and the fact that strategic distortions cannot generate rents in the aggregate across all strategic agents. The first cross-sectional result considers the case of an agent who invests less in autarky than in competitive equilibrium. Since this indicates that risk sharing on financial markets allows for an efficient increase in investment, it follows that distortions from price impact inefficiently lower investment. The second cross-sectional result considers an agent who invests more under autarky than in the constrained-efficient competitive equilibrium. Hence $i$’s technology is dominated, and any increase in investment is indicative of misallocation. Price impact can lead to such misallocation by distorting the transfer of resources to agents with dominant technologies. Note that price impact need not lead to an increase in $k_i$ because the distortion to risk-sharing continues to deter investment.
3.3 Asset Prices

We now study our model’s implications for asset prices. This requires some subtlety because the cross-sectional dispersion of state prices due to price impact induces a divergence between market prices and private valuations based on state prices. This prevents us from pricing redundant securities using state-price implied valuations, as is commonly done in the literature. Specifically, state prices determine the required returns of an asset if agents could trade it without price impact, while market prices determine actually prevailing returns. Both coincide in canonical models of perfect competition. We characterize both classes of equilibrium returns, and show that their divergence is directly linked to the degree of equilibrium market power. The state price-implied risk-free rate is

\[ r^*_f = \left( \sum_{z \in Z} E^* [\Lambda_i(z)] \right)^{-1} \]

where \( E^* \cdot \) is the cross-sectional average across agents. The market-implied risk-free rate constructed from prevailing market prices is

\[ r^m_f = \left( \sum_{z \in Z} q(z) \right)^{-1} \]

Claim \( z \)'s market-implied excess return is

\[ r(z) = \frac{\pi(z)}{q(z)} - r^m_f. \]

Tobin’s \( q \) is defined to be difference between the marginal product of risky investment and the state-price implied risk-free rate, and we define the average market power exerted in state \( z \) to be

\[ \overline{\text{mkt}}(z) = \frac{\partial q(z)}{\partial A(z)} \frac{A(z)}{N} = -\frac{q(z)}{N \eta(z)}. \] (12)

Market power is a function of the inverse price elasticity of demand \( \eta(z) \) as stated in Equation (9). The next proposition states our model’s key asset pricing implications. Taking a cross-sectional average of the first-order condition for \( a_i(z) \) gives the following relationship between state prices, market power, and asset prices:

\[ q(z) = E^* [\Lambda_i(z)] - \overline{\text{mkt}}(z) \] (13)
It follows from Proposition 2 that price impact is irrelevant for all assets that are not traded. Hence will take as given that trade occurs for at least some states. Statements regarding claim $z$ apply only if there is trade in that claim.

**Proposition 6** The following results characterize asset prices under price impact:

(i) If preferences are homogeneous, then $q(z)$ is increasing in price impact and $r^m_f$ is bounded below by $R_f$ and decreasing for any fixed investment policy.

(ii) $r^*_f$ is bounded below by $R_f$ and is lower than under perfect competition.

(iii) The wedge between $r^*_f$ and $r^m_f$ is determined by market power: $\frac{1}{r^*_f} - \frac{1}{r^m_f} = \sum_z \text{mkt} (z)$. 

(iv) The market-implied excess return satisfies the following decomposition for each $z$:

$$r(z) = -\text{Cov} \left( \tilde{u}_2(z), \frac{\delta(z)}{q(z)} \right) + \left( r^m_f - r^*_f - \text{mkt}(z) r^*_f \right) + -\text{Cov} \left( \overline{\text{Lev}}(z), \frac{\delta(z)}{q(z)} \right),$$

where $\overline{\text{Lev}}(z)$ is the average leverage premium defined in the proof.

Sellers have an interest in raising prices, and buyers in lowering them. Statement (i) shows that sellers succeed in raising prices for any fixed investment policy if preferences are homogeneous. This is due a sorting property of equilibrium. An agent chooses to be a seller in state $z$ if his state prices is lower than that of buyers. This requires that the agent has relatively high income in state $z$. Since marginal utility is convex, it then follows that sellers’ marginal utility is also flatter than that of buyers for any given quantity of claim $z$. The optimal distortion trades off the benefit of favorable prices against the cost of trading away from the preferred allocation for any given asset price. Locally, this cost can be measured by the slope of marginal utility. Hence sellers distort more, and prices increase. Hence the market-implied risk-free rate declines.

Figure 1 illustrates the equilibrium determination of asset prices and portfolios. Thick black lines show state prices in a generic state, holding portfolios across all other states fixed. Buyers are buyers because they have a strictly higher state price for any asset position. In the competitive equilibrium without price impact, optimality require setting state prices equal to equilibrium price $q^{CE}$. This leads to an equalization of state prices across buyers and sellers and, thus, perfect risk-sharing. The resulting portfolio
is $a^C_S$ and $a^C_B = -a^C_S$. With price impact, optimality calls for taking into account the fact that buying more (or selling less) of the claim leads to a price appreciation. Hence agents choose as their optimal position the intersection of the state price function with the red pricing function. The resulting portfolio is $a^*_S$ and $a^*_B$. The seller’s state price is less convex than the buyer’s because the seller has more resources in this state for any asset position. Hence it is less costly to distort for the seller, and he responds by adjusting quantities more in response to price impact. In equilibrium, we therefore have that $a^*_S < -a^*_B$, and $q(z)$ rises.

Statement (ii) shows that the state-price implied risk-free rate falls under general conditions. This is because inefficient risk-sharing raises average marginal utility across states and agents. Statement (iii) makes precise the wedge between state price-implied returns and market-implied returns: it is determined by average market power exerted in financial markets. Statement (iv) shows that we can decompose the expected excess return of debt contingent on state $z$ into three components: a risk premium that reflects the covariance between the Arrow-Debreu security and the average marginal utility of purchasing agents, a bias that reflects the average market power of the agents that trade claims, and a collateral premium stemming from limited commitment.

\footnote{A similar phenomenon occurs with idiosyncratic shocks to consumption growth across agents that are multiplicative and i.i.d. log-normally distributed, as in Constantinides and Duffie (1996). Dispersion of marginal utility in their setting, however, results from market incompleteness.}
Corollary 1 offers a characterization of the states most likely to experience an increase in excess returns due to price impact: for any fixed investment policy, it is the state with the highest marginal utility. This state is the most costly for sellers to distort, and hence the decline in the market-implied riskless rate dominates the change in $r(z)$. Consequently, market power can lead to risk compression by raising the expected excess return to high marginal utility states, even though these are the states for which buyers demand the most insurance.

**Corollary 1** Fixing an investment policy, an increase in market power impacts the expected excess return of an A-D security referencing state $z$, $r(z)$, to first-order according to:

$$\frac{\partial r(z)}{\partial \mu} \propto \left( \frac{q(z)}{\sum_z q(z)} \right)^2 + \left( \frac{q(z)}{\sum_z q(z)} - \pi(z) \right) \frac{\partial q(z)}{\partial \mu} \frac{\partial \sum_z q(z)}{\partial \mu}.$$ 

If preferences are homogeneous, then $r(z)$ increases for the highest marginal utility state with the fewest sellers, and falls for at least one lower marginal utility state.

The decomposition shows that whether $r(z)$ is higher or lower for low marginal utility states depends on two forces: the incentive to distort, and market concentration. For the lowest marginal utility states $\left( \frac{q(z)}{\sum_z q(z)} < \pi(z) \right)$, since agents demand a risk premium to hold the asset, while $\frac{\partial q(z)}{\partial \mu} \frac{\partial \sum_z q(z)}{\partial \mu} > 0$, since market power raises all asset prices under homogeneous preferences. As such, the second term is negative, but whether the excess return falls depends on how low sellers’ marginal utility is, and how concentrated markets are. If sellers’ marginal utility is sufficiently low, then $\left( \frac{q(z)}{\sum_z q(z)} - \pi(z) \right)$ is large; if market power is diffuse, then $\frac{\partial q(z)}{\partial \mu} \frac{\partial \sum_z q(z)}{\partial \mu}$ is large. Consequently, the expected excess return rises for low marginal utility states if seller marginal utility is sufficiently low and/or market power is sufficiently concentrated in the state, and falls otherwise. From the corollary, there must be at least one low marginal utility state in which $r(z)$ rises.

### 3.4 Welfare

A comprehensive normative analysis is difficult in this setting because there is endogenous price impact and multiple goods. Nevertheless, we show that the equilibrium with price impact is generally constrained inefficient.

**Proposition 7** The equilibrium with price impact is generically constrained Pareto inefficient.
The source of inefficiency is that some agents remain exposed to diversifiable risk for strategic reasons. That is, even though distortions to market prices alone are not sufficient to lower welfare, dispersion in state prices does lead to inefficiencies. Hence there exists transfers a social planner could implement to improve the allocation of risk. This suggests a potential role for government intervention in concentrated financial markets. An immediate consequence of this argument is that the distribution of productivity across agents matters for capital allocation and risk-sharing. Since agents face endogenous frictions in reallocating resources to the most efficient technologies, this misallocation is reflected in investment, asset prices, and effective aggregate productivity. In the first-best allocation attained by the competitive equilibrium, instead, only the most efficient productivity technologies are relevant for equilibrium outcomes.

**Corollary 2** With limited commitment and/or market power, the distribution of productivity across agents matters for the level of investment and asset prices in the economy. With commitment and perfectly competitive agents, in contrast, equilibrium outcomes depend on only the most efficient production technologies.

Corollary 2 highlights the rich interaction between market power, borrowing constraints, and investment that is absent from endowment economies with limited commitment. Alvarez and Jermann (2000), for instance, highlight an irrelevance result in which risk premia are identical with and without borrowing constraints when agentspecific and aggregate shocks are independent.\(^6\) In our setting, cross-sectional capital misallocation arising from limited commitment and market power distorts the production possibilities frontier. As a consequence, agent heterogeneity has a first-order impact on investment and asset prices.

## 4 A Version with Tractable Heterogeneity

We now consider a number of tractable special cases of our model that illustrate our results. Let there be two types of agents, \( j \in \{1, 2\} \), each with fixed endowment \( e \), and \( \frac{1}{\mu} \) symmetric agents of each type with mass \( \mu \). As explained above, comparative statics

\(^6\)Intuitively, the unconstrained agents that can perfectly share risk are marginal in determining asset prices state-by-state, while the constrained agents that imperfectly share risk are limited in short-selling the assets that they perceive to be overvalued.
with respect to \( \mu \) are informative about the role of market power without affecting other primitives. To simplify notation in this restricted setting, assume that agents care only about consumption in the second period, and that leverage constraints do not bind. The utility function over date-2 consumption is \( u \), and we consider both symmetric and heterogeneous preferences.

The demand of the competitive fringe’s demand is linear, \( D(z) = \frac{1}{\lambda} (q^{CE}(z) - q(z)) \) for all \( z \), where \( \lambda \) parameterizes the price elasticity of demand. The choice of \( q^{CE}(z) \) as the intercept ensures that there are no arbitrage opportunities from trading with the outside fringe, and that \( \lim_{\mu \to 0} A(z) = 0 \). Asset prices and price impact satisfy

\[
q(z) = q^{CE}(z) + \lambda \left( \sum_{i=1}^{N} a_i(z) \right) \quad \text{and} \quad \frac{\partial q(z)}{\partial a_i(z)} = \lambda \mu
\]

Generic state \( z = \{ \theta, d \} \) consists of an aggregate shock \( \theta \in \{ l, h \} \) and a distributional shock \( d \in \{ 1, 2 \} \), where \( \pi(z) = \frac{1}{4} \) for all \( z \). Given expected return \( \bar{Y} > R_f \), the aggregate return process is

\[
Y(\theta) = \begin{cases} 
(1 + \Delta)\bar{Y} & \text{if } \theta = h \\
(1 - \Delta)\bar{Y} & \text{if } \theta = l
\end{cases}
\]

where \( 0 \leq \Delta \leq 1 \).

and the stochastic process for type \( j \)’s idiosyncratic return is

\[
y_j(z) = (1 - a)Y(\theta) + 2aY(\theta)1(d = j) \quad \text{where} \quad 0 \leq a \leq 1.
\]

\( \Delta \) and \( a \) parameterize the degree of aggregate and idiosyncratic risk: there is no aggregate risk if \( \Delta = 0 \), maximal aggregate risk if \( \Delta = 1 \), no idiosyncratic risk if \( a = 0 \), and maximal idiosyncratic risk if \( a = 1 \).

Since agents are symmetric by type, it is without loss to restrict attention to symmetric type-contingent equilibrium. The appropriate state price is

\[
\Lambda_j(z) = \frac{\pi(z)u_j'(c_j(z))}{\sum_{z'} \pi(z')u_j'(c_j(z')) \cdot y_j(z')}
\]

where consumption is \( c_j(z) = y_j(z)e + a_j(z) - y(z) \sum_{z'} q(z')a_j(z') + (R_f - y(z))s_j \).
4.1 Symmetric Preferences

Assume first that both types have symmetric preferences, \( u_j(c) = \log(c) \). To discuss the role of price impact, it is useful to first establish equilibrium outcomes in the hypothetical scenario where all agents take prices as given.

**Observation 1** The competitive equilibrium without price impact has the following properties:

1. Prices satisfy \( p^*([1, 1]) = p^*([1, 2]) \geq p^*([h, 1]) = p^*([h, 2]) \), and \( \Delta > 0 \).
2. Risk-sharing satisfies \( \Lambda_j(z) = \Lambda_{-j}(z) \) for all \( j \) and \( z \).
3. Portfolios satisfy \( a_j([\theta, j]) \leq 0 \) and \( a_j([\theta, -j]) \geq 0 \) for all \( j \) and \( z \), and \( \Delta > 0 \).
4. Investment satisfies \( k_j = e \) and \( s_j = 0 \) for all \( j \) if \( \Delta^2 < 1 - R_f / \bar{Y} \).

These results have the familiar interpretation: only aggregate risk carries a risk premium, and idiosyncratic risk is shared perfectly. Hence no agent invests in the safe technology if aggregate risk is not too severe. Agents hold non-zero positions in financial assets if and only if there is idiosyncratic risk, but there is no trade when all risk is aggregate. In the latter case, there are no distortions from price impact. To illustrate how price impact distorts the sharing of idiosyncratic risk, assume for simplicity that there is no aggregate risk. We can then solve for the equilibrium as if there are only two states, \( d \in \{1, 2\} \). Moreover, agents’ decision problems are symmetric conditional on a re-denomination of state \( d \) to \(-d\). Hence we can look for a symmetric equilibrium where type \( j \) is seller of claims on state \( d = j \) and a buyer of claims on state \( d = -j \). Since both types are of equal mass, the equilibrium must have a unique asset price, \( q(1) = q(2) \).

Taking differences of first-order conditions

\[
\frac{1 - m_j}{m_j(1 + \alpha) + (1 - \alpha)} = \bar{Y} \lambda \mu \left( -a_j(j) + a_j(-j) \right)
\]

where the marginal rate of substitution is \( m_j = u'(c_j(j)) / u'(c_j(-j)) \leq 1 \) and \( m_j = 1 \) if and only if risk sharing is perfect. It follows that price impact (increases in \( \mu \)) particularly hamper risk-sharing when \( \alpha \) is large.

Next consider the effect of price impact on asset prices. By Proposition 6, sellers distort quantities more than buyers, leading to an appreciation of asset prices for a given...
investment policy. We can show this analytically in the limiting case with maximal idiosyncratic risk. The result follows directly from combining the first-order condition for assets with the pricing function.

**Observation 2** \( a_j(-j) < 0 \) and \( a_j(j) = \left( \frac{1}{1+\mu} \right) a_j(-j) \) in the limit as \( \alpha \to 1 \).

That is, agents optimally ration sales of assets by a factor \( \frac{1}{1+\mu} \) relative to purchases. Hence the strategic impact of sellers outweighs that of buyers, and the distortion is strictly increasing in \( \mu \).

Next we illustrate our result on the implied degree of market incompleteness. In the absence of aggregate risk, there are two distinct states. By Proposition 4 we can thus rationalize the observed degree of risk-sharing using a single asset. Since agents are ex-ante symmetric, state prices must satisfy \( \Lambda_j(j) = \Lambda^+ \) and \( \Lambda_j(-j) = \Lambda^- \). The implied dividend-yield of this asset \( \{d_j\}_{j=1,2} \) satisfies

\[
\begin{bmatrix}
\Lambda^h & \Lambda^l \\
\Lambda^l & \Lambda^h
\end{bmatrix}
\begin{bmatrix}
d_1 \\
d_2
\end{bmatrix}
= \begin{bmatrix}
1 \\
1
\end{bmatrix}.
\]

Solving this equation gives

\( d_1 = d_2 = \frac{1}{\Lambda^h + \Lambda^l} = r_f^* \).

An econometrician inferring the degree of market completeness from observed consumption data would conclude that the only asset that agents can trade is a risk-free bond whose rate of return is precisely the state-price implied risk-free rate. This is because any security with a tilt to one state would generate strict gains from trade. Strategic exploitation of price impact thus leads to trading arrangement that look as if they were generated from a model with exogenously incomplete markets of the form most commonly assumed.

Figures 2 and 3 plot comparative statics with respect to \( \mu \) under homogeneous preferences. Figure 2 shows investment in the left panel and equilibrium returns in the right panel. Solid and dashed lines refer to high and low idiosyncratic risk, respectively \((\alpha = 0.6 \text{ versus } \alpha = 0.4)\). Risky investment falls and storage grows as increased price impact hampers risk-sharing, triggering a fall in output and productivity. Moreover, these effects are stronger when idiosyncratic risk is high. The right panel shows equilibrium asset returns. The black line shows that the state-price implied risk-free rate falls as
insurance becomes expensive. The blue line shows the market-implied excess return to holding a claim to aggregate output, $MRP = \frac{\pi Y(h) + \pi Y(l)}{Y(h)q(h) + Y(l)q(l)} - r_f^m$. It falls because the high aggregate state is relatively cheap to distort for sellers, leading to a relative compression of asset prices. Thus, growing price impact (or industry concentration) can lead to a simultaneous compression of risk-free rates and risk premia.

![Figure 2: Parameters: $u(c) = \log(c)$, $e = 1$, $\bar{Y} = 1.1$, $\Delta = 0.15$, $R_f = 1$, $\alpha \in \{0.4, 0.6\}$, $\lambda = 0.5$.](image)

Figure 3 plots claims prices in the left panel and the variance of state prices as a measure of the inefficiency of risk-sharing in the right panel. Asset prices increase if $s^* = 0$ because sellers’ distort more by Proposition 6. However, $q(l)$ begins to fall when agents hold storage because market-based insurance is less valuable if agents self-insure. $q(h)$ continues to rise because high-state output is declining in $s^*$. The right panel shows that price impact strictly hampers risk-sharing as long as agents invest only in risky capital; the variance of state prices declines only once agents rely on storage for self-insurance.

### 4.2 Heterogeneous Risk Aversion

We now consider the case of heterogeneous preferences. In particular, we assume that Type 1 is risk-averse with CRRA utility function $u_1(c) = \frac{c^{1-\gamma_1} - 1}{1-\gamma_1}$, while Type 2 is risk-neutral. To show that price impact matters under preference heterogeneity even when
all risk is aggregate, we assume that there is no idiosyncratic risk ($\alpha = 0$). We can then solve for the equilibrium as if there were only two states, $\theta \in \{l, h\}$.

In the competitive benchmark with a risk-neutral agent, all assets are invested in the risky technology, the risk-averse bear no consumption risk, and asset prices are $q(\theta) = \frac{1}{2\bar{Y}}$ for all $\theta$. In an equilibrium with price impact, risk-neutral agents’ state prices satisfy $\Lambda(\theta) = \frac{1}{2\bar{Y}}$. First-order conditions for the risk-neutral’s asset portfolio are

$$\frac{1}{2} \frac{1}{q(\theta)} = \bar{Y} \left( 1 + \lambda \mu \frac{a_2(\theta)}{q} \right) \quad (14)$$

Observe that the left-hand side is the expected return of a security indexed by $\theta$, whereas the right-hand side is the return on investment $\bar{Y}$ plus an adjustment for price impact. Since asset prices satisfy $q(\theta) = \frac{1}{2\bar{Y}} + \lambda \left( a_1(\theta) + a_2(\theta) \right)$, we have

$$a_2(\theta) = -\frac{1}{1+\mu} a_1(\theta) \quad \Rightarrow \quad q(\theta) = \frac{1}{2\bar{Y}} - \lambda \mu a_2(\theta).$$

Hence risk-neutral agents strategically distort the supply of assets demanded by the risk-averse for insurance purposes, and lower their purchases of assets sold by the risk-averse to fund investment. Prices are tilted to favor the risk-neutral in every state because
they are more willing than the risk-averse to adjust asset holdings. This leads to a
generalization of the insight from Proposition 6 that sellers distort more: it is agents
with flatter marginal utility that always distort more. With homogeneous preferences,
these are sellers; with preference heterogeneity, it may be buyers with low risk aversion.

Taking differences of the risk-averse type’s first-order conditions yields

$$\frac{1 - m_1}{m_1(1 + \Delta) + (1 - \Delta)} = \bar{\lambda} \left( \frac{\mu^2}{1 + \mu} \right) \left[ a_1(l) - a_1(h) \right]$$

where \(m_1 = u'(c_1(h))/u'(c_1(l))\) is risk-averse agents’ marginal rate of substitution across
states. As before, \(m_1 \leq 1\) and \(m_1 = 1\) if and only if there is perfect risk-sharing. The right-
hand side is positive since \(a_1(h) \leq 0\) and \(a_1(l) \geq 0\) in order to transfer risk exposure.
Hence increasing price impact forces the risk-averse agent to bear risk that is costless to bear for risk-neutral agents. This effect is amplified when \(\Delta\) is large because the risk-neutral can extract more rents when risk-averse agents have steeper marginal utility.

Now consider the implied asset span. The risk-neutral agent has a constant state
price \(\Lambda_2(z) = \Lambda^* = 2\pi / \bar{\lambda}\). For short, write the risk-averse agent’s state price for the high aggregate state as \(\Lambda^h < \Lambda^*\), and the state price for the low aggregate state as \(\Lambda^l > \Lambda^*\). Since there are two distinct states, Proposition 4 shows that we rationalize the observed degree of risk-sharing using an incomplete span consisting of one asset. The implied dividend-yield of this asset \(\{d_\theta\}_{\theta = h, l}\) satisfies

$$\begin{bmatrix} \Lambda^* & \Lambda^* \\ \Lambda^h & \Lambda^l \end{bmatrix} \begin{bmatrix} d_h \\ d_l \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$ 

This implies that the only asset traded is a market index. To see this, observe that

$$d_h = \frac{\Lambda^l - \Lambda^*}{\Lambda^* (\Lambda^l - \Lambda^h)} \quad \text{and} \quad d_l = \frac{\Lambda^* - \Lambda^h}{\Lambda^* (\Lambda^l - \Lambda^h)}.$$ 

This asset has expected payoff \(E[d_\theta] = 2\pi (d_l + d_h) = \frac{2\pi}{\Lambda^*} = \bar{\lambda}\), and it provides exposure to aggregate risk since \(\Delta^h + \Delta^l > 2\Lambda^*\) by Jensen’s inequality. At the same time, the spread across states is low enough that risk-averse agents do not want to trade away from their position. Strategic exploitation of price impact just dampens the state-contingency available for hedging purposes by risk-averse traders.
Figure 4: Heterogeneous preferences. $\gamma_1 = 5$, $e = 1$, $\bar{Y} = 1.1$, $\Delta = 0.15$, $R_f = 1$, $\alpha = 0$, $\lambda = 0.5$.

Figure 4 shows comparative statics with respect to price impact for the case of heterogeneous preferences. The top left panel plots asset prices. In sharp contrast to the case of homogeneous preferences, $q(l)$ is strictly increasing and $q(h)$ is strictly decreasing. This is because the risk-neutral agent is a buyer of the high-state claim and a seller of the low-state claim. Since his marginal utility is always flatter than that of the risk-averse agent, he tilts asset prices in his favor in all states. The top right panel shows that this exposes the risk-averse agent to risk that would be costless to bear for the risk-neutral agent. The bottom panels show the underlying asset portfolios. While both types strategically shade quantities, the risk-neutral agent wins by adjusting more quickly. This is because the risk-averse agent has steep marginal utility and is therefore more reluctant to trade away from the preferred allocation.
4.3 Productivity Differences

Lastly, we consider a variant with productivity differences. For simplicity, we assume that there is no risk: Type 1 has a production technology that returns \( y_1 > R_f \) with probability one, and Type 2 has a technology that returns \( y_2 \in (R_f, y_1) \). Without loss for equilibrium allocations, assume that both types are now risk-neutral, and restrict attention to a single risk-free financial claim trading at price \( q \). In the first-best, Type 1 invests the aggregate endowment in the risky technology, \( k_1 = 2e, k_2 = 0 \) and \( s_1 = s_2 = 0 \). This is attained in competitive equilibrium with asset price \( p^* = \frac{1}{y_1} \) and asset allocation \( a_2 = e/p^* \) and \( a_1 = -a_2 \). With price impact, we have

\[
\frac{\partial u_j}{\partial a_j} = y_j \left[ \frac{1}{y_j} - q - a_j \lambda \mu \right] \geq 0 \quad \text{where} \quad q = \frac{1}{y_1} + \lambda(a_1 + a_2).
\]

and with equality for an interior solution. This must be the case for Type 1. It follows that \( a_1 = -\left(\frac{1}{1+\mu}\right) a_2 \). As before, Type 1 shades down supply in order to extract price rents. The equilibrium with price impact is of one of two types: a corner equilibrium where \( k_2 = 0 \) and \( a_2 = \frac{e}{q} \), or an interior equilibrium where Type 2’s first-order condition holds with equality. We focus our exposition the interior equilibrium, which obtains if \( \mu \) is sufficiently large. Type 2’s portfolio is \( a_{2}^{*} = \frac{(y_2^{-1}-y_1^{-1})}{\lambda(2+\mu)} \left( \frac{1+\mu}{\mu} \right) \), and is strictly decreasing in \( \mu \) but increasing in the productivity difference. The asset price is \( q^{*} = y_1^{-1} + \frac{(y_2^{-1}-y_1^{-1})}{2+\mu} \), which is decreasing in \( \mu \) because Type 2 buys fewer assets. Investment satisfies \( k_{1}^{*} = e + \left(\frac{1}{1+\mu}\right) a_{2}^{*} \) and \( k_{2}^{*} = e - q^{*} a_{2}^{*} \), and \( k_{1}^{*} (k_2) \) is strictly decreasing (increasing) in \( \mu \). Hence measured \( TFP = \frac{y_1 k_{1}^{*} + y_2 k_{2}^{*}}{k_{1}^{*} + k_{2}^{*}} \) is decreasing in \( \mu \).

5 Incomplete Network

We now consider an extension in which agents are part of a fixed network \( G \) that determines with whom they can trade claims. We use this environment to study how market decentralization affects price impact. We will draw a sharp contrast between exogenously incomplete markets that prohibit certain contracts and the endogenous incompleteness that stems from having to trade state-contingent contracts through an intermediary who exerts market power.

Portfolio positions are now indexed by counterparty. Hence it is not sufficient
to keep track of one state price per state, and we index securities by issuer and state. The state-
z claim issued by $i$ trades at price $q(i, z)$, and $j$’s holdings of this security are $a_j(i, z)$. To prevent the network from unraveling, we assume that there is a competitive fringe for each security $(i, z)$. Since agents who are not connected directly cannot attempt to align their marginal utilities across states, they need not agree on state prices. Furthermore, since agents internalize their price impact, they have incentive to spread their demand for contingent claims within a state across many issuers to minimize the cost of insurance. Consequently we cannot impose one market-clearing condition and one market-clearing price for claims to output referencing each state of nature.

An undirected link $l_{ij} \in G$ in this graph, with support $\{0, 1\}$, implies that agent $i$ can buy $a_i(j, z) \geq 0$ shares of contingent from agent $j$ for state $z$ at $t = 1$, and sells $a_i(i, z) \leq 0$ shares to other agents. Without loss for our analysis, we assume that all agents are connected to each other, either directly or indirectly through other agents. Since each agent exercises market power, it may profit from trading two securities that reference the same state $z \in Z$. Given that exchanging securities is a zero-sum trade, both agents cannot profit from it and, as such, they will not exchange cross-holdings.

The budget constraint of agent $i$ at $t = 0$ is then:

$$c_{i1} + k_i + \sum_{z \in Z} (\bar{a}_i(z) \odot \bar{l}_i) \bar{q}(z) + s_i \leq e_i - \sum_{z \in Z} q(i, z) a_i(i, z),$$

where $\bar{l}_i$ is the digital vector that summarizes the intermediaries from whom $i$ can purchase output claims. Given its links, the optimization problem for agent $i$ is now:

$$U'_i(e_i, e_{-i}) = \sup_{\{c_{i1}, k_i, s_i, \bar{d}_i, \bar{a}_i(z)\}} u_1(c_{i1}) + \sum_{z \in Z} \pi(z) u_2(c_{i2}(z))$$

$$s.t. : \quad c_{i1} + k_i + \sum_{z \in Z} \bar{a}_{-i}(z) \bar{q}(z) + s_i \leq e_i - \sum_{z \in Z} q(i, z) a_i(i, z),$$

$$\quad : \quad c_{i2}(z) = y_i(z) k_i + \bar{a}_{-i}(z) \bar{l}_i + R s_i + a_i(i, z),$$

$$\quad : \quad -a_i(i, z) \leq \xi(y_i(z) k_i + \bar{a}_{-i}(z) vecl_i + R s_i) \quad (I.C.).$$

Optimal portfolios are similar to Proposition 1 but modified to reflect that agents are restricted in their trading partners. The key novelty of incomplete networks is that agents

\[7\] In this sense, cross-holdings arise in equilibrium only if markets are incomplete. See Donaldson and Piacentino (2017) for a recent variation on this argument.
can earn strict arbitrage rents by intermediating trade between two counterparties.

**Proposition 8** As a result of market power: 1) if agent \( i \) buys debt from \( j \) in state \( z \), then:

\[
q(i, z) > q(j, z),
\]

and \( i \) earns excess profit, \( \Pi_i(z) \), per dollar of equity from trading securities in state \( z \):

\[
\Pi_i(z) = r_i \sum_j \left( \frac{\partial q(i, z)}{\partial a_i(i, z)} a_i(i, z)^2 + \frac{\partial q(j, z)}{\partial a_i(j, z)} a_i(j, z)^2 \right) + \left( 1 - \frac{\Lambda_i(z)}{\sum_{z \in \mathcal{Z}} \Lambda_i(z)} \right) \sum_{j=1}^N a_i(j, z),
\]

(16)

where \( r_i = \frac{1}{\sum_{z \in \mathcal{Z}} \Lambda_i(z)} \) is the effective interest rate for agent \( i \). The first (nonnegative) term is its rent from market power, and is increasing in \( a_i(i, z) \) (\( a_i(j, z) \)) when \( q(i, z) \) (\( q(j, z) \)) is convex in net demand. The second term arises from risk-sharing, is positive if \( i \) is a net lender (\( \sum_{j=1}^N a_i(j, z) > 0 \)), and is smaller if \( i \)’s debt constraint binds.

From Proposition 8, agents create an intermediation spread between the securities they buy and sell in each state, so that they charge a higher rate to borrowers than they pay to creditors. This can be seen from the first term in their trading profit in each state in equation (16). This provides an arbitrage opportunity in an incomplete network, as a central intermediary can arbitrage across segmented markets. There is potential economies of scale in lending, both across counterparties and states of nature. The requirement that the price function be convex in net demand is satisfied (globally) when the price function is linear. It is satisfied (locally) for the other examples given in Appendix B when a risk premium is embedded in the price and, consequently, indirect intermediation is likely to be most profitable when trading claims corresponding to high marginal utility states of nature. In addition, an agent may use storage to increase the pledgeability of its own claims across all states, leading to an inefficient distortion to production. The rent earned from intermediating funds could also potentially give rise to a core-periphery network structure when forming links is costly, as the rents from being a central agent with market power can defray the costs of paying for the additional links to indirectly intermediate funds from lenders to borrowers.

While agents do earn rents from trading on one side of the market as a result of market power, the ability to double margin can introduce strict arbitrage profits in the
form of a "bid-ask" spread, or profits that entail no risk for acting as an intermediary. This double margining also rules out cycles of state-contingent debt holdings, as prices must increase along the intermediation chain. As such, a lender in an intermediation chain cannot be a borrower from a borrower further downstream in the chain, as it must then pay a higher price to borrow than to lend.

A potential tradeoff to this arbitrage, unique to our setting, however, is that it impacts the cost of acquiring insurance for the risk-averse arbitrageur. The second term in the profit expression above, while also present in a competitive setting, reflects the embedded risk premium from buying insurance, which is negative when agent $i$ is a net borrower. The need for insurance against states in which an agent itself has high marginal utility acts as an endogenous limits to arbitrage from arbitraging across segmented markets. As a consequence, there is an interior optimum to the scale of the arbitrage for claims in states of the world in which the central agent itself needs insurance.

In addition, as a result of imperfect competition among agents in an incomplete network, gross flows are also the relevant quantity for balance sheets because of the dispersion of state prices, whereas only net flows (within a state) matter with perfect competition. Whom trades with whom is important for understanding the dispersion in asset prices on otherwise similar securities. To see this, we can express the net market value of debt of agent $i$ in state $z$ as:

$$ - \sum_{j=1}^{N} q(j,z) a_i(j,z) = -\Lambda_i(z) \sum_{j=1}^{N} a_i(j,z) + \sum_{j=1}^{N} \frac{\partial q(j,z)}{ \partial a_i(j,z)} a_i(j,z)^2. $$

While the first term, which is also present with perfect competition, depends only the the net flow, the second two terms reflect the discrepancy in the security valuations between $q(i,z)$ and $q(j,z)$ as a result of market power. These two terms depend on agent $i$'s specific cross-balance sheet linkages, and cannot be netted out into a net flow.

Finally, the topology of the network feeds back into the role of financial markets to efficiently share risks and allocate capital. The ability to intermediate across segmented

---

8There is a vast literature on the limits to arbitrage, including, for instance, De Long et al. (1990), Xiong (2001), Gromb and Vayanos (2002), and Brunnermeier and Pedersen (2009). In contrast to these studies, it is not risk aversion, portfolio constraints, or asymmetric information that gives rise to mispricing in our setting. Instead, it is that the convergence trade for arbitraging agents worsens the price at which they acquire the security for their own consumption.
markets is the key new force that an incomplete network introduces.\footnote{Rahi and Zigrand (2009, 2013) consider endowment economies in which risk-neutral arbitrageurs strategically trade across segmented markets against competitive investors.} This introduces two distortions. First, by arbitraging across markets state-by-state, core agents ration credit to producers and borrower from end providers of capital. This distorts the supply of capital to productive agents. Second, since core agents face borrowing constraints, they may invest in inefficient storage to increase the pledgeability of their balance sheet in all states to be able to more efficiently arbitrage across segmented markets.

Importantly, market power is essential for network effects in pricing and risk-sharing to arise among agents. The following corollary highlights that, while the distortions from limited commitment to investment would also be present in a competitive setting, market power is essential for gross flows to matter and for double-margining.

**Corollary 3** If agents behave competitively: 1) there is no indirect intermediation when forming links is costly or network effects beyond price externalities; and 2) investment in capital is (above) below its efficient level for agents that (do not) invest in the absence of limited commitment.

We next investigate these forces by revisiting our 3 agent example, and characterizing the asymmetric equilibrium that arises with a core-periphery network structure.

### 5.1 Core Periphery Example Economy with Three Agents

To further understand the role of market power, we now study a core-periphery market structure in which a representative core agent trades with two representative peripheral agents, and the periphery trades with the core but not with each other. In contrast to the complete network, the core may now buy and issue claims against a state in which his investment does not pay off to facilitate intermediation. This opens the door to arbitrage opportunities (buying and selling claims on the same state at different prices).

We again look for an equilibrium that is symmetric conditional on the market structure. Hence the core treats the periphery symmetrically, and the periphery holds a symmetric portfolio. There are now three types of asset: the *core asset* issued by the core against the state in which his risky investment pays off, the *periphery asset* issued by each periphery agent against the state in which his asset pays off, and the *arbitrage asset* issued by the core against a state in which a periphery’s investment pays off, and sold
to the periphery agent whose investment does not pay off in that state. We index these assets and the associated states by subscripts $c$, $p$, and $a$, and use $a_z$ and $a_z$ to denote core and periphery holdings of the asset associated with state $z \{c, p, a\}$. The three types of asset trade at prices $q_c$, $q_p$, and $q_a$.

Each periphery agent decides on its issuance of its periphery asset $a_p$, its purchases of the core asset $a_c$ and of the arbitrage asset $a_a$, and its saving $s$. Because there are no direct links between the periphery, it does not buy the other periphery agent’s asset. Consumption may differ across all three states. Hence we will denote periphery consumption levels by $c_c$ (consumption when the core’s investment pays off), $c_p$ (consumption when the own investment pays off), and $c_a$ (consumption when the other peripheral agent’s asset pays off), respectively. The borrowing constraint is the analogue to before. If it does not bind, the first-order conditions with respect to $a_p$, $a_a$, $a_c$ and $s$ are, respectively,

$$
\begin{align*}
 u'(c_p) &= u'(c_p)y(q_p - \mu \lambda a_p) \\
 u'(c_a) &= u'(c_p)y(q_a + \mu \lambda a_a) \\
 u'(c_c) &= u'(c_p)y(q_c + \mu \lambda a_c) \\
 u'(c_a)R_f + u'(c_c)R_f - u'(c_p)(y - R_f) &\leq 0
\end{align*}
$$

Much as before, there is an incentive to shade down asset supply and asset demand to boost $q_p$, the price of the issued asset and to lower $q_c$ and $q_a$, the price of the core and arbitrage asset. The difference is that there is no strategic impact on the price of the purchases peripheral asset. The first-order condition on savings binds only if insurance via asset purchases is too expensive.

The core decides on its own asset issuance $a_c$, its (symmetric) holdings of each of the periphery assets $a_p$, its issuance of each of the arbitrage assets $a_a$, and its savings $s$. In addition to the borrowing constraint on the own asset, the core faces a borrowing constraint on its arbitrage asset given by

$$
\hat{a}_a \leq \xi [\hat{a}_p + R_f \hat{s}] \tag{17}
$$

In a symmetric equilibrium, consumption is $c^+$ when the core’s risky investment pays
off, and \( c^- \) when it does not. Asset prices satisfy

\[
q_c = \frac{1}{y} + \lambda (2a_c - \hat{a}_c) \quad q_p = \frac{1}{y} + \lambda (\hat{a}_p - a_p) \quad q_a = \frac{1}{y} + \lambda (a_a - \hat{a}_a)
\]

Observe that the market structure gives rise to market size effect: because the core has two buyers for its asset, and is the unique buyer or seller of every other asset, it benefits from higher sale prices and can exercise more de-facto market power than periphery agents.

We develop intuition by first focusing states of the world in which the core’s investment does not pay off, and he instead engages in intermediation. In any such state, the core receives assets from periphery agent whose investment pays off, and pays out claims to the other periphery. Hence

\[
c^- = \hat{a}_p - \hat{a}_a + \bar{R}_f \hat{z},
\]

and there is an arbitrage opportunity if \( q_p < q_a \). The portfolio simultaneously determines core consumption. We therefore define the core’s arbitrage problem as the decision problem that determines how to deliver a given target level of consumption \( \bar{c} \) minimal cost. (The consumption target \( \bar{c} \) can be solved for separately.) Given \( \bar{c} \), the arbitrage problem can thus be simply states as

\[
\max_{\hat{a}_p, \hat{a}_a} \Pi^{arb}(\bar{c}) = q_a \hat{a}_a - q_p \hat{a}_p \quad \text{s.t.} \quad \hat{a}_p - \hat{a}_a = \bar{c} \quad \text{and} \quad (17).
\]

If the borrowing constraint does not bind on the arbitrage, first-order conditions reveal that the optimal arbitrage portfolio for a core trader who internalizes his price impact is

\[
\hat{a}_a^*(\bar{c}) = \frac{q_a - q_p}{2\mu\lambda} - \frac{\bar{c}}{2} \quad \text{and} \quad \hat{a}_p^*(\bar{c}) = \frac{q_a - q_p}{2\mu\lambda} + \frac{\bar{c}}{2}
\]

That is, gross positions are increasing in the arbitrage spread \( q_a - q_p \), and are decreasing in market power \( \mu \). The latter effect arises because the spread moves against the arbitrage whenever a trader with market power increases his position. The core’s strategic restraint harms the periphery, who must sell low at \( q_p \) and buy high at \( q_a \). There is no arbitrage spread if and only if the core has no market power.

**Proposition 9** If the borrowing constraint is slack, the core-periphery equilibrium features a strictly positive arbitrage spread \( q_a^* - q_p^* \) for all \( \mu > 0 \). The spread is zero \( (q_a^* = q_p^*) \) and the equilibrium delivers the first-best allocation if and only if \( \mu = 0 \) and \( \lim \lambda \to 0 \). In this limit, moreover, the core acts as a pure pass through and sets \( \hat{a}_p = 2\hat{a}_a = \pi y e. \)
Given the optimal arbitrage portfolio, the core’s consumption in the state in which its own investment pays off can be written as

\[ \hat{c}^+ = ye - \hat{a}(1 - q_c y) + 2y \Pi^* (\bar{c}) + (R_f - y) \bar{s}. \]

Since arbitrage profits are decreasing in \( \bar{c} \), consumption in state \( c \) is strictly decreasing in \( \bar{c} \), while consumption in states \( z \neq c \) is strictly increasing. The first-order conditions for \( \hat{a}_c, \bar{c}, \) and \( \bar{s} \) are

\[
\begin{align*}
\pi u' (\hat{c}^+) &= \pi u' (\hat{c}^+) y (q_c - \lambda \hat{a}_c) \\
2 \pi u' (\hat{c}^-) &= \pi u' (\hat{c}^+) 2y \left( -\frac{\partial \Pi^{arb} (\bar{c})}{\partial \bar{c}} \right).
\end{align*}
\]

The first condition shows the standard decision problem with strategic undersupply of the own asset. The optimal choice of \( \bar{c} \) resolves a trade-off between arbitrage profitability and risk-sharing.

Figure 5 illustrates the resulting equilibrium. Asset prices are in the top left corner. For all \( \mu > 0 \), there is a strictly positive arbitrage spread \( q_a - q_p \). Market power initially serves to strengthen this arbitrage, boosting core utility at the expense of the periphery. Since insurance is expensive for the periphery, it responds by investing in storage. Because the core purposefully underprovides insurance, investment efficiency declines for all agents. This leads to a decline in average expected utility.

Figure 6 shows that the presence of arbitrage rents may sustain a core-periphery network as an equilibrium outcome, despite that it lowers productive efficiency relative to the complete network. To illustrate this, we plot equilibrium utility minus a fixed cost of forming each link required to create the network structure in the left panel, and expected output in the right panel. When linking costs are relatively high, all agents may prefer a core-periphery structure to a complete network, although it generates lower output. The core is willing to pay for additional links because doing so provides arbitrage opportunities. The periphery accepts lower production efficiency in exchange for lower trading costs. The network structure thus endogenously strengthens the harmful effects of market power.

The ability to double margin lenders and borrowers can, consequently, help explain the core-periphery structure often observed in lending in financial markets outside
Figure 5: Equilibrium in core periphery network given $\mu$ assuming the borrowing constraint is slack. Parameters: $e = 1$, $y = 3.3$, $\mathbb{E}[y] = 1.1$, $R_f = 1$, $\lambda = 0.05$ Gray horizontal lines show the first-best equilibrium with $q^* = \frac{1}{y}$, $a_s = 2\pi ye$, $a_b = \pi ye$, and $c^+ = c^- = \pi ye$. of exchanges. This is in contrast to Farboodi (2016), in which a core-periphery structure arises because intermediation spreads decline with distance along the intermediation chain, and Craig and Ma (2018), which rationalizes it with heterogeneity across banks in project quality and link costs.

Lastly, we discuss the effects of borrowing constraints in a core periphery. The borrowing constraint on issuances of the own asset are well understood already. Hence we focus now on a binding borrowing constraint when the core arbitrages across the
periphery. If constraint (17) binds, arbitrage profits in a given state can be written as

$$\Pi = qa_\xi \left[ \hat{a}_p + R_f \hat{s} \right] - q_p \hat{a}_p = (qa_\xi - q_p) \hat{a}_p + qa_\xi R_f \hat{s}$$

Hence savings now raise arbitrage profits by relaxing the borrowing constraint, and the de-facto return to savings is now $R_f(1 + pa_\xi) > R_f$. This means that core agents are more likely to distort their investment towards inefficient savings in order to relax the borrowing constraint that limits arbitrage. Market power thus necessitates the creation of borrowing capacity. More generally, this implies that agents with low risk production capacity or commitment power are more likely to be central in a core periphery network.

## 6 Empirical Implications

Our theoretical investigation gives rise to several empirical predictions as financial markets become more concentrated. First, more downstream agents in segmented intermediation chains are more central, in that changes in their balance sheets have the largest effective sell-side impact on prices and the borrowing capacities of other agents in the economy. Second, one would expect to observe systematic under-investment in risky projects, whether it be C&I or syndicated lending by banks, risky debt purchases by
bond funds, or capital investment by firms as a result of under-insurance, for instance, in wholesale lending, REPO, and OTC markets. Furthermore, the debt of large agents and the firms in which they invest should have lower expected returns as a result of imperfect competition. Third, one would also expect a secular decline in discount rates as agents become larger and internalize their market power.

In addition, leverage constraints and market power act as substitute channels for under-insurance across agents. Since sellers benefit from market power only when they are unconstrained, leverage constraints should bind less, and consequently be less relevant, for larger agents. When sellers are unconstrained, they determine the bias in asset prices, whereas buyers’ asset positions should become more relevant when sellers are constrained.

Our results also have empirical implications for intermediary asset pricing. Cochrane (1991, 1996) emphasize that production outcomes of firms should reflect the marginal rates of transformation (MRT) of firms maximizing shareholder value, just as consumption reflects the marginal rates of substitution (MRS) of consumers. The limitation that firms can transfer production only across time with capital investment, but not across states, however, limits the ability to recover MRTs from investment returns.\(^\text{10}\) Our analysis suggests, if agents in our model can be interpreted as intermediaries, that it is can be the contracts that intermediaries write with each other that reflects state contingency in (effectively joint) production.

Recall that agent \(i\) produces net output \(c_{i2}(z)\) at \(t = 2\) in state \(z \in \mathcal{Z}\). This output reflects both agent \(i\)'s own production, net of what it sends to other agents to honor its contracts from \(t = 1\). These contracts can be financial securities agents trade with each other, such as collateralized debt, equity, and derivatives, or as the delivery of actual goods employable in some alternative production technology. While its choice of capital \(k_i\) at \(t = 1\) suffers from intratemporal rigidity, its net output does not because of its contracts with other agents that reintroduce state contingency. Our analysis suggests that examining earnings and inter-intermediary exposures may be better indicative of not only their market power, but also intermediary state prices relevant for how they would price assets.

\(^{10}\)Belo (2010), for instance, posits that it is the ability of firms to vary their productivity across states, subject to a budget constraint in reassigning conditional probabilities, that makes investment returns informative about MRTs.
We can, for instance, take a Taylor expansion of the implied state price of agent \( i \) around initial earnings to find that

\[
\Lambda \approx \alpha_0 - \alpha_1 \frac{c_{i2}(z)}{c_{i1}} + \alpha_2 \tilde{\psi}_i(z),
\]

(20)

where \( \frac{c_{i2}(z)}{c_{i1}} \) is earnings growth and \( \tilde{\psi}_i(z) \) reflects capacity constraints. One can then test this SDF on test portfolios following a GMM or Fama-MacBeth approach.

Furthermore, the ability for agents to profit from double-margining when trading across segmented markets has implications for the costs of capital in the economy. As lenders internalize how they price the debt of borrowers, our model predicts increasing costs of capital as one moves downstream in the intermediation chain. End lenders, such as depositors or institutional investors, should earn lower returns than the agents who lend on their behalf to other agents or firms.

Finally, our theory can also potentially rationalize both core-periphery network structures and segmented markets for similar products among large agents as stemming from the tradeoffs of strategic interaction.

7 Conclusion

We construct a model of concentrated markets in which large, risk-averse agents with limited commitment invest in risky projects and internalize their price impact when trading contingent claims with each other. Central agents in our setting are those who issue claims to agents who themselves issue claims against output, since their production and savings decisions have the greatest impact on relaxing effective capacity constraints in the economy. As a result of strategic interaction, agents voluntarily under-insure production risk with each other. Such under-insurance adversely impacts capital investment decisions and, despite market completeness, generates dispersion in state prices. This strategic market incompleteness can lead to arbitrage opportunities across segmented markets and a decline in the risk-free rate, and can introduce a role for a government to implement welfare-improving policies. These forces feed back into the benefits for agents to trade with each other, and help rationalize core-periphery networks and market segmentation when links are costly.
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A Proofs of Propositions

A.1 Proposition 1

Since no agents default in equilibrium, the shadow value of an increase in debt issuance by agent $i$ today is absorbed by the appropriate Lagrange multiplier, which increases the asset price despite the lack of change in agent $j$’s marginal utility by Kuhn-Tucker. FONCs are then valid.

Let us normalize the Lagrange multiplier on wealth be $\phi_i$ for (15) to be $\varphi_i e_i^\gamma$, and that on the I.C. constraint to be $\tilde{\xi}_i (z) = \tilde{\xi}_i (z) e_i^\gamma$. The FONCs for $c_{i1}, k_i, s_i,$ and $a_i (z)$ in problem (15) are given by:

$$c_{i1} : u' (c_{i1}) - \varphi_i e_i^\gamma \leq 0 \quad (= \text{if } c_{i1} > 0),$$

$$k_i : \sum_{z \in Z} \pi (z) u'_2 (c_{i2} (z)) y_i (z) - \varphi_i e_i^\gamma + \tilde{\xi}_i e_i^\gamma \sum_{z \in Z} \pi (z) \psi_i (z) y_i (z) \leq 0 \quad (= \text{if } k_i > 0),$$

$$s_i : \sum_{z \in Z} \pi (z) u'_2 (c_{i2} (z)) R - \varphi_i e_i^\gamma + R \tilde{\xi}_i e_i^\gamma \sum_{z \in Z} \pi (z) \psi_i (z) \leq 0 \quad (= \text{if } s_i > 0),$$

$$a_i (z) : -\pi (z) u'_2 (c_{i2} (z)) + \varphi_i e_i^\gamma \left( q (z) + \frac{\partial q (z)}{\partial a_i (i, z)} a_i (i, z) \right) - \pi (z) \psi_i (z) e_i^\gamma = 0 \quad (a_i (z) > \bar{a}_i (z)).$$

Therefore, the above represent the FONCs for agent $i$’s problem. Since $u_1 (\cdot)$ satisfies the Inada condition, $c_{i1} > 0$ and the first FOC binds with equality.

Define $h_i = \frac{\xi_i}{\sum_{i=1}^N \xi_i}$ to be the effective share of scaled equity of agent $i$. Let us conjecture that each agent’s optimal policies are such that, for policy $s_i$, we can decompose $s_i$ into $s_i = \hat{s}_i e_i$, where $\hat{s}_i$ is independent of $e_i$ and the level of equity of any other agent. We can rewrite the stacked incentive compatibility conditions as:

$$\tilde{\delta} (z) = 1 \left\{ Y (z) \overset{\rightarrow}{k} - \overset{\rightarrow}{g} (z) \geq 0 \right\},$$

where $\tilde{\delta} (z)$ is the digital vector of repayments in state $z$ and:

$$\overset{\rightarrow}{g} (z) = \tilde{\xi}_i^{-1} B (z) \overset{\rightarrow}{s} - R \overset{\rightarrow}{s},$$

where $B (z)$ is the diagonal matrix with entries $B_{ii} (z) = -a_i (z)$, which depends on $e_i$ only through $h_i \forall i$. Consequently, the default decision satisfies the conjectured decom-
position.

Furthermore, imposing market-clearing we can express the price of agent $i$'s debt referencing state $z$ as: $D (i, z) = f_i (q (z), z) \sum_{j=1}^{N} e_j$

$$q (z) = f_i^{-1} \left( -\frac{\sum_{j=1}^{N} a_j (i, z)}{\sum_{j=1}^{N} e_j}, z \right) = f_i^{-1} \left( -\sum_{j=1}^{N} h_j \hat{a}_j (i, z), z \right).$$

By the Inverse Function Theorem, the inverse of $f_i (q (z), z)$ exists (locally) wherever $\frac{\partial f_i (x, z)}{\partial x} \neq 0$. Further, $\frac{\partial f_i^{-1} (x, z)}{\partial x} = \left( \frac{\partial f_i (x, z)}{\partial x} \right)^{-1} < 0$. Consequently, it follows that $\frac{\partial q(z)}{\partial a_i (i, z)} > 0 \forall (i, z)$, with the inequalities strict since $f (\cdot, z)$ is a strictly increasing function.

Since utility is homogeneous in degree $e_i^1$, we can then rewrite the FONCs as:

$$\hat{c}_{i1} : u_1' (\hat{c}_{i1}) - \varphi_i = 0,$$

$$\hat{k}_i : \sum_{z \in \mathbb{Z}} \pi (z) \frac{u_1' (\hat{c}_2 (z))}{u_1 (\hat{c}_{i1})} y_i (z) + \hat{c}_i \sum_{z \in \mathbb{Z}} \pi (z) \frac{\psi_i (z)}{u_1 (\hat{c}_{i1})} y_i (z) \leq 1 \quad (= \text{if } \hat{k}_i > 0),$$

$$\hat{s}_i : \sum_{z \in \mathbb{Z}} \pi (z) \frac{u_1' (\hat{c}_2 (z))}{u_1 (\hat{c}_{i1})} R + R \hat{c}_i \sum_{z \in \mathbb{Z}} \pi (z) \frac{\psi_i (z)}{u_1 (\hat{c}_{i1})} \leq 1 \quad (= \text{if } \hat{s}_i > 0),$$

$$\hat{a}_i (i, z) : -\pi (z) \frac{u_1' (\hat{c}_2 (z))}{u_1 (\hat{c}_{i1})} + q (z) - h_i \frac{\partial q (z)}{\partial a_i (i, z)} \hat{a}_i (i, z) - \pi (z) \frac{\psi_i (z)}{u_1 (\hat{c}_{i1})} = 0 \quad (\hat{a}_i (z) > \hat{a}_i e_i),$$

and by definition of the budget constraint at $t = 2$:

$$\hat{c}_i (z) = y_i (z) \hat{k}_i + \sum_{j \neq i} \hat{a}_i (j, z) + R \hat{s}_i + \hat{a}_i (i, z).$$

Finally, the budget constraint at $t = 1$, which will hold with equality by efficiency, can be expressed as:

$$\hat{c}_{i1} + \hat{k}_i + \sum_{z \in \mathbb{Z}} \hat{d}_{-i} (z) \hat{q}_i (z) + s_i = 1 - \sum_{z \in \mathbb{Z}} q (z) \hat{a}_i (i, z).$$

The rewritten FONCs are independent of $e_i \forall i \in \{1, ..., N\}$, and, defining $\Lambda_i (z) = \pi (z) \frac{u_1' (\hat{c}_2 (z)) + \xi_i \psi_i (z)}{u_1 (\hat{c}_{i1})}$ to be the effective state price of agent $i$ in state $z$, confirm our conjecture.

It then follows that the indirect utility of agent $i$ from its trading and repayment
game is:

\[ U_i^t (e_i, \bar{e} - i) = e_i^{1-\gamma} u_1 (\tilde{c}_{i1}) + e_i^{1-\gamma} \sum_{z \in \mathcal{Z}} \pi (z) u_2 (\tilde{c}_{i2} (z)) = \bar{U}_i^t \left( h_i, \bar{h}_{-i} \right) e_i^{1-\gamma}. \]

To see that all the controls, normalized by the equity of agent \( i \), \( e_i \) are bounded, we consider the cases in which agent \( i \) does and does not issue state-contingent claims. When agent \( i \) does not issue any state-contingent claims, then by the balance-sheet constraint, it can only self-finance its uses of funds, and therefore \( \tilde{c}_{i1}, -\hat{a}_i (i,z), \hat{k}_{i}, \hat{a}_i (i,z) \leq 1 \). When agent \( i \) issues state-contingent claims, then the total normalized resources of all agents in the network is \( N \) plus borrowing from the outside creditor, agent \( 0 \).

By assumption, the credit that can be extracted from the outside investor \( N+1 \) in any state, \( q (z) f_i (q (z), z) \sum_j e_{jr} \) is bounded from above. It then follows that:

\[ -\tilde{a}_i (i,z) \leq \left( 1 + \sup_{q(z) b_{N+1}(z)} q (z) f_i (q (z), z) \right) \sum_{j \neq i} e_{jr}, \]

which is bounded, and can be rewritten as:

\[ -\tilde{a}_i (i,z) \leq \frac{1}{h_i} \left( 1 + \sup_{q(z) b_{N+1}(z)} q (z) f_i (q (z), z) \right) \leq \frac{1}{\min \{ h_j \}} \left( 1 + \sup_{q(z) b_{N+1}(z)} q (z) f_i (q (z), z) \right) = -\tilde{a}_i (i,z). \]

Since borrowing in each state is bounded from above, total borrowing by agent \( i \) is bounded from above.

Therefore, \( \hat{a}_i (i,z) \geq \tilde{a}_i (i,z), \) and \( \hat{c}_{i1}, \hat{k}_{i}, \hat{a}_i (j,z) \leq 1 - \tilde{a}_i (i,z). \) The controls are trivially bounded from below by \( 0 \) because of the investment constraints on agent \( i \)'s problem. Since the controls lie in a closed and bounded set, they lie in a compact set by the Heine-Borel Theorem.

Define \( \mathcal{X}_i \) to be the state space at date \( t \) for agent \( i \) \( \mathcal{X}_i = \mathbb{R}_+^{N} \times S_i^{N \times |\mathcal{Z}|} \times \Sigma (-i), \) where \( \Sigma (-i) \) is the space of strategies of the other agents in the network. Notice that, given the homotheticity of agent utility, it follows that \( U_0^t (e_i, \bar{e} - i) = U_0^t \left( h_i, \bar{h}_{-i} \right) e_i^{1-\gamma}, \) where \( h_j \in [0,1] \) \( \forall j \in \{1, \ldots, N\}, \) and the range of the controls \( \hat{k}_{i}, \tilde{s}_{i}, \hat{a}_i (i,z), \) and \( \hat{a}_i (j,z) \) lies in a compact, convex set. As such, the set of admissible controls normalized by equity \( \mathcal{M} \subseteq \mathbb{R}_+^{N \times |\mathcal{Z}|+2} \) is compact and convex, where \( g_i \in \mathcal{M} \) is a \( N \times |\mathcal{Z}|+2 \)
tuple \( g_i = \left[ \hat{c}_i \right. \hat{k}_i \left. \hat{a}_i (i, z_1) \ldots \hat{a}_i (z_{|Z|}) \right. \left. \ldots \hat{\tilde{a}}_{-i} (z_1) \ldots \hat{\tilde{a}}_{-i} (z_{|Z|}) \right]' \), which maps to the controls in the original problem as \( g_i e_i \). Then \( G_i : \mathcal{X}_i \rightarrow \mathcal{M} \) is a compact-valued correspondence.

Since the objective \( u_2 (c_{i2} (z)) \) is continuous in the states, and consequently in the controls, and the correspondence \( G_i \) is compact-valued, then by Berges’ Theory of the Maximum there exists a solution to the agent’s problem and the optimal \( G_i^* \) is an upper-hemicontinuous correspondence. Importantly, it is the homogeneity of the optimal controls with respect to the equity of agent \( i \) \( e_i \) that allows us to express the space of controls as a compact-valued correspondence to satisfy the theorem.

Given that \( G_i^* \) is an upper-hemicontinuous correspondence \( \forall \ i \in \{1, \ldots, N\} \), applying Kakutani’s Fixed Point Theorem on the market-clearing conditions for agent debt establishes existence of a fixed point that is an equilibrium.

### A.2 Proof of Proposition 2

Let CE refer to the competitive equilibrium without price impact.

For the only if part, if the competitive and strategic equilibria coincide, then \( q (z) \) and \( \Lambda_i (z) \) must coincide. From the FONCs for asset positions, \( a_i^{CE} (z) \), this can only be the case if either price impact is zero \( \partial q (z) / \partial a_i (z) \) or, more generically, when \( a_i^{CE} (z) = 0 \) in the competitive equilibrium. When \( a_i^{CE} (z) = 0 \), however, then there are no gains from trade, and the only equilibrium is autarky.

For the if part, if there are no gains from trade in the competitive equilibrium, then \( a_i^{CE} (z) = 0 \) and the competitive and strategic equilibria trivially coincide.

### A.3 Proof of Proposition 3

Since there are no cross-holdings, suppose that agent \( i \) buys claims from agent \( j \) in state \( z \). Then, from the FONCs for the respective agents:

\[
\Lambda_i (z) - \frac{\partial q (z)}{\partial a_i (j, z)} a_i (j, z) = q (z) = \Lambda_j (z) - \frac{\partial q (z)}{\partial a_j (j, z)} a_j (j, z),
\]

from which follows that:

\[
\Lambda_i (z) - \Lambda_j (z) = -\frac{\partial q (z)}{\partial a_j (j, z)} a_j (j, z) + \frac{\partial q (z)}{\partial a_i (j, z)} a_i (j, z) > 0,
\]
and consequently $\Lambda_i(z) > \Lambda_j(z)$. Consequently, as a result of market power, risk-sharing is imperfect beyond the impairment from the endogenous leverage constraints arising from limited commitment.

To see that market power and limited commitment are substitutive effects, we recognize that both market power and limited commitment lead to an underprovision of state contingent securities. Since market power shades down how many state contingent claims agent $i$ issues on state $z$, and therefore borrows less against state $z$, it is less likely that $i$’s borrowing constraint will bind.

### A.4 Proof of Proposition 4

Consider the Nash Equilibrium in demand schedules allocation in the economy with imperfect competition, $\{(c_{i1}, k_i, s_i, \{c_i(z)\}_{z \in Z})\}_{i=1}^{N}$. Define

$$\Lambda_i(z) = q(j, z) + \frac{\partial q(j, z)}{\partial a_i(j, z)} a_i(i, z) \forall (i, j, z),$$

to be the implied state price deflator of agent $i$ in state $z$. Notice the implied state price deflator can include a collateral premium.

Now consider a fictitious incomplete markets economy in which all agents are instead competitive. We will try to derive the implied market structure, indexed by a set of $M$ securities with arbitrary $M \times K$ payoff matrix $D$. Since there are $K$ possible states of the world, it follows that we need at most $K$ linearly independent securities for markets to be complete, and consequently $M \leq K$. If markets are incomplete, then these $M$ securities are rank deficient, and consequently specifying at most $K$ potentially tradeable securities is without loss. Since assets cannot distinguish between some (subset of) states, the market-implied asset span spans $M$ (linear combinations of the $K$) states.

Let the price vector for these $M$ assets be $\vec{p}$ and define $\frac{D}{p}$ to be the dividend yield matrix with elements $\frac{d_{ij}}{p_i}$, where $d_{ij}$ is the payoff of the $i^{th}$ security in state $j$, and $q(\cdot)$ is the price of security $i$. We follow the convention that, since by no arbitrage $p_i = 0$ iff $d_{ij} = 0 \forall j \in \{1, ..., M\}$, that $\frac{d_{ij}}{p_i} = 0$ in this contingency.

No arbitrage then requires that

$$\mathcal{M} \frac{D}{p} = i_N N'_{M},$$

49
where $\mathcal{M}$ is the $N \times K$ matrix with elements $\mathcal{M}_{ij} = \Lambda_i (j)$ for intermediary $i$ and state $j$, and $t_M$ is the $M \times 1$ vector of one.

For fixed $M$, this equation can be rewritten as a matrix equation:

$$(I_M \otimes \mathcal{M}) \text{vec} \left( \frac{D}{p} \right) = \text{vec} \left( t_M' N t_M \right).$$

Since $\text{rank} (I_M \otimes \mathcal{M}) = M + \text{rank} (\mathcal{M})$, it follows that there is a unique solution for $\frac{D}{p}$ if $N \geq K$, while the system is under-identified if $N < K$.

As the synthetic assets are potentially derivatives, $\frac{D}{p}$ can have negative entries. We then choose the largest $M$ such that the recovered $\frac{D}{p}$ has rank $M$. We choose the largest $M$ since, if there were an additional asset that replicated the asset span that we ignored, then introducing this additional asset would not initiate trade, since it would already be priced at its no arbitrage value by all intermediaries. Consequently, $M \in \max_{M' \in \{0, \ldots, K\}} \left\{ \text{rank} \left( \frac{D}{p} \right) = M' \right\}$.

With this payoff matrix, we have constructed an incomplete market structure that measures the degree of market incompleteness by replicating the effective asset span of the Nash Equilibrium with complete markets. Since at least one state price is misaligned among intermediaries, the rank of $\frac{D}{p}$ must be less than $K$, and markets must be incomplete.

Finally, we recognize that the state prices of agent $i$ and $j$ are related by:

$$\Lambda_i (z) = \Lambda_j (z) + \frac{\partial q (z)}{\partial a_i (z)} (a_j (z) - a_i (z)).$$

Substituting for $\Lambda_j (z)$ for each $j \neq i$, we can rewrite the condition:

$$\mathcal{M} \frac{D}{p} = t_M' N t_M,$$

as $\frac{D'}{p} \Lambda_i = t_M$ and:

$$\frac{D'}{p} \left( \frac{\partial q}{\partial a'_j} \otimes (\bar{a}_j - \bar{a}_i) \right) = 0_{M \times 1} \forall j,$$

where $\frac{\partial q}{\partial a'_j}$ is the $K \times 1$ vector with entries $\frac{\partial q (z)}{\partial a_j (z)}$ and $\bar{a}_j$ is the the $K \times 1$ vector with entries $a_j (z)$.

For the second part of the claim, consider the Hansen and Jagannathan (1991)
decomposition of an admissible state price deflator, $\Lambda_i(z)$, into:

$$\Lambda_i(z) = \Lambda^*(z) + (\Lambda_i(z) - \Lambda(z)),$$

where a generic $\Lambda(z)$ is a state price deflator implied by market prices:

$$\Lambda(z) = \bar{\Lambda} + \left( \frac{D}{p} - \Pi \frac{D}{p} \right) \beta(\Lambda),$$

$\bar{\Lambda}$ is the market-implied mean of the state price deflator, $\bar{\Lambda} = [\sum_{z \in \mathcal{Z}} q(z)]$, or the inverse of the market-implied riskfree rate, and $\Pi$ is the diagonal matrix with diagonal entries $\pi(z)$. The minimum variance choice of $\Lambda(z)$, $\Lambda^*(z)$, satisfies:

$$\beta(\bar{\Lambda}) = \Sigma^{-1} \left( i_M - \Pi \frac{D}{p} \bar{\Lambda} \right),$$

where $\Sigma$ is the covariance matrix of returns. By construction, one has that:

$$\text{Cov} (\Lambda^*(z), \Lambda_i(z) - \Lambda^*(z)) = 0. \quad (21)$$

It then follows for two admissible $\Lambda_i(z)$ and $\Lambda_j(z)$ by simple manipulation of the above relation:

$$\text{Cov} (\Lambda^*(z), \Lambda_i(z) - \Lambda_j(z)) = 0. \quad (22)$$

Since:

$$\Lambda^* = \bar{\Lambda} + \left( \frac{D}{p} - \Pi \frac{D}{p} \right) \Sigma^{-1} \left( i_M - \Pi \frac{D}{p} \bar{\Lambda} \right),$$

it follows that:

$$\text{Cov} \left( \frac{D}{p}, \Lambda_i(z) - \Lambda_j(z) \right) = 0, \quad (23)$$

which is our orthogonality condition identifying $\frac{D}{p}$ along with $\Lambda^* i_M = [\sum_{z \in \mathcal{Z}} q(z)]$. Consequently, we can express the implied market incompleteness by equation $\text{(22)}$. 

51
A.5 Proof of Proposition 5:

Let $k_{CE}^i$ be the scale of capital an agent would choose in the competitive equilibrium without price impact. Further, let $k_{Aut}^i$ be the scale of capital an agent would choose in autarky, or if it could not trade state contingent claims with other agents.

There are two forces impacting the choice of capital with price impact, the first indirect and the second direct. First, since $\Lambda_i(z) > \Lambda_j(z)$ in the presence of strategic interaction, it follows that $j$ cannot sell as many claims to $i$ as it would, all else equal, in a competitive environment. By a symmetric argument, $j$ is not able to buy as claims from $i$ as it would in any state $z'$ in which it would like to buy, such that $\Lambda_j(z) > \Lambda_i(z)$, as compared to a competitive environment. Consequently, $j$ is more exposed to the output of its own production than, all else equal, in a competitive environment. Since its state price $\Lambda_j(z)$ is more exposed to its own output, $y_j(z) k_j$, because of diminished risk-sharing opportunities ($\Lambda_j(z)$ lower when $y_j(z)$ is higher, and vice versa), it follows from the FONC for capital that agent $j$ chooses a level of capital $k_j$ that approaches its level under autarky, $k_{Aut}^j$. Second, as a result of price impact, agent $j$ has incentive to restrict its supply of state contingent securities to earn oligopoly rents as a seller. This force also lowers the optimal choice of capital.

Consequently, given these two forces, if $k_{CE}^j > k_{Aut}^j$, then $k_j$ decreases because of price impact.

If instead $k_{CE}^j \leq k_{Aut}^j$, then agent $j$’s capital choice is more ambiguous. The first force leads $k_j$ to increase toward the autarky capital choice, while the second reduces $k_j$ because of strategic rationing. As a result, $k_j$ may be increasing (first force), decreasing (second force), or U-shaped (both forces) in $\mu$.

Given these cross-sectional results, we now characterize the aggregate implications. For agents for whom $k_{CE}^j > k_{Aut}^j$, the set $J$, their total capital investment, $\sum_{j \in J} k_j$, is decreasing in price impact. For agents for which $k_{CE}^j \leq k_{Aut}^j$, the set $1,...,N \setminus J$, their total capital investment, $\sum_{j \in 1,...,N \setminus J} k_j$, may be increasing in price impact. Since price impact acts as an effective tax on the joint production among agents, capital misallocation rises, which lowers total output in each state and, consequently, productivity. That total capital investment also falls follows from the asymmetry that more productive agents, $k_{CE}^j > k_{Aut}^j$, rely on risk-sharing for financing their capital investment, while less productive agents, $k_{CE}^j \leq k_{Aut}^j$, rely on risk-sharing to reduce their scale.
When capital investment is (weakly) lower, savings is (weakly) higher.

A.6 Proof of Proposition 6:

(i) Equation 13 states that \( q(z) = E^*[\Lambda_i(z)] - \frac{\partial q(z)}{\partial A(z)} \frac{A(z)}{N} \) or, equivalently,

\[
 f^{-1}(-A(z), z) + \frac{\partial q(z)}{\partial A(z)} \frac{A(z)}{N} = E^*[\Lambda_i(z)].
\]

Given any fixed investment policy, the distribution of income is fixed for all \( z \). By Proposition 3, price impact hampers risk-sharing for all \( z \). Hence \( E^*[\Lambda_i(z)] \) is increasing in \( \mu \) because marginal utility is convex. The left-hand side is strictly increasing in \( A(z) \). Hence we must have \( \frac{\partial A(z)}{\partial \mu} > 0 \). This implies that sellers restrict supply more than buyers restrict demand. Hence \( \frac{\partial q(z)}{\partial \mu} > 0 \). Comparative statics for \( r^m \) follow immediately because it is a summation over market prices. Moreover, \( r^m \geq R \) because storage can be traded without market impact.

(ii) For the second claim, we first recognize from the FOCs in the trading game that, if there is storage, then \( E \left[ u'_2 \left( \frac{c_2(z)}{\tilde{c}_1(z)} \right) \right] = \frac{1}{R} \), and, if there is no storage, then \( E \left[ u'_2 \left( \frac{c_2(z)}{\tilde{c}_1(z)} \right) \right] < \frac{1}{R} \). Consequently, if all agents invest only in storage, then the state-price implied risk-free rate, \( r^* \), is \( R \). To see that this is a lower bound, a necessary condition that agent \( i \) to invest in capital in its production technology is \( E[y_i(z)] > R \), with the strict inequality necessary to embed a risk premium for the agent. Consequently, as long as agent \( i \) holds storage, then \( E \left[ u'_2 \left( \frac{c_2(z)}{\tilde{c}_1(z)} \right) \right] = \frac{1}{R} \), and only once it exhausts all its resources in state contingent claims and capital, then \( E \left[ u'_2 \left( \frac{c_2(z)}{\tilde{c}_1(z)} \right) \right] < \frac{1}{R} \). Since this holds for all agents, it follows that \( r^* \geq R \).\[11\]

We next recognize that, in the first-best equilibrium without leverage constraints or market power, all agents consume fixed fractions of the aggregate resources each period, \( \tilde{c}_{1,agg} \) and \( \tilde{c}_{2,agg}(z) \), and their marginal utilities are aligned state-by-state

\[
 \left( E^* \left[ \frac{c_{FB}^i(z)}{\tilde{c}_{FB}^i(z)} \right] = \frac{\tilde{c}_{2,agg}(z)}{\tilde{c}_{1,agg}} \right) \forall i). \]

This is confirmed in Proposition 7.

As an intermediate step, we next compare the first-best economy to a pseudo-economy that has the same capital and storage choices as in the decentralized

\[11\] Though agents will not trade risk-free claims with each other because of market power, they may still trade with outside investors as a result of their difference in preferences.
economy, viewed now as agent endowments, but with perfect risk-sharing. In this pseudo-economy, all agents consume a fixed fraction of their aggregate output each period, \( c_{1,agg}^{*} \) and \( c_{z,agg}^{*}(z) \), but this endowment reflects the capital and savings decisions of the true noncompetitive economy.

From Proposition 5 and Corollary 5, agents that would invest in capital in the first-best underinvest in the noncompetitive economy because of strategic frictions, while those that do not invest in capital may start to invest because of diminished opportunities to invest. For the same resources transferred intertemporally, \( \sum_{i=0}^{N} e_i - \tilde{c}_{1,agg} \), the first-best employs more efficient technologies without cross-sectional misallocation. The first-best optimally increases the average level of consumption in all states, and shifts it from less probable to more likely states of the world, lowering marginal utility in those states and the average marginal utility, and consequently raising the risk-free rate. Since the decentralized production frontier is in the interior of the first-best production possibilities set, it is less efficient in shifting consumption across states of the world, leading to higher average marginal utility, and a lower riskless rate.

What remains for us to establish now is that the risk-free rate in the noncompetitive economy is (weakly) lower than in the pseudo-economy with perfect risk-sharing, since then the result follows by transitivity. We now recognize that:

\[
\sum_{z \in Z} E^* [\Lambda_i(z)] \geq E \left[ E^* \left[ u' \left( \frac{c_{i2}(z)}{c_{i1}} \right) \right] \right],
\]

where the inequality, similar to Alvarez and Jermann (2000), follows from the possibility of binding leverage constraints and the homotheticity of agent utility. As a result of market power, these inequalities can hold strictly even when no leverage constraints bind. Since \( \sum_{i=1}^{N} c_{i2}(z) < c_{z,agg}^{*}(z) \) and \( \sum_{i=1}^{N} c_{i1} = c_{1,agg}^{*} \),

\[
\frac{E^* [c_{i2}(z)]}{E^* [c_{i1}]} < \frac{c_{z,agg}^{*}(z)}{c_{1,agg}^{*}} \quad \forall \quad z \in Z,
\]

and therefore

\[
E \left[ u'_2 \left( \frac{E^* [c_{i2}(z)]}{E^* [c_{i1}]} \right) \right] > E \left[ u'_2 \left( \frac{c_{z,agg}^{*}(z)}{c_{1,agg}^{*}} \right) \right]
\]

since marginal utility is convex. It then follows that it is sufficient that:

\[
E \left[ E^* \left[ u' \left( \frac{c_{i2}(z)}{c_{i1}} \right) \right] \right] \geq E \left[ u'_2 \left( \frac{E^* [c_{i2}(z)]}{E^* [c_{i1}]} \right) \right],
\]

for the state-price implied riskless rate rate, \( r^* \), to be less than that in the first-best
economy. Notice now, since the function $1/x$ is convex, it follows that:

$$E \left[ E^* \left[ u'_2 \left( \frac{c_{i2}(z)}{c_{i1}} \right) \right] \right] \geq E \left[ u'_2 \left( E^* \left[ \frac{c_{i2}(z)}{c_{i1}} \right] \right) \right] \geq E \left[ u'_2 \left( E^* \left[ \frac{c_{i2}(z)}{E^* [c_{i1}]} \right] \right) \right] = E \left[ u'_2 \left( \frac{E^* [c_{i2}(z)]}{E^* [c_{i1}]} \right) \right]$$

and therefore the risk-free rate is lower than in the first-best economy.

Furthermore, notice that the expected gross return to capital for agent $i$ is $E [y_i (z) - 1]$, since capital is supplied elastically to all agents at unit cost. It then follows that, if the riskless rate is depressed, that the expected excess return to capital, $E [A (z) - r^*]$, increases in the economy. Consequently, the expected excess return on investment increases in the economy, raising Tobin’s $q$.

(iii) Follows from a summation over states of Equation (13). Notice that $R (z)$ is the expected excess payoff in state $z$. Summing across all states, it follows that:

$$\sum_{z \in Z} R (z) = \sum_{z \in Z} E^* \left[ \frac{\partial q (z)}{\partial a_i (i, z)} a_i (i, z) \right] \geq 0,$$

since $\sum_{z \in Z} \text{Cov} (\tilde{u}'_2 (z) + \tilde{L} \tilde{v} (z), \delta_i (z)) = 0$. Consequently, $\sum_{z \in Z} R (z)$ is nonzero despite being a riskless expected excess return derived from assembling all state contingent securities. It must therefore be the case that the market-implied riskless rate $\left[ \sum_{z \in Z} q (z) \right]^{-1}$ is biased based on the average market power, $\sum_{z \in Z} E^* \left[ \frac{\partial q (z)}{\partial a_i (i, z)} a_i (i, z) \right]$, in financial markets. Since there is anonymity in financial markets, in that price impact is symmetric across agents ($\frac{\partial q (z)}{\partial a_i (i, z)} = \frac{\partial q (z)}{\partial a_j (j, z)}$), the bias is determined by the probability-weighted average of the net demand of agents, $\sum_{z \in Z} \frac{\partial q (z)}{\partial A (z)} E^* \left[ a_i (i, z) \right] = \sum_{z \in Z} \frac{\partial q (z)}{\partial A (z)} A (z)$.

The bias is positive (negative) when agent net demand is positive (negative), which is the supply of the outside agent. [12]

(iv) We start by defining the expected excess payoff to state $z$ be $R^e (z) = \pi (z) - r^*_f q (z)$, where $\frac{1}{N} = E \left[ E^* [A_i (z)] \right] = E \left[ E^* \left[ \frac{u'_2 (c_{i2}(z)) + \xi \Psi_i (z)}{u'_2 (c_{i1})} \right] \right]$ and $E^* [:]$ is the cross-sectional average $\frac{1}{|N|} \sum_{i=1}^{|N|} [:]$. $r^*$ is the state-price implied risk-free rate constructed from the cross-sectional average of agent state prices. Then, it follows that we can express

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[12] If instead there were internal market-clearing among the large agents, then asymmetry in price impact would again give rise to a nontrivial bias.
this return from the FONCs for $a_i(z)$ as:

$$R^e(z) = \pi(z) - r_f^* \pi(z) E^* \left[ \frac{u'_2(c_2(z)) + \zeta \psi_i(z)}{u'_1(c_1(z))} \right] + r_f^* E^* \left[ \frac{\partial q(z)}{\partial a_i(i,z)} a_i(i,z) \right]$$

$$= -Cov \left( \frac{E^* \left[ \frac{u'_2(c_2(z)) + \zeta \psi_i(z)}{u'_1(c_1(z))} \right]}{E^* \left[ \frac{\partial q(z)}{\partial a_i(i,z)} \right]}, \delta(z) \right) + r_f^* E^* \left[ \frac{\partial q(z)}{\partial a_i(i,z)} a_i(i,z) \right],$$

since $E \left[ \frac{E^* [\Lambda_i(z)]}{\sum_{z \in Z} E^* [\Lambda_i(z)]} \right] E[\delta(z)] = \pi(z)$ and $E \left[ \frac{E^* [\Lambda_i(z)]}{\sum_{z \in Z} E^* [\Lambda_i(z)]} \delta(z) \right] = \frac{E^* [\Lambda_i(z)]}{\sum_{z \in Z} E^* [\Lambda_i(z)]} \pi(z)$.

We can further decompose this average excess payoff as:

$$R^e(z) = -Cov \left( \frac{E^* \left[ \frac{u'_2(c_2(z)) + \zeta \psi_i(z)}{u'_1(c_1(z))} \right]}{E^* \left[ \frac{\partial q(z)}{\partial a_i(i,z)} \right]}, \delta(z) \right) - Cov \left( \frac{E^* \left[ \zeta \psi_i(z) \right] - E^* \left[ \frac{u'_2(c_2(z))}{u'_1(c_1(z))} \right]}{E^* \left[ \frac{\partial q(z)}{\partial a_i(i,z)} \right]}, \delta(z) \right) + r_f^* E^* \left[ \frac{\partial q(z)}{\partial a_i(i,z)} \right].$$

Dividing by the price of the claim, $q(z)$, and substituting with the market-implied interest rate, we arrive at the expected excess market-implied return, $r(z)$:

$$r(z) = -Cov \left( \bar{u}'_2(z), \delta(z) \right) + \frac{\overline{mkt}(z)}{q(z)} - Cov \left( \bar{Lev}(z), \delta(z) \right),$$

where:

$$\overline{mkt}(z) = r_f^* E^* \left[ \frac{\partial q(z)}{\partial a_i(i,z)} a_i(z) \right] + r_f^* - \frac{\sum_{z \in Z} q(z)}{E^* [\Lambda_i(z) - \xi \psi_i(z)]}$$

$$= \left( E^* \left[ \frac{\partial q(z)}{\partial a_i(z)} a_i(z) \right] + 1 - \frac{E^* [\Lambda_i(z)]}{\sum_{z \in Z} q(z)} \right) r_f^*$$

$$= \left( E^* \left[ \frac{\partial q(z)}{\partial a_i(z)} a_i(z) \right] - \frac{\sum_{z \in Z} E^* \left[ \frac{\partial q(z)}{\partial a_i(z)} a_i(z) \right]}{\sum_{z \in Z} q(z)} \right) r_f^*$$

$$= \left( r_f^* \sum_{z \in Z} q(z) \frac{\eta}{N \eta} - \frac{q(z)}{N \eta(z)} \right) r_f^*$$

with $\bar{u}'(z) = \frac{u'_2(c_2(z))}{E^* \left[ \frac{\partial q(z)}{\partial a_i(i,z)} \right]}$, and $\bar{Lev}(z) = \frac{E^* [\zeta \psi_i(z)] - E^* \left[ \frac{u'_2(c_2(z))}{u'_1(c_1(z))} \right]}{E^* \left[ \frac{\partial q(z)}{\partial a_i(i,z)} \right]}$.  

56
A.7 Proof of Corollary 1

We start with market-implied expected excess return in state $z$, $r(z)$:

$$r(z) = \frac{1}{q(z)} \left( \pi(z) - \frac{q(z)}{\sum q(z)} \right),$$

and we recognize that:

$$\frac{\partial r(z)}{\partial q(z)} = \frac{1}{q(z)^2} \left( \left( \frac{q(z)}{\sum q(z)} \right)^2 + \frac{q(z)}{\sum q(z)} - \pi(z) \right),$$

$$\frac{\partial r(z)}{\partial q(z')} = \frac{1}{q(z')^2} \left( \left( \frac{q(z')}{\sum q(z)} \right)^2 \right).$$

It follows, to first-order, an increase in all A-D prices because of an increase in market power impacts the expected excess return according to:

$$\Delta r(z) \approx \frac{\partial r(z)}{\partial q(z)} \Delta q(z) + \sum_{z' \neq z} \frac{\partial r(z)}{\partial q(z')} \Delta q(z') + \sum_{z' \neq z} \frac{\partial r(z)}{\partial q(z')} \frac{\partial q(z')}{\partial \mu} \Delta q(z) / \Delta \sum q(z).$$

Notice that $\Delta (\sum q(z)) / q(z) \geq 0$ since $\Delta (\sum q(z)) > 0$ from Proposition 6.

It then follows, when we parameterize market power with $\mu$ that;

$$\frac{\partial r(z)}{\partial \mu} \propto \left( \frac{q(z)}{\sum q(z)} \right)^2 + \left( \frac{q(z)}{\sum q(z)} - \pi(z) \right) \frac{\partial q(z)}{\partial \mu} / \partial \sum q(z).$$

With impaired risk-sharing, an increase in $\mu$ raises the price of A-D securities referencing high marginal utility states, as insurance becomes more scarce. With symmetric preferences, from Proposition 6 it also raises the price of A-D securities referencing low marginal utility states, since sellers' supply curves for all states are more inelastic.

Fix an investment policy. Consider the highest marginal utility state with the fewest sellers, so that market power is sufficiently concentrated. For this state, it must
be the case that \( \frac{q(z)}{\sum z \cdot q(z)} > \pi(z) \), since this state trades at a premium because of risk aversion, and market power inflates its price even moreso. Consequently, we expect the high marginal utility aggregate state risk premia to go up.\(^{13}\)

Finally, we recognize that the sum of all expected excess returns across states is zero, since then the portfolio of A-D securities is riskfree:

\[
\sum z \cdot r(z) = 0.
\]

As such, it must be the case that:

\[
\sum z \cdot \Delta r(z) = 0.
\]

Since \( r(z) \) increases for the highest marginal utility state, it must be the case that it falls for at least one other state. Since all other states have lower marginal utility than the highest marginal utility state, the claim follows.

### A.8 Proof of Proposition 7

We first characterize the first-best allocation. Since all agent links allow for both the sale and purchase of state contingent claims, resources can potentially flow to any agent.

Without constraints on allocations, the Social Planner solves the problem:

\[
\sup_{\{c_1(z), k\}} \sum_{i=1}^{N} u_i' \left(1 / \eta_i \right) u_1 (c_{i1}) + \sum_{z \in Z} \pi(z) u_2' \left(1 / \eta_i \right) u_2 (c_{i2}(z)) + \sum_{z \in Z} \pi(z) u_2' \left(1 / \eta_0 \right) F(c_{02}(z)),
\]

s.t. : \[
\sum_{i=0}^{N} (c_{i2}(z) - y_i(z) k_i - R s_i) \leq 0 \forall z \in Z,
\]

\[
\sum_{i=1}^{N} k_i + c_{i1} + s_i - e_i - e_0 \leq 0,
\]

where \( u_i' \left(1 / \eta_i \right) \) are the Pareto weights and \( s_0 = k_0 = 0 \). In the above, we have implicitly given equal Pareto weights to the cross-section of outside investors by issuer with a state \( z \).

\(^{13}\)Although we fix an investment policy, since market power shifts investment toward the risk-free and lower productivity technologies, it follows that production amplifies this trend of risk premia compression as long as marginal utility states are not reordered as investment shifts to less efficient technologies.
With Lagrange multipliers on the feasibility conditions \( \lambda \) and \( \mu(z) \pi(z) \), it follows that the FOC for \( c_{i1}, c_{i2}(z) \), and \( k_i \) are:

\[
\begin{align*}
c_{i1} & : \quad u_2'(1/\eta_i) u_1'(c_{i1}) - \lambda \leq 0, \quad (= \text{if } c_{i1} > 0), \\
c_{i2}(z) & : \quad u_2'(1/\eta_i) u_2'(c_{i1}(z)) - \mu(z) \leq 0, \quad (= \text{if } c_{i2}(z) > 0), \\
c_{02}(z) & : \quad u_2'(1/\eta_0) F'(c_{02}(z)) - \mu(z) \leq 0, \quad (= \text{if } c_{02}(z) > 0) \\
k_i & : \quad \sum_{z \in Z} \pi(z) \mu(z) y_i(z) - \lambda \leq 0 \quad (= \text{if } k_i > 0), \\
s_i & : \quad R \sum_{z \in Z} \pi(z) \mu(z) - \lambda \leq 0 \quad (= \text{if } s_i > 0),
\end{align*}
\]

Since the utility function satisfies the Inada condition, in equilibrium \( c_{i1}, c_{i2}(z) > 0 \).

Let \( \hat{c}_{i1} = \hat{c}_{i1} \sum_{i=0}^{N} e_i, c_{i2}(z) = \hat{c}_{i2}(z) \sum_{i=0}^{N} e_i \) and \( k_i = \hat{k}_i \sum_{i=0}^{N} e_i \). It is straightforward to see from the FOC for consumption, the homotheticity of the utility functions, and the feasibility constraint that:

\[
\begin{align*}
c_{i1} & = \frac{\eta_i}{\sum_{i=1}^{N} \eta_i} \sum_{i=1}^{N} c_i = \frac{\eta_i}{\sum_{i=1}^{N} \eta_i} \hat{c}_{1,agg}, \\
c_{02}(z) & = F^{-1'} \cdot u_2'(\frac{\eta_i}{\sum_{i=0}^{N} \eta_i} \hat{c}_{2,agg}(z)), \\
c_{i2}(z) & = \frac{\eta_i}{\sum_{i=0}^{N} \eta_i} \hat{c}_{2,agg}(z), \quad i \in \{1, ..., N\},
\end{align*}
\]

where:

\[
\hat{c}_{2,agg}(z) = u_2'(F(c_{02}(z))) - c_{02}(z) + \sum_{i=1}^{N} R (1 - c_{i1}) + (y_i(z) - R) k_i,
\]

is the aggregate effective resources in each state \( z \). The first FOC implies that \( \lambda = u_1'(\hat{c}_{1,agg} / \sum_{i=1}^{N} \eta_i) \), while the second that \( \mu(z) = u_2'(\hat{c}_{2,agg}(z) / \sum_{i=0}^{N} \eta_i) \), from which follows that the fourth and fifth FOCs reduce to:

\[
\begin{align*}
\sum_{z \in Z} \pi(z) u_2' \left( \frac{\sum_{i=1}^{N} \eta_i}{\sum_{i=0}^{N} \eta_i} \right) \frac{u_2'(\hat{c}_{2,agg}(z))}{u_1'(\hat{c}_{1,agg})} y_i(z) & \leq 1 \quad (= \text{if } k_i > 0), \\
R \sum_{z \in Z} \pi(z) u_2' \left( \frac{\sum_{i=1}^{N} \eta_i}{\sum_{i=0}^{N} \eta_i} \right) \frac{u_2'(\hat{c}_{2,agg}(z))}{u_1'(\hat{c}_{1,agg})} & \leq 1 \quad (= \text{if } s > 0),
\end{align*}
\]
where we drop the subscript for storage since only the aggregate level of storage is determinate.

Suppose that production technology \( y_i(z) \) is employed. Since the technology is employed, it must be the case that \( \sum_{z \in Z} \pi(z) u'_2(c_{j2}(z)) y_i(z) \geq R \sum_{z \in Z} \pi(z) u'_2(c_{j2}(z)) \), and consequently:

\[
\pi(z) u'_2(c_{j2}(z)) y_i(z) \geq R \sum_{z \in Z} \pi(z) u'_2(c_{j2}(z)) ,
\]

and therefore consumption is a submartingale.

Notice that, if \( k_i > 0 \), then it follows, since \( \sum_{z \in Z} \pi(z) u'_2 \left( \frac{\sum_{i=1}^{N} \eta_i}{\sum_{i=0}^{N} \eta_i} \right) \frac{u'_2(\tilde{c}_{2,agg}(z))}{u'_1(\tilde{c}_{1,agg})} \left( y_i(z) - y_j(z) \right) < 0 \), that:

\[
\sum_{z \in Z} \pi(z) y_i(z) > R,
\]

since all production technologies \( y_i(z) \) are (weakly) positively correlated, and therefore \( \text{Cov}(u'_2(\tilde{c}_{2,agg}(z)), y_i(z)) < 0 \).

Notice that, if \( \bar{y}_i = \bar{y}_j \), with at least one inequality strict, then, if \( k_i > 0 \), it must be the case that \( k_j = 0 \), since:

\[
\sum_{z \in Z} \pi(z) u'_2 \left( \frac{\sum_{i=1}^{N} \eta_i}{\sum_{i=0}^{N} \eta_i} \right) \frac{u'_2(\tilde{c}_{2,agg}(z))}{u'_1(\tilde{c}_{1,agg})} \left( y_i(z) - y_j(z) \right) < 0 ,
\]

given that \( \frac{u'_2(\tilde{c}_{2,agg}(z))}{u'_1(\tilde{c}_{1,agg})} \geq 0 \) and \( \bar{y}_i \geq \bar{y}_j \), and therefore:

\[
1 = \sum_{z \in Z} \pi(z) u'_2 \left( \frac{\sum_{i=1}^{N} \eta_i}{\sum_{i=0}^{N} \eta_i} \right) \frac{u'_2(\tilde{c}_{2,agg}(z))}{u'_1(\tilde{c}_{1,agg})} y_i(z) > \sum_{z \in Z} \pi(z) u'_2 \left( \frac{\sum_{i=1}^{N} \eta_i}{\sum_{i=0}^{N} \eta_i} \right) \frac{u'_2(\tilde{c}_{2,agg}(z))}{u'_1(\tilde{c}_{1,agg})} y_j(z) .
\]

We now demonstrate that this first-best allocation is infeasible in the decentralized economy because of market power. Rewriting FOCs with complementary slackness, summing over the states \( z \in Z \) for the first two FONCs, and substituting them into the budget constraint reveals that:

\[
\sum_{z \in Z} \pi(z) \frac{u'_2(c_{j2}(z))}{u'_1(c_{i1})} c_{i2}(z) = e_i - c_{i1} + e_i \sum_{z \in Z} \left( - \frac{\partial q(z)}{\partial a_i(i,z)} a_i(i,z) + \frac{\partial \bar{q}(z)}{\partial \bar{a}_{-i}(z)} \bar{a}_{-i}(z) \right) \geq e_i - c_{i1} ,
\]

where the \( \psi_i(z) \) terms cancel because of the complementary slackness condition on the
I.C. constraint. The above condition implies that value of the wealth portfolio exceeds initial invested wealth \( e_i - c_{i1} \) because of market power, implying the agent earns a higher return because it internalizes its price impact.

If there is perfect-risk sharing, then the marginal utilities of all agents are aligned state-by-state, and there is a unique pricing kernel. Adding the equilibrium condition across agents implies that:

\[
\sum_{i=1}^{N} c_{i1} + \sum_{z \in Z} \Lambda(z) \sum_{i=1}^{N} c_{i2}(z) = \sum_{i=1}^{N} e_i + \sum_{i=1}^{N} e_i \sum_{z \in Z} \left( -\frac{\partial q(z)}{\partial a_i(i,z)} a_i(i,z) + \frac{\partial \hat{q}(z)}{\partial \hat{a}_{-i}(z)} \hat{a}_{-i}(z) \right),
\]

where \( \Lambda(z) \) is the unique state price deflator. In contrast, for the outside investors:

\[
\pi(z) F'(\frac{1}{|Z|}c_{02}(z) + (1 - rq(z)(z)) D(i,z), z) \sum_{z \in Z} \pi(z) F'\left(\frac{1}{|Z|}c_{02}(z) + (1 - rq(z)(z)) D(i,z), z\right) = rq(z)(z), \forall (i,z)
\]

which implies, by complete markets, that:

\[
\sum_{z \in Z} \Lambda(z) c_{02}(z) = \frac{1}{R} e_0,
\]

where \( R = \frac{1}{\sum_{z \in Z} \Lambda(z)} \) is the effective risk-free rate. Consequently, outside investors’ portfolios are correctly priced.

It then follows, since \(-\frac{\partial q(z)}{\partial a_i(i,z)} a_i(i,z) + \frac{\partial \hat{q}(z)}{\partial \hat{a}_{-i}(z)} \hat{a}_{-i}(z)\) is nonnegative for all \( i \) and \( z \), that:

\[
\sum_{i=1}^{N} c_{i1} + \sum_{z \in Z} \pi(z) c_{02}(z) + \sum_{z \in Z} \Lambda(z) \sum_{i=1}^{N} c_{i2}(z) > \frac{1}{R} e_0 + \sum_{i=1}^{N} e_i,
\]

and that the present discounted value of aggregate consumption exceeds the total value of all equity in the economy at \( t = 1 \). This is a contradiction since it implies the aggregate wealth portfolio is mispriced, or that there is an aggregate economic profit from trading.

For the final part of the claim, we consider the perspective of a social planner who can design a mechanism to implement any allocation that respects the FOCs of all agents and the market structure that features imperfect competition. Expanding the differential
of the indirect utility function for agent \( i \), \( U^i_1 \), we see that:

\[
\Delta U_i = u^i_1(c_{i1}) \Delta c_{i1} + E \left[ u^i_2(c_{i2}) \Delta c_{i2} \right] + \Delta \left( \sum_{z \in Z} \pi(z) \psi_i(z) \left( a_i(i,z) + \xi y_i(z) k_i + \xi b_i(z) \iota + \xi R_{si} \right) \right) \\
+ \Delta \left( \phi^k_i k_i \right) + \Delta \left( \phi^s_i s_i \right) - \Delta \left( \phi^a_i a_i(i,z) \right) + \sum_{z \in Z} \Delta \left( \phi^{a_s}_{-i} a_{-i}(z) \right),
\]

where \( \phi^k_i \) is the nonnegativity Lagrange multiplier on \( k_i \), and similarly for the other \( \phi^j_i \)'s.

Substituting with the FOCs in Proposition 1, the above reduces to:

\[
\Delta U_i = u^i_1(c_{i1}) \sum_{z \in Z} \left( -\Delta q(z) + \frac{\partial q(z)}{\partial a_i(i,z)} \Delta a_i(i,z) \right) a_i(i,z) \\
+ u^i_1(c_{i1}) \sum_{z \in Z} \left( \sum_{j \neq i} a_i(j,z) \left( -\Delta q(z) + \frac{\partial q(z)}{\partial a_i(j,z)} \Delta a_i(j,z) \right) \right) \\
+ \sum_{z \in Z} \pi(z) \Delta \psi_i(z) (a_i(i,z) + \xi y_i(z) k_i + \xi a_{-i} \iota + \xi R_{si}) \\
+ \Delta \phi^k_i k_i + \Delta \phi^s_i s_i - \Delta \phi^a_i a_i(i,z) + \sum_{z \in Z} \Delta \phi^{a_s}_{-i} a_{-i}(z)
\]

Finally, invoking complementary slackness and the definition of \( q(z) \), we arrive at:

\[
\Delta U_i = -u^i_1(c_{i1}) \sum_{z \in Z} \Delta \Lambda_i(z) (a_i(i,z) + \bar{a}_{-i}(z) \iota) \\
- u^i_1(c_{i1}) \sum_{z \in Z} \left( \sum_{j \neq i} \Delta \left( \frac{\partial q(z)}{\partial a_i(j,z)} \right) a_i(j,z) + \Delta \left( \frac{\partial q(z)}{\partial a_i(i,z)} \right) a_i(i,z) \right).
\]

When markets are competitive, the second term vanishes, and \( \Lambda_i(z) = \Lambda(z) \), since state contingent claim prices align state prices across agents. It then follows, since \( a_i(i,z) + \bar{a}_{-i}(z) \iota > 0 \) for some agent \( i \) and \( a_j(j,z) + \bar{a}_{-j}(z) \iota < 0 \) for some other agent \( j \), generically, and \( \Delta \Lambda_i(z) = \Delta \Lambda(z) \) is symmetric across agents, it follows that generically the trading game is constrained Pareto efficient.\(^{14}\)

Autarky is an equilibrium in which \( q(z) = \max_i \Lambda_i(z) \) to induce all agents to want to sell claims, which leads to excess supply in all markets. As a result, state prices of agents are not aligned state-by-state. If another competitive equilibrium exists, the Planner can improve upon the allocation by lowering asset prices sufficiently to induce

\(^{14}\)This is a stronger statement than the Planner cannot improve utilitarian welfare, which also follows since summing across agents reveals that \( \sum_i \Delta U_i = 0 \), when the outside agents are included in the calculation.
trade. This would lower the marginal utility of buyers ($\Delta \Lambda_i (z) a_i (j, z) < 0$), and raise the marginal utility of sellers ($\Delta \Lambda_i (z) a_i (i, z) < 0$) as state prices are aligned, resulting in $\Delta U_i > 0$ for now trading agents.

With imperfect competition, we now recognize that the second term reveals that a Planner would like to raise (lower) prices in securities for which agent $i$ borrows (lends). Since the pricing functions are symmetric across agents, $\frac{\partial q(z)}{\partial a_i (j, z)} = \frac{\partial q(z)}{\partial a_j (j, z)}$, it follows that manipulating prices is zero-sum, and would necessarily have to hurt the seller’s (buyer’s) utility at the expense of the buyer’s (seller’s). Consequently, the second term is compatible with constrained Pareto efficiency.

We focus now on the first term. As a result of market power, it is no longer true that $\Lambda_i (z) = \Lambda_j (z)$ across agents, since they intentionally under diversify risk to earn monopoly rents. Since $u_2' (c_{11}) > 0$, it follows that policies by a Planner that raise $\Lambda_i (z), \Delta \Lambda_i (z) > 0$, when $i$ is a net borrower, $a_i (i, z) + \bar{a}_{-i} (z) \ell < 0$, raises $i$'s utility by raising the price at which $i$ sells its securities, and similarly with $\Delta \Lambda_i (z) < 0$ when $i$ is a net lender, $a_i (i, z) + \bar{a}_{-i} (z) \ell > 0$. Since state prices are not aligned across agents, there can exist policies for which it is possible to improve $i$'s utility without harming that of other agents. Generically, then, the equilibrium in the trading game with imperfect competition is constrained Pareto inefficient.

### A.9 Proof of Corollary 4:

The claim follows from Proposition 7. Since the decentralized economies with limited commitment and/or market power cannot achieve the first-best allocation, the distribution of productivities across agents matters for investment and, consequently, asset prices. As such, any redistribution of productivities across agents state-by-state has a material impact on the production possibilities frontier, and therefore equilibrium outcomes, with the constrained efficient frontier being achieved with only limited commitment.

In contrast, with commitment and perfectly competitive agents, the competitive equilibrium attains the first-best allocation. It then follows from Proposition 7 that all equilibrium outcomes depend on only the most efficient production technologies.
A.10 Proof of Proposition 8:

Suppose \( i \) indirectly intermediates funds to agent \( j \). Comparing the expressions for the price of a state contingent claim referencing state \( z \) issued by agent \( j \) from Proposition 8, we see that if agent \( i \) both issues claims \( a_i (i, z) \) and buys claims \( a_i (j, z) \) from agent \( j \), then it must be the case that:

\[
q(i, z) - q(j, z) = -h_i \frac{\partial q(i, z)}{\partial a_i(i, z)} a_i(i, z) + h_i \frac{\partial q(j, z)}{\partial a_i(j, z)} a_i(j, z) > 0.
\]

Agent \( i \) earns an intermediation spread \(-h_i \frac{\partial q(i, z)}{\partial a_i(i, z)} a_i(i, z) + h_i \frac{\partial q(j, z)}{\partial a_i(j, z)} a_i(j, z)\) that acts as a wedge between the price at which it sells claims and that at which it buys claims to the same state, and this is true regardless of whether agent \( i \)'s constraint on \( a_i (i, z) \) binds. Even though the claims \( a_i (i, z) \) and \( a_i (j, z) \) net, it is their sum and not their difference that determines market power since agent \( i \) is trading in segmented markets.

Notice now that the intermediation spread is increasing in both how many claims agent \( i \) issues, \( a_i (i, z) \), and how much it buys, \( a_i (j, z) \). Furthermore, buying more claims \( a_i (j, z) \) has two effects: 1) since these obligations net at \( t = 2 \), such that agent \( i \) receives \( a_i (j, z) + a_i (i, z) \) in net transfers in state \( z \); and 2) buying more claims \( a_i (j, z) \) relaxes the I.C. constraint of agent \( i \), allowing it to issue \( \xi a_i (j, z) \) more claims to its creditors.

We can further examine the profit to exploiting this trading opportunity. Notice that, by buying \( a_i (j, z) \) and issuing \( a_i (i, z) \), the agent owes a net cash flow of \(-a_i (i, z) - a_i (j, z)\) at \( t = 2 \), and receives \(-q(i, z)a_i(i, z) - q(j, z)a_i(j, z)\) at \( t = 1 \). Consequently, the excess profit to agent \( i \), \( \Pi_{ij} (z) \), per dollar of equity is:

\[
\Pi_{ij} (z) = -r_i q(i, z) a_i (i, z) - r_i q(j, z) a_i (j, z) + a_i (j, z) + a_i (i, z)
\]

\[
= -r_i \left( -h_i \frac{\partial q(i, z)}{\partial a_i (i, z)} a_i (i, z) + h_i \frac{\partial q(j, z)}{\partial a_i (j, z)} a_i (j, z) \right) a_i (i, z) + r_i q(j, z) - 1 \right) \right) (a_i (i, z) - a_i (j, z))
\]

\[
= r_i h_i \left( \frac{\partial q(i, z)}{\partial a_i (i, z)} a_i (i, z)^2 + \frac{\partial q(j, z)}{\partial a_i (j, z)} a_i (j, z)^2 \right)
\]

\[
+ \left( 1 - \pi (z) \frac{u_2' (c_2 (z)) + \xi \psi_i (z)}{E [u_2' (c_2 (z)) + \xi \psi_i (z)]} \right) \right) \right) (a_i (j, z) + a_i (i, z)),
\]

where \( r_i = \frac{1}{\sum_{z \in Z} N_i (z)} \) is the effective risk-free rate for agent \( i \), and we have substituted for \( q(j, z) \) with the FONC from Proposition 8. Adding across agents \( j \), and recognizing
that \( \frac{\pi(z)(u'_2(c_2(z)) + \tilde{\zeta} \psi_i(z))}{E[u'_2(c_2(z)) + \tilde{\zeta} \psi_i(z)\]} = \frac{\Lambda_i(z)}{\sum_{z \in Z} \Lambda_i(z)} \), we arrive at the expression in the statement of the proposition.

Since the second term in parentheses is nonnegative, the second term is positive whenever \( \sum_{i=1}^{N} a_i (j, z) \geq 0 \). Agent \( i \) therefore earns a positive profit from being a net lender in state \( z \) and, since \( \frac{x}{a+z} \) is increasing in \( x \), this profit is smaller if the debt constraint binds in state \( z \).

The first term is a profit (strict arbitrage) from agent \( i \)'s market power. This term is increasing in lending activity when \( \frac{\partial q(j,z)}{\partial a_i(j,z)} \) is an increasing function, which is satisfied when \( f_i(\cdot, \cdot) \) is a convex function of its first argument. To see this, we recognize that, letting \( x_j = -\sum_{i=1}^{N} a_i (j, z) \):

\[
\frac{\partial^2 q(j,z)}{\partial a_i (j,z)^2} = \frac{\partial}{\partial a_i (j,z)} \left( \frac{\partial f_j(x_j,z)^{-1}}{\partial x_j} \frac{\partial x_j}{\partial a_i (j,z)} \right) = -\frac{\partial x_j}{\partial a_i (j,z)} \frac{\partial f_j(x_j,z)}{\partial x_j} \frac{\partial^2 f_j(x_j,z)}{\partial x_j^2}
\]

from which follows that \( \frac{\partial^2 q(j,z)}{\partial a_i (j,z)^2} \geq 0 \) whenever \( \frac{\partial^2 f_i(x_j,z)}{\partial x_j^2} \geq 0 \) \( \forall (j, z) \), and similarly:

\[
\frac{\partial^2 q(j,z)}{\partial a_i (j,z)^2} = -\frac{\partial x_i}{\partial a_i (j,z)} \frac{\partial f_i(x_i,z)}{\partial x_i} \frac{\partial^2 f_i(x_i,z)}{\partial x_i^2} \geq 0,
\]

whenever \( \frac{\partial^2 f_i(x_i,z)}{\partial x_i^2} \geq 0 \) \( \forall (i, z) \)

\[\text{Since } q(i,z) = f_i^{-1}(\cdot, z), \text{ and similarly with } q(j,z), \text{ this requirement is equivalent to requiring that } q(i,z) \text{ and } q(j,z) \text{ are convex in net demand.}
\]

### A.11 Proof of Corollary

Suppose all agents are competitive. Then, \( q(i,z) = q(z) \) \( \forall i \) because agents are price takers, and the profit to agent \( i \) from intermediating funds to \( j \) in state \( z \) is

\[
\Pi_i(z) = \left( 1 - \pi(z) \frac{u'_2(c_2(z)) + \tilde{\zeta} \psi_i(z)}{E[u'_2(c_2(z)) + \tilde{\zeta} \psi_i(z)]} \right) (a_i(j,z) + a_i(i,z)) = \left( 1 - \pi(z) \frac{u'_2(c_2(z)) + \tilde{\zeta} \psi_i(z)}{E[u'_2(c_2(z)) + \tilde{\zeta} \psi_i(z)]} \right) \tilde{a}_i(j,z),
\]

\[\text{Essentially, we require } f_i^{-1}(x,z) \text{ to be concave in its first argument } \forall (i,z). \text{ This is the case if } f_i(x,z) \text{ is convex, since } f_i^{-1}(x,z) \text{ is the inverse of } f_i(x,z).\]
where $a_i(j, z)$ is the net debt agent $i$ buys. It then follows that the extent of indirect intermediation is irrelevant. Consequently, agent $i$ would earn the same profit from lending to $j$ without borrowing funds $a_i(i, z)$, and incurring a cost for forming the additional link with its lender. As such, no agent would indirectly intermediate funds in the network if forming links is costly.

Furthermore, since the identity of $i$’s counterparties no longer impacts its optimization in the trading game, as state-contingent claims prices are equated within the network, there are no network effects beyond price externalities. There are still network linkages since subsets of agents may not be able to trade with each other.

Finally, consider the FONC for capital for agent $i$:

$$
\sum_{z \in Z} \pi(z) \left( \frac{u'_2(c_{i2}(z)) + \xi e^{-\gamma \psi_i(z)}}{u'_1(c_{ii})} \right) y_i(z) \leq 1 \quad (= \text{if } k_i > 0).
$$

If capital were not collateralizable ($\pi(z)$ of the state price omitted), then when the borrowing constraint binds, agent $i$ is limited in its ability to sell claims to states in which it has a lot of resources, while it is always limited in its ability to buy claims to states in which it has few resources. As a result, agent $i$ is more exposed to its own production, reducing its optimal choice of capital. Since capital is collateralizable, however, having capital is also valuable because it relaxes the borrowing constraint across all states. Given that agent $i$ must be constrained for this offsetting value to capital to be in effect, it follows agent $i$ must still face rationed risk-sharing at its optimal choice of capital.

Consequently, agent $i$ that produces in the competitive equilibrium chooses a lower level of capital than in the absence of the borrowing constraint. Similar to the case of market power, we also expect all agents that do not produce in a competitive environment to choose a (weakly) higher level of capital as a result of limited commitment.
B Derivation of Outside Investor Demand

Outside investors purchasing claims on state $z$ have initial resources $e_0$, can freely borrow at interest rate $r$, and collectively choose demand $D(z)$ to maximize utility $F(\cdot)$ over final consumption:

$$w_0 = \sup_{D(z)} E \left[ F \left( r \frac{e_0}{|NZ|} + \left( 1_{\{z'=z\}} - rq(z) \right) D(z), z \right) \right],$$

The optimization program gives rise to the FONC:

$$\frac{\pi(z)}{1 - \pi(z)} \frac{F'(r \frac{e_0}{|NZ|} + (1 - rq(z)) D(z), z)}{F'(r \frac{e_0}{|NZ|} - rq(z) D(z), z)} \frac{1 - rq(z)}{rq(z)} = 1,$$

Mean-Variance: In the case that outside investors have MV utility with coefficient of risk aversion $\lambda / \sum_{j=1}^N e_j$, it follows from their optimization program that:

$$D(i,z) = \frac{\pi(z) - rq(i,z)(z)}{\lambda \pi(z) (1 - \pi(z))} \sum_{j=1}^N e_j,$$

and $f(q(i,z),z) = \frac{\pi(z) - rq(i,z)(z)}{\lambda \pi(z) (1 - \pi(z))}$. This gives rise to the linear pricing function:

$$q(i,z) = \frac{1}{r} \pi(z) - \frac{\lambda}{r} \pi(z) (1 - \pi(z)) \frac{-\sum_{j=1}^N a_j(i,z)}{\sum_{j=1}^N e_j},$$

where $\hat{\lambda} = \frac{\lambda}{r} \frac{\pi(z)(1 - \pi(z))}{\sum_{j=1}^N e_j}$ has the interpretation of Kyle (1985)’s Lambda for purchasing agents, which is constant in this setting.

CARA Utility: In the case that outside investors have CARA utility with risk aversion $\beta / \sum_{j=1}^N e_j$, which scales inversely with total agent equity, then their demand is given by:

$$D(i,z) = \left( \frac{1}{\beta} \log \frac{\pi(z)}{1 - \pi(z)} + \frac{1}{\beta} \log \left( \frac{1}{rq(i,z)} - 1 \right) \right) \sum_{j=1}^N e_j,$$

and $f(q(i,z),z) = \frac{1}{\beta} \log \frac{\pi(z)}{1 - \pi(z)} + \frac{1}{\beta} \log \left( \frac{1}{rq(i,z)} - 1 \right)$. This gives rise to the pricing func-
where we have selected the appropriate root of the quadratic form by imposing that

\[ q(i, z) = \frac{1}{r} \left( 1 + \frac{1 - \pi(z)}{\pi(z)} \exp \left( \beta \frac{\sum_{j=1}^{N} a_j(i, z)}{\sum_{j=1}^{N} e_j} \right) \right), \]

which is quasi-concave in net supply \(-\sum_{j=1}^{N} a_j(i, z)\) (convex for \(\sum_{j=1}^{N} a_j(i, z) < 0\) and concave for \(\sum_{j=1}^{N} a_j(i, z) \in (0, 1/r)\)).

**CRRA Utility:** In the case that outside investors instead have CRRA utility with coefficient of relative risk aversion \(\gamma_0\), and assuming that \(e_0 = \frac{1}{N|Z|} \sum_{j=1}^{N} e_j\), it follows their demand is given by:

\[ D(i, z) = \frac{1}{rq(i, z) + \frac{r}{\left( \frac{\pi(z) - rq(i, z)}{1 - \pi(z)} \right) \frac{1}{\gamma_0} - 1}} \sum_{j=1}^{N} e_j, \]

and \(f(q(i, z), z) = \left( rq(i, z) + \frac{1}{\left( \frac{\pi(z) - rq(i, z)}{1 - \pi(z)} \right) \frac{1}{\gamma_0} - 1} \right)^{-1} r\). In the special case of log utility, then:

\[ D(i, z) = \frac{r}{1 - rq(i, z)} \left( \frac{\pi(z)}{rq(i, z)} - 1 \right) \frac{1}{N|Z|} \sum_{j=1}^{N} e_j, \]

which gives rise to the pricing function:

\[ q(i, z) = \begin{cases} \frac{1}{2r} \left( 1 + \frac{r}{N|Z|} \sum_{j=1}^{N} e_j \right) + \frac{1}{2r} \sqrt{\left( 1 + \frac{r}{N|Z|} \sum_{j=1}^{N} e_j \right) \left( 1 + \frac{r}{N|Z|} \sum_{j=1}^{N} e_j \right) - \frac{4\pi(z)}{N|Z|} \sum_{j=1}^{N} e_j - \frac{4\pi(z)}{N|Z|} \sum_{j=1}^{N} e_j}, & \sum_{j=1}^{N} a_j(i, z) \geq 0 \\ \frac{1}{2r} \left( 1 + \frac{r}{N|Z|} \sum_{j=1}^{N} e_j \right) - \frac{1}{2r} \sqrt{\left( 1 + \frac{r}{N|Z|} \sum_{j=1}^{N} e_j \right) \left( 1 + \frac{r}{N|Z|} \sum_{j=1}^{N} e_j \right) - \frac{4\pi(z)}{N|Z|} \sum_{j=1}^{N} e_j - \frac{4\pi(z)}{N|Z|} \sum_{j=1}^{N} e_j}, & \sum_{j=1}^{N} a_j(i, z) < 0 \end{cases}, \]

where we have selected the appropriate root of the quadratic form by imposing that \(\pi(z) \to 0 \implies q(i, z) \to 0^+\) and \(\pi(z) \to 1 \implies q(i, z) \to \frac{1}{r}\).