How Large are Pre-Default Costs of Financial Distress?
Estimates from a Dynamic Model

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Abstract

We estimate the costs of financial distress prior to default (pre-default costs) separately from the loss incurred at default (loss given default) using a dynamic trade-off model of capital structure. We find that accounting for pre-default costs significantly improves the ability of a trade-off model to match the levels of leverage, default rates and loss incurred at default observed in the data. We document that pre-default costs are large, and on average equal to 6.5% of firm value per year during times of financial distress. We also show that omitting pre-default costs in a trade-off model leads to a large bias in the estimates of the loss given default. Last, our study documents that the expected pre-default costs of financial distress are higher for firms that produce more durable products.

Keywords: Dynamic Capital Structure, Financial Distress, Structural Estimation

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1 Introduction

According to the trade-off theory of capital structure, firms choose their leverage ratio by balancing tax benefits and costs of financial distress. Despite its wide-spread use, the empirical validity of the trade-off theory has often been questioned: when models are calibrated to the empirical estimates of tax benefits (Graham, 2000) and bankruptcy costs incurred at default (Andrade and Kaplan, 1998; Davydenko, Strebulaev, and Zhao, 2012), the predicted leverage ratios are much higher than the empirically observed ones. A key assumption that is common to many of these dynamic trade-off models is that financial distress costs are incurred only at the time of default. However, firms may experience financial distress costs prior to default (Titman, 1984).

The purpose of this paper is to (1) quantify the costs of financial distress that firms expect to experience prior to default (pre-default costs) in addition to the loss given default (i.e. the expected loss at the time of default), (2) analyze the bias in the estimates of costs of financial distress if pre-default costs are omitted from a dynamic trade-off model, and (3) analyze how such costs vary amongst different industries and firm’s characteristics. To provide an answer to the above questions, we develop and estimate a dynamic trade-off model in the style of Goldstein, Ju, and Leland (2001) which features corporate and personal taxes that give rise to the tax advantage of debt. In our model, firms may experience financial distress prior to default and they are also subject to refinancing and liquidation costs. To the best of our knowledge, this is the first study that attempts to separately estimate pre-default costs of financial distress from loss given default, and to show the different implications that such pre-default costs have for leverage and probability.

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1 Other theories of capital structure include pecking order, signaling and market timing. We do not consider them because this paper focuses on costs of financial distress which are suited to be studied in a trade-off framework. We refer to Parsons and Titman (2009) and Graham and Leary (2011) for reviews on capital structure research.

2 For example, there is anecdotal evidence that customers might not buy the products of a highly levered firm because they fear that the firm will not be able to honour warranty, or managers might not focus on the core business because they have to deal with creditors when the firm is in distress. We refer to Hotchkiss et al. (2008) and Senbet and Wang (2012) for excellent reviews of the indirect costs of financial distress. Two papers that address pre-default costs in a theoretical model are Elkamhi, Ericsson, and Parsons (2012) and Titman and Tsyplakov (2007). In Elkamhi, Ericsson, and Parsons (2012), the authors show that pre-default costs can greatly increase the net present value of financial distress costs and help match the observed leverage ratios. We differentiate from their study since we separately estimate both the loss given default and the pre-default costs that make our model consistent with the empirical data.
of default. Our findings also shed light on the bias of estimating the loss given default in a model that ignores the costs of financial distress experienced prior to default.

We highlight how pre-default costs and loss given default differently affect the firm’s financing choices and probability of default. In our setting, pre-default costs are captured by letting the firm “leak” a percentage of its value during times of financial distress. On the one hand, increasing the loss given default raises the present value of financial distress costs, thus leading the firm to choose a lower optimal leverage at the time of issuing debt. The lower optimal leverage also implies a lower probability of default. On the other hand, an increase in pre-default costs slightly reduces the optimal leverage and it leads to a higher probability of default since the firm will be losing more value (e.g. lose customers) during times of financial distress. The identification of our model relies on this different effect that pre-default costs and loss given default have on the behavior of leverage and probability of default.

We estimate our model by fitting it to observed financing choices as well as actual default rates using Simulated Method of Moments. This estimation methodology does not restrict our sample to defaulted firms only, but it allows to estimate the expected pre-default costs and loss given default for firms that are still alive. Assuming that firms start incurring pre-default costs when their cash flows fall below their coupon payments (Titman and Tsyplakov, 2007; Chen, Hackbarth, and Strebuliaev, 2019), we show that the average pre-default cost – expressed as the percentage of cash flows lost during times of financial distress – is 6.5% per year and it implies an ex-ante loss in firm value of 5.0%. While we assume that firms incur pre-default costs from the moment that net income becomes lower than required coupon payments, it is possible to specify alternative definitions. We run robustness tests by estimating different versions of our model in which firms incur pre-default costs when their EBIT is above or below the current coupon. We show that the ex-ante value of estimated pre-default costs is constant regardless of the assumption about the onset of financial

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3This is the same mechanism used in Elkamhi et al. (2012) but we allow firms to restructure their debt during their life-time similarly to Chen, Hackbarth, and Strebuliaev (2019).
4The final sample consists of U.S. public industrial firms between 1990 and 2018.
5For example, in Elkamhi et al. (2012) firms start incurring pre-default costs from the moment they become speculative grade rather than investment grade. Their definition implies that firms start incurring pre-default distress costs sooner compared to the definition that we employ in this study.
distress. The estimated average loss given default is 22.4%, a value that is consistent with empirical
evidence (Davydenko et al., 2012; Andrade and Kaplan, 1998).

Since pre-default costs of financial distress and loss given default are incurred at different times,
we calculate their net present values (NPV) in order to understand how they compare to each
other. We show that the NPV of the pre-default costs is approximately 5.0% on average, while
the NPV of the loss given default is 1.26%. This result confirms that pre-default costs are large
and strongly affect the ex-ante value of the firm. We estimate our model on subsamples split by
the durability of output. Consistent with the intuition that firms whose clients might be worried
about replacement of parts and warranties (Titman, 1984; Hortaçsu et al., 2013) are expected to
have high pre-default costs, we find that the firms with durables output Durables have net present
value of pre-default costs equal to 6.61% while the NPV is only 5.77% for Non Durables.

Next, we show the importance of accounting for pre-default costs. First, including pre-default
costs allows a trade-off model to simultaneously fit leverage, probability of default and volatility of
equity. Our results show that a model that omits pre-default costs might match two moments (e.g.
leverage and probability of default) but it is not be able to match the third (e.g. volatility of equity).
Second, as shown by Chen et al. (2019), including pre-default costs of financial distress allows a
trade-off model of capital structure to explain two apparently contradicting empirical puzzles: the
negative relationship between probability of default and stock returns (Campbell et al., 2008) and
the positive distress risk premium (Friewald et al., 2014). Our paper contributes to this debate by
providing quantitative estimates of pre-default costs of financial distress that allow the model to
fit the data.

Third, we show that there exists a bias in the estimation of the loss given default when
pre-default costs are omitted. Understanding this bias has practical importance because of the
widespread practice of calibrating trade-off models to gauge insights of the firms’ responses to a
policy change or to evaluate the behaviour of credit spreads, leverage and probability of default.6

6Examples of papers that use a calibration of structural models to study various phenomena include Huang and
and Bai, Goldstein, and Yang (2018).
To quantify this bias, we estimate the model again, but we fix the pre-default costs to zero. We then compare the results from this exercise with the ones from the model that includes pre-default costs. While the model with pre-default costs requires an average loss given default of 22.4%, the same model where we fix pre-default costs to zero needs an average loss given default of 55.4% in order to fit the data.\(^7\) Without pre-default costs, the model needs a loss at default which is approximately 2.5 times higher than in the model with pre-default costs. This difference suggests that omitting pre-default costs from trade-off models can lead to a significant bias when one calibrates them to the empirically observed values of loss at default. This happens because empirical estimates of loss at default are measured after the firm has experienced (potentially large) pre-default costs which might be unobservable to the econometrician. Therefore, such estimates might not be appropriate for the calibration of trade-off models where the parameter that measures the loss given default is in fact capturing the total costs of financial distress. We provide a tractable method to include pre-default costs in a standard dynamic trade-off model of capital structure as well as an estimate of the average pre-default costs that can be used for the calibration of models in future research.

Estimating costs of financial distress presents a challenge. Leverage is endogenous and variables that serve as a proxy for pre-default or at-default costs are scarce. Therefore, some studies have relied on a clear identification of financial distress at the cost of having small samples: for example, Andrade and Kaplan (1998) estimate that, in a sample of 31 leveraged buyout firms, the default costs are between 10-23% of the value that the firm had immediately prior to default; Hortaçsu, Matvos, Syverson, and Venkataraman (2013) document a price drop for used cars when the CDS spread for the car manufacturer increases; Pulvino (1998) documents that distressed airlines are forced to liquidate their assets at prices lower than their non-distressed competitors, consistent with the asset fire sales hypothesis of Shleifer and Vishny (1992).

Others have estimated theoretical models to infer the net benefits of leverage (Korteweg, 2010)\(^7\) Glover (2016) estimates the expected loss given default to be approximately 45% without requiring the model to match the probability of default. In our estimation, we require our model to simultaneously match the volatility of equity, leverage and probability of default. To compare our estimation results to those in Glover (2016), we estimate a version of our model where (i) pre-default costs are omitted and (ii) we fit it only to leverage and volatility of equity (not the probability of default). In this specification, our estimate loss given default is 49.818%, a value that is consistent with Glover (2016).
or the loss given default. Davydenko et al. (2012) estimate that the average costs of default are approximately 21.7% using a sample of 175 firms that defaulted between 1997 and 2010. Glover (2016) estimates a dynamic trade-off model to the data and documents that the expected costs at default for the average U.S. public firm are 45%. We contribute to this literature by explicitly modeling pre-default costs in a dynamic model in the spirit of Elkamhi et al. (2012), and we separately estimate the pre- and at-default costs of financial distress by exploiting the different effects that they have on the behaviour of leverage and default rates.

Academics and practitioners use dynamic models of capital structure (commonly referred to as structural models) to gauge insights of the firms’ responses to a policy change or to evaluate the behaviour of credit spreads, leverage and probability of default. For example, Huang and Huang (2012) and Feldhütter and Schaefer (2018) study the credit spread puzzle, Schaefer and Strebulaev (2008) analyze the relation between corporate bond returns and changes in the value of equity, Du, Elkamhi, and Ericsson (2018) show that adding stochastic volatility to dynamic trade-off models greatly improves their ability to fit the data, and Bai, Goldstein, and Yang (2018) discuss the importance of including jumps in the dynamics of asset values. We contribute to this literature by providing estimates of pre-default costs that allow the model to better fit both leverage and probabilities of default.

Another strand of the literature has studied which firm’s characteristics affect financial distress costs. There is evidence that financial distress can disrupt the ability of a producer of durable goods to provide complementary services such as warranties, spare parts and maintenance (Hortaçsu et al., 2013; Lee et al., 2018). Others show that financially distressed firms might lose the most talented employees and might not be able to attract new human capital (Berk et al., 2010; Brown and Matsa, 2016; Graham et al., 2016). Cookson (2017) shows that financially distressed casinos cannot respond to aggressive investment strategies by their competitors. We contribute to this literature by providing evidence that our estimates of pre-default costs are positively associated with the durability of a firm’s product.
Methodologically, this paper is close to the structural estimation literature. For example, Hennessy and Whited (2005) and Hennessy and Whited (2007) estimate a discrete-time dynamic capital structure model and show that it is able to generate several empirical facts (e.g. path-dependency of leverage and its relation with liquidity). Nikolov and Whited (2014) estimate a dynamic model and show that agency conflicts between managers and shareholders enhance the ability of the model to explain the dynamics of cash. Morellec, Nikolov, and Schürhoff (2012) show that small conflicts of interests between managers and shareholders improve the ability of a trade-off model to explain the dynamics of leverage. We contribute to this literature by estimating the costs of financial distress incurred prior to default separately from the costs incurred at the moment the firm defaults.

The rest of the paper is structured as follows. Section 2 develops a trade-off model of capital structure which allows for pre-default costs of financial distress. Section 3 presents the comparative statistics of the model, while the details of the identification and the structural estimation methodology are presented in Section 4. Section 5 discusses the results of the estimation via SMM and robustness tests. Section 6 concludes.

2 Model

This section presents a model of firm’s financing decisions in a dynamic trade-off framework that incorporates costs of financial distress incurred before the firm goes default. The model follows Elkamhi, Ericsson, and Parsons (2012), Goldstein, Ju, and Leland (2001) and Chen, Hackbarth, and Strebulaev (2019). As standard in the literature, we assume that assets are continuously traded in complete and arbitrage-free markets.

Firms operating cash flows (EBIT) at time $t$ are denoted by $X_t$ and their process is specified exogenously by a Geometric Brownian Motion which is described below. As in Goldstein et al. (2001), we setup a framework that is consistent with a general equilibrium model with rational

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8 Please see Strebulaev and Whited (2012) for a comprehensive review of papers that use a structural estimation approach in Corporate Finance.
expectations where firms are exposed to both systematic and idiosyncratic risk. We let the aggregate economy follow a Geometric Brownian Motion

\[
dX_{At} = \mu^P_A dt + \sigma^A_A dB^A_t^P
\]

where \(\mu^P_A\) is the expected growth rate of the economy under the physical probability space, \(\sigma^A_A\) is the volatility parameter, and \(dB^A_t^P\) is a standard Brownian Motion. A common assumption in the literature (Leland, 1994; Abel, 2018) is that firms experience a deadweight loss at the moment they default. Creditors receive a fraction \(1 - \alpha\) of the continuation value of the firm in the event of default, so the total social cost is a fraction \(\alpha\) of the continuation value, where \(\alpha \in [0, 1]\).

Following Elkamhi et al. (2012), we relax the assumption that default costs are incurred exclusively as a lump sum when firms default. In our model, we keep the assumption that firms experience a deadweight loss at the moment they default (i.e. a proportion \(\alpha\) of firm’s assets is lost at default). However, we also assume that there exists a threshold \(X_D\) such that when firm \(i\)’s EBIT drops below such threshold, the firm experiences financial distress costs proportional to the value of its assets before declaring default. More specifically, a firm’s EBIT is governed under the physical probability measure \(\mathcal{P}\) by the process

\[
dX_t = \begin{cases} 
(\mu + \beta(\mu^P_A - r)) dt + \beta \sigma^A_A dB^A_t^P + \sigma_F dB^F_t & \text{for } X_B \leq X_D \leq X_t \\
(\mu + \beta(\mu^P_A - r)) dt + \beta \sigma^A_A dB^A_t^P + \sigma_F dB^F_t - \gamma dt & \text{for } X_B < X_t < X_D \end{cases}
\]

where \(\mu\) is the (risk-neutral) expected growth rate of EBIT, \(\beta\) is the exposure to market risk, \(\gamma\) is the constant rate at which the firm loses value when it is in financial distress (i.e. \(X_B < X_t < X_D\)), \(r\) is the constant risk-free rate, \(dB^F_t\) is a standard Brownian Motion (independent from \(dB^A_t^P\)).

\textsuperscript{9}We do not include time-varying macroeconomic conditions in our model which have been shown to be an important component of this class of models to explain observed average leverage ratios and credit spreads (Bhamra et al., 2009; Chen, 2010; Bhamra et al., 2010). However, we study a different mechanism (pre-default financial distress costs) that helps reconcile the model with the data. Our approach is similar in spirit to Morellec et al. (2012) who study the effect of agency conflicts between managers and shareholders on financing decisions; they also omit time-varying macroeconomic conditions to focus on the role of agency conflicts.
that governs the idiosyncratic firm shocks, $\sigma_F$ is the idiosyncratic volatility of the firm-specific shocks, $X_D$ and $X_B$ are the distress and default thresholds, respectively. As it will be explained in Section 2.3, both the default and distress thresholds are endogenous. In Figure 1 we plot two EBIT paths using Equation (2). The two paths have been generated using the exact same parameters except for the level of pre-default costs $\gamma$. The blue solid line depicts the EBIT path for a firm that does not suffer any pre-default costs (i.e. $\gamma = 0$). The red dashed line shows the EBIT path for a firm that suffers pre-default costs equal to a loss of 2% per year for the value of its assets during financial distress (i.e. $\gamma = 2\%$). The key intuition from Figure 1 is that, all else equal, firms that experience pre-default costs of financial distress have a higher probability of default than firms that do not have such costs.

[Insert Figure 1 here]

Under the risk-neutral measure $Q$, the firm’s EBIT is governed by the process

$$\frac{dX_t}{X_t} = \begin{cases} 
\mu dt + \sigma_X dB_t & \text{for } X_B \leq X_D \leq X_t \\
(\mu - \gamma) dt + \sigma_X dB_t & \text{for } X_B < X_t < X_D 
\end{cases}$$

(3)

where $\sigma_X = \sqrt{(\beta \sigma_A)^2 + \sigma_F^2}$ is the total volatility of the firm. We provide the derivation of Equation (3) in Appendix A.

Earnings are taxed at a constant rate $\tau_c$. Therefore, firms have an incentive to issue debt to benefit from its tax-shield. As in Leland (1994), we ensure a time-homogeneous setting by assuming that firms issue an infinitely lived debt which pays a continuous flow of coupons $C$. To allow for dynamic leverage, we assume that debt is callable and issued at par. Following Goldstein et al. (2001), we assume that firms can adjust their capital structure upwards at any point in time by incurring a proportional cost $\lambda$ but they cannot reduce their debt downward. The firm’s

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10 Equation (2) shows that pre-default costs are proportional to the value of $X_t$. Since the value of assets is a monotonically increasing function of $X_t$ (as shown in Equation (4)) then pre-default are also proportional to the value of assets.
initial debt structure remains fixed until either the firm goes default or calls its debt at par and restructures with newly issued debt. The proceeds from debt issuance are distributed proportionally to shareholders. The personal tax rate on dividends is $\tau^e$ and coupon payments’ tax rate is $\tau^d$. We define $\tau = 1 - (1 - \tau^c)(1 - \tau^e)$ as the effective tax rate which includes both corporate and personal taxes. All investors face the same tax rates.

The value of (after-tax) unlevered assets is

$$V(X_t) = E^Q \left[ \int_t^\infty (1 - \tau)X_s e^{-rs} ds \right] = (1 - \tau) \frac{X_t}{r - \mu} \quad (4)$$

## 2.1 Pricing of Debt and Equity

Before discussing optimal capital structure decisions, we compute the value of debt and equity for fixed levels of coupon ($C$), distress ($X_D$) and default ($X_B$) thresholds as well as the restructuring boundary ($X_U$). The value of EBIT at the time when the firm makes its decision if $X_0$. When the firm’s EBIT reaches $X_U$, the firm retires its previously issued debt at par and it issues new one. In Section 2.2, we discuss the optimal capital structure decision made by the firm conditional on respecting limited liability (smooth-pasting condition for the default threshold). In Section 2.3 we describe the endogenous distress boundary $X_D$.

### 2.1.1 Net Income

We start by computing the value of a claim on net income, which represents the cash flows continuously accruing to shareholders at each time $t$, $(1 - \tau)(X_t - C)$. We define a refinancing cycle as the period of time over which the firm’s capital structure does not change. That is, the period of time when $X_t$, remains between $X_U$ and $X_B$. Formally, let $T = \min \{T^U, T^B\}$ where $T^U = \inf \{t > 0 : X_t \geq X_U \}$ and, $\forall s < t, X_s > X_B$ is the first time EBIT reaches the restructuring

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11This is a standard and mild assumption when one is not studying the dynamics of cash while it would be restrictive when studying precautionary saving and cash dynamics as shown, for example, in Hennessey and Whited (2005) and Nikolov and Whited (2014).
12While Section 2.3 discussed the endogenous distress boundary, we refer to Section 4.2 for the implications that such a choice has for the structural estimation of our model.
boundary conditional on not having hit the default threshold, and \( T^B = \inf\{t > 0 : X_t \leq X_B \text{ and } \forall s < t, X_s < X_U\} \) is the first time \( X_t \) reaches the default threshold conditional on not having hit the restructuring boundary. The refinancing cycle is defined as the period of time until time \( T\), \( \{t : 0 < t < T\}\).

Consider a claim over net income over one refinancing cycle. This claim depends on the level of EBIT \( X_t \) and the coupon paid on the firm’s outstanding debt. Intuitively, this claim is simply the expected net present value of the cash flows accrued to shareholders between time \( t \) and the first time the firm changes its capital structure (either by declaring default or restructuring its debt). Its value is

\[
n(X_t, C) = \begin{cases} 
\mathbb{E}^Q \left[ \int_t^T (1 - \tau)(X_s - C)e^{-r(s-t)}ds \mid X_D \leq X_t \leq X_U \right] \\
\mathbb{E}^Q \left[ \int_t^T (1 - \tau)(X_s - C)e^{-r(s-t)}ds \mid X_B < X_t < X_D \right]
\end{cases}
\]  
(5)

Equation (5) considers two cases: for \( X_D \leq X_t \leq X_U \), the firm does not incur pre-default costs and its EBIT is governed by the process described in the top expression in Equation (2); for \( X_B < X_t < X_D \), the firm experiences pre-default costs and it loses a fraction \( \gamma \) of its EBIT per period of time as described in the bottom expression in Equation (2).

To simplify notation, denote the state when the firm is not distressed, \( X_D \leq X_t \leq X_U \), as \( ND \) (No Distress state) and denote the state when it is distressed, \( X_B < X_t < X_D \), as \( DS \) (Distressed State). Let \( p_{ND}^U(X_t) \) and \( p_{DS}^U(X_t) \) be the present value of $1 to be received at the time of restructuring, contingent on restructuring occurring before default when the state is \( ND \) and \( DS \), respectively. Similarly, let \( p_{ND}^B(X_t) \) and \( p_{DS}^B(X_t) \) be the present value of $1 to be received at the time of default, contingent on default occurring before restructuring when the state is \( ND \) and \( DS \), respectively. It follows that the solutions to Equation (5) can be written as follows

\[
n(X_t, C) = \begin{cases} 
(1 - \tau) \left[ \frac{X_U}{r-\mu} - \frac{C}{r} - p_{ND}^U(X_t) \left( \frac{X_U}{r-\mu} - \frac{C}{r} \right) - p_{ND}^B(X_t) \left( \frac{X_B}{r-\mu} - \frac{C}{r} \right) \right] \\
(1 - \tau) \left[ \frac{X_U}{r-\mu} - \frac{C}{r} - p_{DS}^U(X_t) \left( \frac{X_U}{r-\mu} - \frac{C}{r} \right) - p_{DS}^B(X_t) \left( \frac{X_B}{r-\mu} - \frac{C}{r} \right) \right]
\end{cases}
\]  
(6)
where the expressions for $p_{ND}^U(X_t)$, $p_{ND}^B(X_t)$, $p_{DS}^U(X_t)$, and $p_{DS}^B(X_t)$ are provided in Appendix B.

To calculate the value of a claim on net income over all refinancing cycles we need an intermediate result. We need to show that, at the time of restructuring $T_U$, all claims are scaled up by the same proportion $\rho = X_U/X_0$ that EBIT has increased. We refer to this feature of the model as the scaling property. We provide an intuition of this property while a formal proof that shows the validity of the scaling property in this type of models can be found in Goldstein et al. (2001), pages 509-511. If we do not allow firms to restructure their debt, our model would have a static capital structure and the default threshold $X_B$ would be linear in the coupon $C$. This can be easily proved by noting that our model with static capital structure would reduce to a version of Leland (1994) in which the state variable is EBIT rather than unlevered assets. Leland (1994) shows that the default threshold is linear in the coupon.$^{13}$ In addition, the optimal coupon rate chosen by the firm at the time of issuing debt, $C^*$, is linear in $X_t$ as shown in Leland (1994).$^{14}$ Assume that $X_0$ is the time at which the last restructuring occurred. It follows that if the two firms $i$ and $j$ are identical, except that $X_{i0} = \rho X_{j0}$, then the optimal coupon rate and default threshold satisfy $C_i^* = \rho C_j^*$ and $X_{iB} = \rho X_{jB}$. Any claim will be scaled by the same factor $\rho$ and subsequent restructurings scale up these variables again by the same ratio $\rho$. For example, the of debt for firm $i$, $D(X_i, C_i^*)$, is proportional to that of firm $j$, $D(X_j, C_j^*)$, by the factor $\rho$: $D(X_i, C_i^*) = \rho D(X_j, C_j^*)$. The same property applies to all other claims (e.g. net income, etc.).

The value of net income over all refinancing cycles is equal to the expected net present value of cash flows accrued to shareholders over the entire life of the firm which we can write as follows

$$
\text{NI}(X_t, C) = \begin{cases} 
\text{n}(X_t, C) + p_{ND}^U(X_t) \cdot \text{NI}(X_U, C_U) & \text{for } X_D \leq X_t \leq X_U \\
\text{n}(X_t, C) + p_{DS}^U(X_t) \cdot \text{NI}(X_U, C_U) & \text{for } X_B < X_t < X_D 
\end{cases}
$$  

(7)

where $X_U$ is the restructuring boundary that defines when the firm issues new debt and retires the

$^{13}$Please see Leland (1994), equation (14), page 1222.

$^{14}$Please see Leland (1994), equation (21), page 1229. Leland shows that the optimal coupon $C^*$ is linear in the value of unlevered assets $V(X)$, which we proved to be linear in $X$ in Equation (4).
existing one and $C_U$ is the new coupon paid by the firm after having restructured its debt. The scaling property implies that $NI(X_U, C_U) = \rho NI(X_0, C)$ and $C_U = \rho C$ where $\rho = X_U/X_0$. At the time of debt issuance, the value of net income over all refinancing cycles simplifies to

$$NI(X_0, C) = \begin{cases} \frac{n(X_0, C)}{1-\rho p_{ND}(X_0)} & \text{for } X_D \leq X_0 \leq X_U \\ \frac{n(X_0, C)}{1-\rho p_{DS}(X_0)} & \text{for } X_B < X_0 < X_D \end{cases} \quad (8)$$

### 2.1.2 Debt

The value of a claim on coupon payments over one refinancing cycle includes the expected present value of all coupons to be received until the firm either restructures its debt or goes default plus the recovery value in the event of default. Accounting for taxes, we can write the value of this claim as follows

$$d(x_t, C) = \begin{cases} \mathbb{E}^Q \left[ \int_t^T (1-\tau^d)Ce^{-r(s-t)}ds + (1-\alpha) \cdot V(X_B) \cdot e^{-rT_B} \mid X_D \leq x_t \leq X_U \right] \\ \mathbb{E}^Q \left[ \int_t^T (1-\tau^d)Ce^{-r(s-t)}ds + (1-\alpha) \cdot V(X_B) \cdot e^{-rT_B} \mid X_B < x_t < X_D \right] \end{cases} \quad (9)$$

where $\alpha$ represents the default costs which are proportional to the value of the unlevered assets.

Similar to the analysis for net income, Equation (9) contains two cases: the expression above evaluates the value of debt over one refinancing cycle when the firm is not in financial distress ($X_D \leq x_t \leq X_U$, see top expression in Equation (2)) while the one below provides the value of debt when the firm is experiencing pre-default costs ($X_B < x_t < X_D$, see bottom expression in Equation (2)).

We can write the solution to Equation (9) as follows

$$d(x_t, C) = \begin{cases} (1 - p_{ND}^U(x_t) - p_{ND}^B(x_t)) \frac{(1-\tau^d)C}{r} + p_{ND}^B(x_t)(1-\alpha) \cdot V(X_B) \\ (1 - p_{DS}^U(x_t) - p_{DS}^B(x_t)) \frac{(1-\tau^d)C}{r} + p_{DS}^B(x_t)(1-\alpha) \cdot V(X_B) \end{cases} \quad (10)$$
Debt is retired at par when the firm restructures its debt. Assuming that the last time the firm issued debt was at time $t = 0$, the value of outstanding debt, $D(X_t, C)$, is equal to the value of debt over one refinancing cycle $d(X_t, C)$ (provided in Equation (10)) plus the expected net present value of the repayment when the firm retires the debt at par. Since debt is issued at par, the face value of debt is equal to $D(X_0, C)$. We can write the value of outstanding debt as follows

$$D(X_t, C) = \begin{cases} 
  d(X_t, C) + p_{UD}^U(X_t) \cdot D(X_0, C) & \text{for } X_D \leq X_t \leq X_U \\
  d(X_t, C) + p_{DS}^U(X_t) \cdot D(X_0, C) & \text{for } X_B < X_t < X_D
\end{cases}$$

(11)

Equation (11) holds for any $X_t \in [X_B, X_U]$ therefore at the time of issuance, when the value of EBIT is $X_0$, the value of debt is

$$D(X_0, C) = \begin{cases} 
  d(X_0) & \text{for } X_D \leq X_0 \leq X_U \\
  d(X_0) & \text{for } X_B < X_0 < X_D
\end{cases}$$

(12)

The value of outstanding debt reflects the value of debt for current debtholders but it does not include the value of debt for future debt issues. To account for new debt issued in the future, we calculate the value of a claim on the coupons that the firm will pay over its entire life (i.e. including the increased coupons following debt restructurings). We denote this claim as $TD(X_t, C)$. Intuitively, $TD(X_t, C)$ should reflect the expected net present value of coupons paid to debtholders over the entire life of the firm plus the expected recovery received at the time of default. We can write its value as follows

$$TD(X_t, C) = \begin{cases} 
  d(X_t, C) + p_{UD}^U(X_t) \cdot TD(X_U, C_U) & \text{for } X_D \leq X_t \leq X_U \\
  d(X_t, C) + p_{DS}^U(X_t) \cdot TD(X_U, C_U) & \text{for } X_B < X_t < X_D
\end{cases}$$

(13)

where $X_U$ is the restructuring boundary and $C_U$ is the new coupon paid by the firm after having
restructured its debt. The scaling property implies that \( \text{TD}(X_U, C_U) = \rho \text{TD}(X_0, C) \) where \( \rho = X_U/X_0 \). It follows that the value of total debt at \( X_0 \) is

\[
\text{TD}(X_0) = \begin{cases} 
\frac{d(X_0)}{1 - \rho p_{ND}(X_0)} & \text{for } X_D \leq X_0 \leq X_U \\
\frac{d(X_0)}{1 - \rho p_{DS}(X_0)} & \text{for } X_B < X_0 < X_D
\end{cases}
\]  

(14)

### 2.1.3 Adjustment Costs

We assume that the firm incurs adjustment costs each time it changes its capital structure. These adjustment costs are equal to a percentage \( \lambda \) of the debt being issued. At the time of refinancing, the total value of adjustment costs are equal to the flotation costs for the debt currently being issued plus the expected adjustment costs that the firm will pay for subsequent debt issues. We can write the value of adjustment costs at the time of issuance as follows

\[
\text{AC}(X_0, C) = \begin{cases} 
\lambda \text{D}(X_0, C) + p_{ND}^U(X_0) \cdot \text{AC}(X_U, C_U) & \text{for } X_D \leq X_0 < X_U \\
\lambda \text{D}(X_0, C) + p_{DS}^U(X_0) \cdot \text{AC}(X_U, C_U) & \text{for } X_B < X_0 < X_D
\end{cases}
\]  

(15)

where \( X_U \) is the restructuring boundary and \( C_U \) is the new coupon paid by the firm after having adjusted its capital structure. As for any claim in our model, Equation (15) differentiates between the time when the firm is not distressed \( (X_D \leq X < X_U) \) and when it is distressed and is incurring pre-default costs \( (X_B < X_0 < X_D) \).

By the scaling property, \( \text{AC}(X_U, C_U) = \rho \text{AC}(X_0, C) \). We can simplify Equation (15) as follows

\[
\text{AC}(X_0, C) = \begin{cases} 
\frac{\lambda \text{D}(X_0, C)}{1 - \rho p_{ND}(X_0)} & \text{for } X_D \leq X_0 < X_U \\
\frac{\lambda \text{D}(X_0, C)}{1 - \rho p_{DS}(X_0)} & \text{for } X_B < X_0 < X_D
\end{cases}
\]  

(16)

After having issued debt, the total value of adjustment costs is equal to the expected adjustment
costs that the firm will incur over its entire life which we can write as follows

$$AC(X_t, C) = \begin{cases} 
p_U \rho AC(X_0, C) & \text{for } X_D \leq X_t < X_U \\
p_U \rho AC(X_0, C) & \text{for } X_B < X_t < X_D \\
v \rho AC(X_0, C) & \text{for } AC(X_U, C_U) \\
\text{NPV of future adjustment costs} & \end{cases}$$

(17)

2.1.4 Firm and Equity Value

At any time $t$, the value of the firm, $v(X_t, C)$, is the sum of the present value of cash flows to shareholders plus cash flows to all debtholders minus the net present value of adjustment costs. We can write the value of the firm as follows

$$v(X_t, C) = NI(X_t, C) + \underbrace{TD(X_t, C)}_{\text{NPV of claim on total debt}} - \underbrace{AC(X_t, C)}_{\text{Adjustment costs}}$$

(18)

Equity is a residual claim and its value is the difference between the total value of the firm $v(X_t, C)$ and the value of current debt $D(X_t, C)$:

$$E(X_t, C) = v(X_t, C) - D(X_t, C)$$

(19)

2.2 Optimal Policies

We assume that financing decisions are made by shareholders. This assumption is equivalent to assuming that there are no conflicts of interests between firms’ managers and shareholders. When the firm decides its capital structure, it faces a trade-off between the tax benefits of debt and the expected costs of financial distress (both pre-default costs and deadweight loss at the time of default). Since proceeds from debt issuance are distributed proportionally to shareholders and the debt is fairly priced due to complete and arbitrage-free markets, shareholders’ objective is to maximize the value of the firm.

Shareholders choose the optimal default threshold $X_B$ by maximizing the value of equity. This
is equivalent to applying the smooth-pasting condition as in Leland (1994) to find the optimal default threshold $X_B$ such that

$$\frac{\partial \mathbb{E}(x, C)}{\partial x} \bigg|_{x=X_B} = 0$$

(20)

The total value of the firm $v(\cdot)$ is a function of the amount of debt that is issued which affects the coupon level $C$, and the restructuring boundary $X_U$ which affects how often the firm restructures its debt. At time 0 shareholders choose the coupon $C$ and the restructuring boundary $X_U$ to maximize the ex-ante value of the firm. Formally, shareholders solve the following problem

$$\max_{C, X_U} v(\cdot) \text{ subject to Equation (20)}$$

(21)

The above problem can be solved using standard numerical procedures. For any given set of parameters, solving the problem in Equation (21) yields the optimal coupon $C^*$, the optimal restructuring boundary $X_U^*$ and the optimal default threshold $X_B^*$.

2.3 Distress Boundary

Figure 1 shows that the distress threshold $X_D$ is a key determinant for the behavior of the firm. The role of the distress boundary is to act as a trigger for the pre-default costs of financial distress. The higher $X_D$ the sooner the firm will experience pre-default costs. Figure 1 also shows that firms can emerge from distress when the value of their EBIT improves and becomes higher than the distress threshold $X_D$. Deciding a distress threshold is admittedly an arbitrary choice. On the one hand, if $X_D$ is set too low (e.g. close to the default threshold $X_B$), then pre-default costs would not have a strong effect on financing decisions since they are experienced when the probability of default is already extremely high. On the other hand, if $X_D$ is set too high, pre-default costs would play a crucial role in deciding the optimal capital structure for the firm and would overshadow the costs at default $\alpha$.

Following Andrade and Kaplan (1998), Titman and Tsyplakov (2007) and Chen, Hackbarth, and Strebulaev (2019), we define the onset of financial distress to be when $X_t$ drops below the
coupon $C$ paid on outstanding debt. This choice has two desirable features for our study. First, this choice leads to an endogenous distress boundary because the optimal coupon is an endogenous decision that the firm makes. Second, it implies that financial distress begins when firms have high leverage which is what we would expect in reality. Indeed, using this definition we show in Table 1 that the firm becomes financially distressed when its leverage increases above 46.92%. On average, a firm with this leverage would have a rating of BB (Schaefer and Strebulaev, 2008). To further support our argument that this choice is conservative, Elkamhi, Ericsson, and Parsons (2012) choose the onset of financial distress when the firm’s debt loses its investment-grade status. Their decision matches well with their empirical strategy which consists of fitting the model using average probabilities of default by rating but it also implies a higher $X_D$ compared to our choice.

3 Comparative Statics

We now examine the predictions of our model for financing decisions and provide a first look at the importance of pre-default costs in capital structure choice. In Table 1 we report the comparative statics describing the effects of the main parameters of the model on: (i) target leverage which is the optimal leverage chosen by the firm at the time of issuing debt; (ii) the leverage at restructuring, which is informative of the restructuring boundary $X_U$; (iii) the leverage at the time the firm becomes distressed, which is informative of the distress threshold $X_D$; (iv) the recovery rate for debtholders at the time of default.

We set the base case parameters as follows. Using the estimates in Graham (1999) and Graham (2000), we set the personal tax rate on dividends $\tau^e = 11.6\%$ and the tax rate on interest income $\tau^d = 29.3\%$. The corporate tax rate is set to the average marginal tax rate $\tau^c = 35.0\%$. The effective tax rate for the company is $\tau = 1 - (1 - \tau^c)(1 - \tau^e) = 42.54\%$. The aggregate earnings growth rate and volatility are set to the average growth rate and volatility of the Net Operating Surplus series from NIPA $\mu_A = 5.84\%$ and $\sigma_A = 9.471\%$ for the period 1990-2018, respectively.\footnote{All rates and volatilities reported in this paper are annualized. A detailed description of the data is provided in Section 4.1.}
The risk-free rate is $r = 2.27\%$. We set the growth rate and idiosyncratic volatility of cash flows to $\mu = 0.5\%$ and $\sigma_F = 14.8\%$. The exposure to market risk parameter is set to $\beta = 0.9$. This calibration implies a total volatility for the firm’s EBIT $\sqrt{\beta^2 \sigma_A^2 + \sigma_F^2} = 17.08\%$. The loss given default parameter is $\alpha = 22.44\%$, the pre-default costs are set to $\gamma = 6.5\%$. The proportional adjustment costs is set to $\lambda = 1.0\%$ which is in line with empirical evidence (Kim, Palia, and Saunders, 2008). We set the distress boundary $X_D$ equal to the value of coupon $C$. We normalize the initial value of operating cash flows $X_0 = 50.0$.

Table 1 shows that an increase in the loss given default $\alpha$ lowers the target leverage, the restructuring boundary and the distress trigger point. Consistent with Goldstein et al. (2001) (Table 2, page 506), an increase in $\alpha$ also lowers the recovery rate for debtholders. An increase in pre-default costs $\gamma$ has qualitatively a similar effect on target leverage, restructuring boundary and distress trigger. However, we show in Section 4.2 that $\gamma$ is positively correlated with the probability of default contrary to $\alpha$. More specifically, an increase in $\gamma$ has two opposing effects on the probability of default. On the one hand, it increases the probability of default by reducing the firm’s EBIT growth rate when it is distressed. On the other hand, an increase in $\gamma$ makes debt more costly and less attractive for shareholders, thus leading them to choose a lower optimal leverage. Our model shows that the former effect dominates the latter. Also, $\gamma$ affects the recovery value only through the change in optimal leverage and lower default threshold. The intuition underlying this result is as follows. A lower optimal leverage implies a lower default threshold which lowers the recovery value. An increase in $\alpha$ also leads to a lower leverage and, hence, a lower default threshold; however, in addition to this effect, an increase in $\alpha$ also directly lowers the portion $(1 - \alpha)$ of the unlevered asset values that is recovered at default.

The rest of Table 1 shows the effect on leverage and recovery rate of EBIT growth rate $\mu$ and volatility $\sigma_F$, firm’s beta $\beta$, and adjustment costs $\lambda$. The results are consistent with those previously reported in the literature (see, for example, Strebulaev (2007)).

[Insert Table 1 here]
4 Identification and Structural Estimation

We estimate the model parameters using Simulated Method of Moments (SMM). Before presenting the results of our estimations in Section 5, we discuss the empirical data in Section 4.1, and then proceed to discuss the identification of our model in Section 4.2. A detailed description of the SMM methodology is provided in Appendix C.

4.1 Empirical Data

We collect financial statement from Compustat Fundamentals Quarterly (library “compd”, file “fundq”). Following the literature (see, for example, Hennessy and Whited (2005) and Hennessy and Whited (2007)), we drop financial firms (SIC codes 6000 - 6999), utilities (4900 - 4999), and public administration firms (9000 - 9999). We gather firms’ equity returns from the Center for Research in Security Prices (CRSP). We define a firm’s quasi-market leverage for quarter $t$ as the book debt at time $t$ ($d_{lcq} + d_{lttq}$) divided by the sum of book debt and market value of equity ($pr_{ccq} \times c_{shoq}$). Since our model is developed based on operating cash flows $X_t$, we define the operating return on assets (Operating ROA) as the ratio between operating profits and lagged total assets. A detailed description of the variables is provided in Table 2. We get the average probabilities of default by rating and year from the Moody’s Default & Recovery Database (DRD). For firm $i$ with rating $R$ observed in any quarter of year $y$, we assign the average probability of default for rating $R$ in year $y$ published by Moody’s DRD.

Observations with missing total assets, quasi-market leverage, common shares outstanding, closing price or equity returns for the end of the quarter are excluded. Also, we keep firms with total value of assets of at least $10$ million and a market value of equity of at least $5$ million. We winsorize all variables at the 1% level to avoid the influence of outliers. After applying the aforementioned selection criteria, we obtain a panel data set with $101,032$ firm-quarter observations for $N = 4,603$ individual firms between 1990 and 2018 at the quarterly frequency.

[Insert Table 2 here]
For the calibration of the aggregate economy, we use the quarterly aggregate earnings series from NIPA (Series “Net value added of corporate business: Net operating surplus”, Table 1.14, Line 8). We calculate the quarterly returns and calibrate the parameters of the aggregate economy, $\mu_A^P$ and $\sigma_A$, to the mean and standard deviation of such returns.

Table 3 provides the descriptive statistics for our sample, which are representative of generic samples from Compustat, except that our sample is slightly biased towards large firms. The median firm assets is $1.97$ billion.

[Insert Table 3 here]

4.2 Identification

We estimate 5 parameters: the parameter $\gamma$ that governs the pre-default costs of financial distress, the loss given default $\alpha$, the expected (risk neutral) growth rate of EBIT $\mu$, the idiosyncratic volatility $\sigma_F$, and the exposure to market risk $\beta$. The selection of moments used in the SMM estimation is important to ensure that the parameters of interest are identified. Therefore, it is important to choose moments that are a priori informative about the unknown structural parameters. Intuitively, a moment is informative about an unknown parameter if that moment is sensitive to changes in the parameter.

We use six empirical moments for the identification of our model: the operating ROA, the average equity returns, the quasi-market leverage$^{16}$, the excess return of firm’s equity with respect to the risk-free rate, the probability of default at 5 years, the variance of equity returns. We estimate the rest of the model parameters (for example, the risk-free rate $r$) separately and set them equal to the values of the Base Scenario which are set to the values described in Section 3. For each scenario, we simulate the model 5,000 times for a time period of 150 years. We keep only the last 80 quarters of data to remove the effect of the initial conditions. For each simulation, we calculate model moments and then we average across all simulations.

$^{16}$For simplicity, we often use the word “leverage” to refer to “quasi-market leverage”.
Every moment is affected by all parameters of our model. However, some parameters are more important than others for a particular moment. In Table 4 we describe which moment is most important to identify each parameter using the elasticities of the model moments with respect to each parameter. The elasticity of moment \( m \) with respect to parameters \( p \) is defined as \( \frac{dm}{dp} \). The elasticities are calculated at the estimated parameter values discussed in Section 5.

The idiosyncratic volatility \( \sigma_F \) is directly related to the variance of equity returns. This can be seen from the diffusion parameter of the process described in Equation (2) and from the strong effect that \( \sigma_F \) has on the variance equity returns with an elasticity of 8.45 as shown in Table 4. The parameter \( \mu \) is identified mainly through the Operating ROA. As shown in Table 4, \( \mu \) has the strongest effect on Operating ROA compared to the other estimated parameters. The exposure to market risk (\( \beta \)) is primarily identified by the equity returns. This is intuitive because we expect high beta stocks to have higher expected returns, a feature that is confirmed by our model. To confirm that we are able to separately identify both \( \sigma_F \) and \( \beta \), we run a robustness test in Figure 2. This figure shows that \( \sigma_F \) is positively related to both variance of equity and excess equity returns while \( \beta \) is positively related to excess returns and has almost no effect on the variance of equity. This result confirms that \( \sigma_F \) and \( \beta \) are identified.

Disentangling pre-default costs of financial distress from the loss given default requires us to separately identify the parameters \( \alpha \) and \( \gamma \). In Table 4, we show that both \( \alpha \) and \( \gamma \) negatively affect leverage which is consistent with other models in the literature (see for example, Goldstein et al. (2001)). The interesting result related is that \( \alpha \) has a little negative effect on the probability of default while \( \gamma \) is strongly and positively related to the probability of default. Therefore, the combination of leverage and probability of default at 5 years should provide enough information to identify \( \alpha \) and \( \gamma \) because the two parameters have different effects on the simultaneous behaviour of leverage and probability of default.
Figure 3 provides additional evidence for the identification of $\alpha$ and $\gamma$. In Panel A, we show how leverage and probability of default are affected by $\alpha$. Specifically, we simulate the model 5,000 times for a time period of 150 years. We keep only the last 80 quarters of data to remove the effect of the initial conditions. For each simulation, we calculate the average Quasi-Market Leverage and Probability of Default at 5 years. We then average across all simulations. We plot Quasi-Market Leverage (left y-axis) and the Probability of Default (right y-axis) as a function of $\alpha$ for a fixed level of $\gamma = 6.5\%$ and the other model parameters are set to the Base Case scenario described in Table 1. Panel A shows that increasing $\alpha$ leads to lower leverage (blue solid line) and lower probability of default (red dashed line). In Panel B we repeat the same exercise but we vary $\gamma$ for a fixed level $\alpha = 22\%$. Panel B shows that increasing $\gamma$ leads to a decrease in leverage (blue solid line) but a sharp increase in the probability of default (red dashed line).

In Panel C of Figure 3, we choose combinations of $\alpha$ and $\gamma$ that yield the same leverage ratio and we check whether the probability of default changes. For each value of $\alpha$, we look for the level of $\gamma$ that keeps Quasi-Market Leverage constant at 32.0%. The blue solid line in Panel C plots the pair of $\alpha$ (left y-axis) and $\gamma$ (x-axis) that yield a constant leverage ratio of 32%. There is a strong negative relation between $\alpha$ and $\gamma$ meaning that if we lower $\alpha$, the model needs a higher $\gamma$ in order to keep leverage constant. The red dashed line in Panel C shows the probability of default at 5 years (right y-axis) corresponding to the various pairs of $\alpha$ and $\gamma$ that keep leverage constant at 32%. The red dashed line shows a strong positive relation between $\gamma$ and the probability of default. This implication stems from the fact that both $\alpha$ and $\gamma$ negatively affect leverage because an increase in any of these two parameters increases the overall cost of debt (sum of pre-default and at-default costs). However, while an increase in $\alpha$ leads to a lower optimal leverage and lower probability of default, the effect of $\gamma$ on leverage and probability of default is different. An increase in $\gamma$ lowers the optimal leverage which in turn has a negative effect on the probability of default. However, at the same time, it increases the pre-default costs, thus increasing the probability of default. Panel C of Figure 3 shows that the latter effect dominates the former and leads to a higher probability of default for an increase in $\gamma$. 

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In Section 2.3, we described our choice of distress boundary. Here we discuss what are the implications of such a choice for the estimation of the parameter $\gamma$. We define the onset of financial distress to be when the operating cash flows $X_t$ drop below the coupon $C$ paid on outstanding debt. This choice is consistent with Andrade and Kaplan (1998), Titman and Tsyplakov (2007) and Chen et al. (2019). How does the choice of the distress boundary affect the estimation of $\gamma$? On the one hand, if $X_D$ is set too low (e.g. close to the default threshold $X_B$), then pre-default costs would not have a strong effect on financing decisions since they are experienced when the probability of default is already very high. This would result in an underestimation of the pre-default costs. On the other hand, if $X_D$ is set too high, pre-default costs would play a crucial role in deciding the optimal capital structure for the firm and would overshadow the costs at default $\alpha$. In this case, we would end up overestimating the pre-default costs.

We argue that our choice is conservative because, as explained in Section 2.3, it implies that financial distress begins when firms have very high leverage. In the Base Scenario in Table 1, we show the firm becomes financially distressed when its leverage increases above 46% which is the average leverage for firms rated BB (Schaefer and Strebulaev, 2008). Further, in Section 5.2 we re-estimate our model with various distress boundaries and show that the total value of pre-default costs of financial distress is stable across different specifications.

5 SMM Estimation Results

We first present the results of the estimation of our model using SMM on the entire sample of firms from 1990 to 2018 under the assumption that financial distress starts when the operating cash flows of the firm are lower than the required coupon payments. Next, we present the results for subsamples split by durability of firms’ output. We then show the results of the estimation under alternative definitions of financial distress and calculate the net present value of expected financial distress costs.
5.1 Estimations for the entire sample

We estimate three different specifications of our model on our sample of firms from 1990 to 2018. We label them (1) Model With Pre-Default Costs, (2) Model Without Pre-Default Costs, and (3) Not Matching Prob. of Default. The Model With Pre-Default Costs estimates 5 parameters using the 6 moments described in details in Section 4.2. This model includes both parameter $\gamma$ to capture pre-default costs of financial distress and parameter $\alpha$ that captures the loss given default. Table 5 shows the model fit. We report the empirical moments (column “Data”) as well as the simulated moments from the model. The $t$-statistics for the difference between empirical and simulated moments are reported in parenthesis. The Model With Pre-Default Costs fits the data well. The simulated leverage and probability of default at 5 years are 34.949% and 5.370%, respectively.\footnote{As confirmed from the low $t$-statics, these values are not statistically different from their empirical counterparts which are 34.98\% for leverage and 5.428\% for the probability of default. Table 5 also confirms that the other four simulated moments are not statistically different from their empirical counterparts.}

Table 5 also confirms that the other four simulated moments are not statistically different from their empirical counterparts.

[Insert Table 5 here]

Table 6 presents the estimated parameters for the estimation of the Model With Pre-Default Costs. The estimated the parameters $\beta$ (exposure of the firm’s cash flows to market risk) is equal to $\approx 0.897$.\footnote{We report the parameters multiplied by 100 in our tables for ease of readability.} The parameter $\mu$ (risk-neutral expected growth rate of the firm) is 0.517\% and the volatility of the expected earnings’ growth rate $\sigma_F$ is estimated at 14.818\%.

The two main parameters of interest for our study are $\gamma$ and $\alpha$. The parameter $\gamma$ captures pre-default costs of financial distress (i.e. constant rate at which the firm loses value when it is in financial distress but has not defaulted yet) while $\alpha$ represents the loss given default. We estimate $\gamma = 6.531\%$ and $\alpha = 22.444\%$. Our estimate of $\gamma$ implies that, on average, when firms
are in financial distress they are expected to lose approximately 6.5% of their value per year. This value is large and, as we explain further below, is such that pre-default costs of financial distress constitute a large proportion of the total costs of financial distress. The estimated parameter $\alpha$ implies that firms are expected to lose 22.444% at the time of default. Such a value is consistent with empirical values estimated by Davydenko et al. (2012) and Andrade and Kaplan (1998) that estimate the average costs to be 21.7% and 10-23%, respectively.

Glover (2016) argues that firms that have lower expected default costs use more leverage and, consequently, they end up defaulting more often. He estimates a trade-off model of capital structure where firms experience costs of financial distress only at the time of default. He finds that the average firm expects to lose 45% of firm value in default, a value that is much higher than empirical estimates. However, Glover (2016) shows that when distress costs are calculated for the sub-sample of defaulted firms they are approximately 25%. This selection bias is consistent with our study because firms with low total distress costs (sum of pre-default costs and loss given default) optimally choose a higher leverage to benefit from tax benefits as shown in Table 1. Our model shows a different channel which is consistent with the selection bias but does not require a loss given default that is much higher than empirical estimates. Our intuition is as follows. The loss given default estimated by our model is 22.444% but this is only a fraction of the total costs of financial distress which include also pre-default costs. While for the sample of defaulted firms previous studies have estimated the loss given default (see for example Davydenko et al. (2012)), firms that are still alive are expected not only to incur a deadweight loss at the time of default but also pre-default costs. This implies that the total distress costs are higher than the loss given default which makes our study consistent with Glover (2016). In other words, our results confirm that the estimated loss given default is consistent with empirical evidence (Davydenko et al., 2012; Andrade and Kaplan, 1998) but that firms also experience additional costs prior to default.\footnote{Our estimated $\alpha \approx 22.4\%$ is consistent with the sub-sample of defaulted firms in Glover (2016) for which $\alpha \approx 25\%$.}

Having established that pre-default costs are large and that our model is consistent with prior
studies (Glover, 2016; Andrade and Kaplan, 1998; Davydenko et al., 2012), we address the following concern regarding the calibration of structural models using empirical estimates of loss given default. Many trade-off models used to study corporate financing decisions assume that distress costs are incurred only at default. The standard procedure to calibrate these models consists of using the empirical estimates of tax benefits (Graham, 2000) and default costs (Andrade and Kaplan, 1998; Davydenko et al., 2012). However, in order to calculate empirical estimates of default costs, previous studies estimate the change in firm value that happens immediately after default, when the firm has already incurred pre-default costs. Therefore, applying such estimates to firms that are not close to default would underestimate the effective costs of financial distress (Elkamhi et al., 2012). How large would this bias be? To answer this question, we estimate a modified version of our model which we label Model Without Pre-Default Costs. In this specification, we exogenously set $\gamma = 0$ so the costs of financial distress are experienced only at the time of default when the firm is expected to lose a portion $\alpha$ of its value. We fit this specification to the data using the same six moments that we used to estimate the Model With Pre-Default Costs. This exercise allows us to answer the question: if we omit pre-default costs from a structural model, how would the estimation of the loss given default $\alpha$ change? Given the widespread practice of calibrating trade-off models to gauge insights of the firms’ responses to a policy change or to evaluate the behaviour of credit spreads, leverage and probability of default, answering this question is important for the literature.

In column Model Without Pre-Default Costs of Table 5, we discuss the results for the specification where pre-default costs are omitted and we use the six moments to fit the model to the data. The model fits the data generally well except for the variance of equity. The $t$-statistic for the difference between the empirical moment of the variance of equity and its simulated counterpart is 4.216. This is the only $t$-statistic in this specification that signals a statistical difference between empirical and simulated moments. Why do we observe this difference? Our explanation is that,
absent pre-default costs of financial distress, the Model Without Pre-Default Costs needs a higher variance of equity in order to generate probabilities of default that are consistent with empirical evidence.\textsuperscript{22} This is confirmed in Table 6 which shows that the estimated parameter $\sigma_F$ (volatility of EBIT) is 20.251\% for the Model Without Pre-Default Costs while it is only 14.818\% for the Model With Pre-Default Costs.

![Insert Table 6 here]

Table 6 also shows that the estimated loss given default $\alpha$ in the Model Without Pre-Default Costs is 55.44\%. This value is approximately 2.5 times than the 22.444\% that we had estimated with the Model With Pre-Default Costs. This suggests that there is a large bias and that practitioners and researchers should be using a higher value than empirical estimates of loss given default in the calibration of their models if they omit pre-default costs.

Our estimate of $\alpha = 55.44\%$ in the Model Without Pre-Default Costs is also higher than the one presented by Glover (2016) when he fitted a structural model to observed leverage choices but not probabilities of default. To understand whether this difference is driven by our choice to fit the model also to probabilities of default in addition to leverage, we estimate a version of our model which we label Not Matching Prob. of Default. In this specification, we exogenously set $\gamma = 0$ as in the Model Without Pre-Default Costs but we exclude the Probability of Default moment (i.e. deviation from the empirical probability of default are not penalized in this specification) to make it comparable to Glover (2016). The results of this estimation are presented in column Not Matching Prob. of Default of Table 6. In this specification the estimated loss given default is $\alpha = 49.818$. While still slightly higher than the value of 45\% estimated by Glover (2016), such a small difference can easily be attributed to the difference in the samples used. Indeed, while Glover estimates his model on a panel of 2,505 firms between 1990 and 2010, we estimate our model on a panel of 4,603 firms for the period 1990 and 2020.

\textsuperscript{22}Recall from Section 4.2 that the parameter that capture pre-default costs $\gamma$ strongly affects the probability of default. Absent this parameter, the model needs a higher volatility in order to match the probability of default.
5.2 Splits by Durability

Financial distress for a supplier of durable goods might compromise its ability to provide warranties, service and spare parts. Customers of such firms factor these risks into their decision making process and either demand a discount or avoid the firm’s product all together (Titman, 1984). Either ways the firm suffers a loss that would not have incurred if it did not have debt. Hortaçsu, Matvos, Syverson, and Venkataraman (2013) use a panel data set for used car sales and provide evidence that increases in the CDS spread of a car manufacturer leads to a sharp decrease in the price of the used cars of the same brand. Their paper supports the hypothesis that customers and suppliers are reluctant to buy and sell, respectively, products from a distressed firm because they are worried about the void of warranty, replacement of parts and services.

In order to split the data on the durability of their product, we need to define which industries are subject to distress due to customers being concerned for warranty voidance, etc. We start by looking at the classification of industries by durability of a firm’s product in Gomes et al. (2009). They categorize the industries in 6 different buckets: Personal consumption expenditures on durable goods (d.), Personal consumption expenditures on nondurable goods (n.d.), Personal consumption expenditures (PCE) on services (s.), Gross private domestic investment (I), Government consumption expenditures and gross investment (G), and Net exports of goods and services (NX). For the purpose of our study, we are only interested in splitting the data into two categories: firms that produce products subject to warranties, services and parts (e.g. a producer of cars and trucks) and those firms whose output is immediately consumed (e.g. a coffee at Starbucks). We therefore categorize the industries into **Durables** and **Non Durables** by manually looking at the definition of the 4 digit SIC codes.\(^{23}\) We start from the categories in Gomes et al. (2009) and then adjust them according the our categorization of durables vs. non-durables. After applying our categorization, we obtain a panel of 46,167 firm-quarter observations. The decrease in the number of firms compared to the full sample is due to the fact that not all industries in our full sample are categorized by Gomes et al. (2009) which we use as the starting point of our classification.

\(^{23}\text{Our classification is available upon request and will be published on the authors’ website.}\)
Table 7 shows the model fit for the estimation of the Model With Pre-Default Costs on the subsample of firms split by durability of their output (Durables vs. Non Durables). Panel A shows the empirical moments, the simulated moments, and the $t$-statistics for the difference between empirical and simulated moments. The model fits both durables’ and non durables’ firms well since none of the $t$-statistics signal a statistically significant difference between empirical and simulated moments. We present the estimated parameters and their standard errors (in parentheses) based on the splits by durability in Panel B of Table 7. The parameter $\gamma$ is equal to 6.585% for Durables and 6.171% for Non Durables. The loss given default for Durables and Non Durables are 13.375% and 15.187%. The (risk-neutral) expected growth rate $\mu$ is higher for Durables ($\mu = 0.664\%$) compared to Non Durables ($\mu = 0.313\%$) while the estimated $\beta$ parameters are very similar. The volatilities of EBIT $\sigma_F$ are similar amongst the two sub-samples: $\sigma_F \approx 16\%$ for Durables and $\sigma_F \approx 15.5\%$ for Non Durables.

How large are the expected net present values of pre-default costs between Durables and Non Durables? To answer this question we compare the ex-ante value of the firm for the Model With Pre-Default Costs (Full Model) and two counterfactual scenarios: (i) when there are no pre-default costs of financial distress $\gamma = 0$, and (ii) when there is no expected loss at the time of default $\alpha = 0$. We present the results for this exercise in Panel C. In column Full Model we standardize the value of the firm to $100.00. In columns $\gamma = 0$ and $\alpha = 0$ we report the increase in firm value that there would have been if the parameters $\gamma$ and $\alpha$ are set to zero, respectively. The increases in firm value when we set $\gamma = 0$ and $\alpha = 0$ are informative of the expected NPV of pre-default costs and loss given default, respectively. Consistent with the intuition that Durables should have higher expected pre-default costs since customers might be worried about services, warranty and

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24Note that these values are lower than that of the average firm which is equal to 22.444%. However, as we explained above in this section, our sample of Durables’ plus Non Durables’ firms is composed of a panel of 46,167 firm-quarter observations. Such value is considerably lower than the full sample which has 101,032 firm-quarter observations.
replacement of parts, Panel C shows that the NPV of pre-default costs is higher for \textit{Durables} than \textit{Non Durables}. The increase in firm value when $\gamma = 0$ for \textit{Durables} is 6.61\% while the increase for \textit{Non Durables} is more modest at 5.77\%.

5.3 Alternative Distress Boundaries

We have so far allowed firms to experience pre-default costs of financial distress when operating cash flows are lower than the required coupon payments. This modeling choice is justified by empirical evidence (Andrade and Kaplan, 1998), and it has been used by other authors (Titman and Tsyplakov, 2007; Chen et al., 2019). We recognize that this choice is arbitrary but we provide evidence here that it does not impact our estimation. Intuitively, if we allowed firms to experience pre-default costs of financial distress sooner (e.g. when EBIT is twice as high as coupon payments), we would expect that the our model needed a lower $\gamma$ in order to fit the data. This is because the firm “leaks” less value per year but it becomes distressed sooner. Conversely, if firms experience pre-default costs of financial distress very late (e.g. when EBIT is one half of coupon payments) then we would expect a higher $\gamma$. Even if the estimated $\gamma$ might vary if we assume different distress boundaries, we intuitively expect that the net present value of expected pre-default costs of financial distress would not vary much with the choice of distress boundary. We conduct our robustness tests for various distress boundaries below.

First, we check that the estimated $\gamma$ is negatively related with the level of the distress boundary. Specifically, the endogenous distress boundary is defined as $X_D$, and it represents the level of EBIT below which the firm experiences pre-default costs of financial distress. In Table 8 we estimate four different versions of the \textit{Model With Pre-Default Costs} with various distress boundaries: $X_D = k \times C$ for $k \in \{2.0, 1.5, 1.0, 0.5\}$. Panel A reports the estimated model parameters with standard errors in parentheses. Consistent with our intuition, $\gamma$ is inversely related to the level of the distress boundary $X_D$. When $X_D = 2 \times C$ our model estimates a $\gamma = 3.718\%$, for $X_D = 1.5 \times C$ the model requires a $\gamma = 4.154\%$ while the estimated $\gamma = 14.308$ for $X_D = 0.5 \times C$. Panel B shows the model fit of the various estimations. Overall, the choice of distress boundary does not have a strong effect
on the ability of the model to fit the data except for the moment of variance of equity. For this moment, only the model with $X_D = 1 \times C$ is able to match the variance of equity while the other choices of distress boundary generate simulated moments of variance of equity that are statistically different from the empirical counterparts as shown by the $t$-statistics which are higher than 6 (six). This confirms that choosing $X_D = 1$ is not only supported by empirical evidence (Andrade and Kaplan, 1998) but it is also the choice that allows the model to best fit the data.

[Insert Table 8 here]

Second, we check how the net present value of expected pre-default costs vary with the choice of distress boundary. Table 9 compares the ex-ante value of the firm for the Model With Pre-Default Costs and two counterfactual scenarios: (i) when there are no pre-default costs of financial distress $\gamma = 0$, and (ii) when there is no expected loss at the time of default $\alpha = 0$. In column Model With Pre-Default Costs we standardize the value of the firm to $100.00$. In columns $\gamma = 0$ and $\alpha = 0$ we report the increase in firm value that there would have been if the parameters $\gamma$ and $\alpha$ are set to zero, respectively. This increase represents the NPV of expected pre-default costs. We present the results for various specifications of the distress boundary: $X_D = k \times C$ for $k \in \{2.0, 1.5, 1.0, 0.5\}$. For $X_D = 2.0$, the NPV of pre-default costs is 5.00% of firm value, for $X_D = 2.0$, the NPV of pre-default costs is 5.54% of firm value, and finally the NPV is equal to 5.87% of firm value when $X_D = 0.5 \times C$. These values are similar considering that they are the result of estimating the model for very different distress boundaries.

[Insert Table 9 here]

6 Conclusion

This paper quantifies the expected costs of financial distress that firms experience prior to default (pre-default costs) separately for the loss given default. We develop a dynamic model of capital
structure which includes pre-default costs and we fit it to the data. We find that pre-default costs are large and on average equal to 6.5% of firm value per year during times of financial distress. We document that omitting pre-default costs leads to a large bias in the estimates of loss at default when using dynamic models of capital structure: while the average loss given default in our model with pre-default costs of financial distress is 22.444%, the same model where we exogenously fix pre-default costs to zero needs a loss given default of 55.44% in order to match both leverage and probability of default or, alternatively, a loss given default of 49.18% in order to match leverage alone. This value is consistent with previous studies such as Glover (2016).

Our estimates confirm the intuition that firms with more durable output expect to lose more during times of financial distress. When we fit our model on samples split by the durability of output, we find that that the net present value of pre-default costs of financial distress for Durables’ firms are 6.61% of firm value while they are 5.77% for Non Durables’ firms.

Our study has implications for academics and practitioners using calibrated dynamic capital structure models to gauge insights of the firms’ responses to a policy change or to evaluate the behaviour of credit spreads, leverage and probability of default. First, we show that omitting pre-default costs and using empirical estimates of the costs at default underestimates the total costs of financial distress. Second, we show that some industries and firms with a high durability of output are more exposed to pre-default costs of financial distress.

The next step is to study the effects that pre-default costs have on the ability of dynamic trade-off models to match credit spreads for bonds and other firms’ policies (e.g. dividend payout policy). While in this paper we focus on the estimation of expected pre-default costs and we analyze their effect on the probability of default, the precise role that pre-default costs have on credit spreads, investment or dividend payouts is a question open for further research.
References


Chen, Zhiyao, Dirk Hack Barth, and Ilya A Strebulaev, 2019, A unified model of distress risk puzzles.


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Figures

Figure 1
Model Intuition

This figure shows the impact that pre-default costs have on the path of EBIT. EBIT follows a geometric brownian motion as described in Equation (2):

\[
\frac{dX_t}{X_t} = \begin{cases} 
  (\mu + \beta(\mu_A^P - r)) \ dt + \beta \sigma_A dB_t^{A,P} + \sigma_F dB_t^F & \text{for } X_B \leq X_D \leq X_t \\
  (\mu + \beta(\mu_A^P - r)) \ dt + \beta \sigma_A dB_t^{A,P} + \sigma_F dB_t^F - \gamma dt & \text{for } X_B < X_t < X_D 
\end{cases}
\]

The figure shows two scenarios. The blue solid line shows the path of the firm’s EBIT if it had no pre-default costs of financial distress (\(\gamma = 0\)) while the red dashed line shows the scenario when the firm’s has pre-default costs (\(\gamma = 2\%\)). The rest of the parameters are the same in the two scenarios. Both paths start from \(X_0\) and they evolve according to the law of motion described above. When \(X_t\) is below \(X_D\), the firm experiences pre-default costs. When \(X_t\) reaches \(X_B\) the firm defaults.
Figure 2
Identification $\beta$ and $\sigma_F$

This figure depicts the relation between the firm’s exposure to market risk $\beta$ and the idiosyncratic volatility of EBIT $\sigma_F$ with respect to the moments of Variance of Equity (Panel A and Panel B), and Excess Return (Panel C and Panel D). The data used to create these figures have been generate as follows. We simulate the model 5,000 times for a time period of 150 years. We keep only the last 80 quarters of data to remove the effect of the initial conditions, and we calculate the Variance of Equity and the Excess Return (equity return - risk-free rate). The model parameters are set to the base case scenario described in Table 1. Panel A and Panel B plot the Variance of Equity as a function of $\sigma_F$ and $\beta$, respectively. Panel C and Panel D plot the Excess Return as a function of $\sigma_F$ and $\beta$, respectively. The model parameters are set to the base case scenario described in Table 1.
Figure 3
Identification of $\alpha$ and $\gamma$

This figure depicts the relation between the default-loss $\alpha$, the pre-default costs $\gamma$, and the moments of leverage and probability of default. The data used to create these figures have been generated as follows. We simulate the model 5,000 times for a time period of 150 years. We keep only the last 80 quarters of data to remove the effect of the initial conditions. For each simulation, we calculate the Quasi-Market Leverage and Probability of Default at 5 years. We then average across all simulations. Panel A plots the Quasi-Market Leverage (left $y$-axis) and the Probability of Default (right $y$-axis) as a function of $\alpha$ for a fixed level of $\gamma = 6.5\%$. Panel B plots Quasi-Market Leverage (left $y$-axis) and the Probability of Default (right $y$-axis) as a function of $\gamma$ for a fixed level of $\alpha = 22\%$. The other model parameters are set to the base case scenario described in Table 1. Panel C plots the probability of default for various combinations of $\alpha$ and $\gamma$ that yield a constant Quasi-Market Leverage. The blue solid line in Panel A plots the value of $\alpha$ (left $y$-axis) and $\gamma$ (x-axis) that yield a constant Quasi-Market Leverage. Specifically, for each value of $\alpha$, we look for the level of $\gamma$ that keeps Quasi-Market Leverage constant at 32.0%. The red dashed line plots the probability of default (right $y$-axis) corresponding to the pair of $\alpha$ and $\gamma$ that yield a constant leverage of 32%. 

Panel A 

Panel B 

Panel C
Tables

Table 1
Comparative Statics

This table presents the comparative statics of the model with regards to financing decisions. We set the base case parameters as follows. The personal tax rate on dividends $\tau_e = 11.6\%$, the tax rate on interest income $\tau_d = 29.3\%$, the corporate tax rate $\tau_c = 35.0\%$, the aggregate earnings growth rate and volatility are $\mu_A = 5.84\%$ and $\sigma_A = 9.471\%$, the risk-free rate is $r = 2.27\%$, the growth rate and volatility of cash flows are $\mu = 0.5\%$ and $\sigma_F = 14.8\%$, the exposure to market risk parameter is set to $\beta = 0.9$, the loss given default parameter is $\alpha = 22.44\%$, the pre-default costs are set to $\gamma = 6.5\%$, the proportional adjustment costs is set to $\lambda = 1.0\%$. We normalize the initial value of operating cash flows $X_0 = 5.0$ and set the distress boundary $X_D$ equal to the value of coupon $C$.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Quasi-Market Leverage (%) at</th>
<th>Recovery Rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X_B/X_0</td>
<td>Distress Target</td>
</tr>
<tr>
<td>Base</td>
<td>0.092</td>
<td>46.92</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-Default Costs (Base: $\gamma = 6.5%$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma = 8.0%$</td>
<td>0.078</td>
<td>44.11</td>
</tr>
<tr>
<td>$\gamma = 5.0%$</td>
<td>0.112</td>
<td>50.19</td>
</tr>
<tr>
<td>Costs at Default (Base: $\alpha = 22.44%$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha = 34.0%$</td>
<td>0.086</td>
<td>46.54</td>
</tr>
<tr>
<td>$\alpha = 12.0%$</td>
<td>0.099</td>
<td>47.23</td>
</tr>
<tr>
<td>Expected Growth Rate of EBIT (Base: $\mu = 0.5%$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu = 0.8%$</td>
<td>0.085</td>
<td>41.97</td>
</tr>
<tr>
<td>$\mu = 0.26%$</td>
<td>0.104</td>
<td>50.87</td>
</tr>
<tr>
<td>Idiosyncratic Volatility (Base: $\sigma_F = 14.8%$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_F = 18.0%$</td>
<td>0.085</td>
<td>45.11</td>
</tr>
<tr>
<td>$\sigma_F = 12.0%$</td>
<td>0.099</td>
<td>48.33</td>
</tr>
<tr>
<td>Exposure to market risk (Base: $\beta = 0.9$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta = 1.35$</td>
<td>0.086</td>
<td>45.33</td>
</tr>
<tr>
<td>$\beta = 0.5$</td>
<td>0.097</td>
<td>47.84</td>
</tr>
<tr>
<td>Cost of debt issuance (Base: $\lambda = 1.0%$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda = 1.5%$</td>
<td>0.094</td>
<td>47.02</td>
</tr>
<tr>
<td>$\lambda = 0.5%$</td>
<td>0.089</td>
<td>46.80</td>
</tr>
</tbody>
</table>
Table 2  
Variables Definition  
This table presents a description of the empirical variables.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Compustat</strong></td>
<td></td>
</tr>
<tr>
<td>Book Debt</td>
<td>Debt in Current Liabilities (dlcq) + Long Term Debt (dlttq)</td>
</tr>
<tr>
<td>Market Value</td>
<td>Market price (prccq) $\times$ Common Shares Outstanding (cshoq)</td>
</tr>
<tr>
<td>Quasi-Market</td>
<td></td>
</tr>
<tr>
<td>Leverage</td>
<td>Book Debt / (Book Debt + Market Value)</td>
</tr>
<tr>
<td>Operating Profit</td>
<td></td>
</tr>
<tr>
<td>Operating ROA</td>
<td>Operating Profit / Lagged Total Asset (atq)</td>
</tr>
<tr>
<td><strong>CRSP and FED</strong></td>
<td></td>
</tr>
<tr>
<td>Risk free rate</td>
<td>10-Year Constant Maturity Government Bond Yield</td>
</tr>
<tr>
<td>Equity Return</td>
<td>CRSP Total Return (trt1m)</td>
</tr>
<tr>
<td>Excess Return</td>
<td>Equity Returns - Risk-Free Rate</td>
</tr>
<tr>
<td><strong>Moody’s</strong></td>
<td></td>
</tr>
<tr>
<td>Prob. of Default</td>
<td>Average Probability of Default at 5 years by year and rating</td>
</tr>
<tr>
<td>5 Years</td>
<td></td>
</tr>
</tbody>
</table>
Table 3
Descriptive Statistics

This table presents descriptive statistics for the variables used in the estimation. The sample is based on financial statements from Compustat quarterly Industrial Files and returns from CRSP. The Probability of Default are taken from the average default rates provided by Moody’s. Table 2 provides a detailed definition of the moments.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>St.Dev</th>
<th>25th</th>
<th>Median</th>
<th>75th</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Assets (Billions)</td>
<td>8.23</td>
<td>23.94</td>
<td>0.70</td>
<td>1.97</td>
<td>5.93</td>
</tr>
<tr>
<td>Operating ROA (%)</td>
<td>12.18</td>
<td>13.85</td>
<td>8.10</td>
<td>13.30</td>
<td>18.95</td>
</tr>
<tr>
<td>Quasi-Market Leverage (%)</td>
<td>34.98</td>
<td>25.97</td>
<td>13.26</td>
<td>34.38</td>
<td>54.16</td>
</tr>
<tr>
<td>Prob. of Default 5 years (%)</td>
<td>5.43</td>
<td>8.21</td>
<td>0.00</td>
<td>1.72</td>
<td>7.57</td>
</tr>
<tr>
<td>Excess Returns (%)</td>
<td>4.85</td>
<td>91.39</td>
<td>-45.69</td>
<td>4.47</td>
<td>54.37</td>
</tr>
<tr>
<td>Equity Returns (%)</td>
<td>9.57</td>
<td>91.37</td>
<td>-40.91</td>
<td>9.13</td>
<td>58.97</td>
</tr>
</tbody>
</table>
Table 4
Elasticity of Moments to Parameters

This table shows the elasticity of model-implied moments (in columns) with respect to model parameters (in rows). The elasticity of moment $m$ with respect to parameters $p$ is defined as $\frac{dm}{m} \frac{dp}{p}$. The elasticities are calculated at the estimated parameter values from Table 6. Parameter $\sigma_F$ is the idiosyncratic EBIT volatility, $\mu$ is the risk-neutral expected growth rate of EBIT, $\beta$ is the firm’s exposure to market risk, $\gamma$ is the parameter that captures pre-default costs (i.e., constant rate at which the firm loses value when it is in financial distress but has not defaulted yet), $\alpha$ is the parameter that captures the expected loss given default. A description of the moments is provided in Table 2.

<table>
<thead>
<tr>
<th>Moments</th>
<th>Variance of Equity</th>
<th>Operating ROA</th>
<th>Equity Returns</th>
<th>Probability of Default 5 years</th>
<th>Quasi-Market Leverage</th>
<th>Excess Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_F$</td>
<td>8.45</td>
<td>0.02</td>
<td>1.23</td>
<td>11.40</td>
<td>0.42</td>
<td>1.85</td>
</tr>
<tr>
<td>$\mu$</td>
<td>-1.62</td>
<td>-0.58</td>
<td>-0.21</td>
<td>0.49</td>
<td>-0.29</td>
<td>-0.35</td>
</tr>
<tr>
<td>$\beta$</td>
<td>-1.41</td>
<td>-0.01</td>
<td>1.72</td>
<td>-4.83</td>
<td>-0.60</td>
<td>2.51</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1.12</td>
<td>0.00</td>
<td>-0.30</td>
<td>1.35</td>
<td>-0.49</td>
<td>-0.40</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-4.26</td>
<td>0.00</td>
<td>-0.73</td>
<td>-0.48</td>
<td>-0.27</td>
<td>-1.07</td>
</tr>
</tbody>
</table>
Table 5
Simulated Moments Estimation

The estimation is conducted via Simulated Method of Moments (SMM) which searches for the model parameters that minimize the distance between the empirical moments and the moments calculated from a simulated panel of firms. This table shows the empirical moments (column Data) as well as the simulated moments. The $t$-statistics for the difference between empirical and simulated moments are reported in parenthesis. We estimate three different specifications of our model: Model With Pre-Default Costs includes both the parameter $\gamma$ to capture pre-default costs of financial distress and the parameter $\alpha$ that captures the expected loss given default, Model Without Pre-Default Costs exogenously sets $\gamma = 0$ so the costs of financial distress are experienced only at the time of default when the firm is expected to lose a portion $\alpha$ of its value, Not Matching Prob. of Default estimates the same parameters as the Model Without Pre-Default Costs but it excludes the Probability of Default moment (i.e. deviation from the empirical probability of default are not penalized in this specification). The Not Matching Prob. of Default allows for comparability with previous studies such as Glover (2016).

<table>
<thead>
<tr>
<th>Data</th>
<th>Model With Pre-Default Costs</th>
<th>Model Without Pre-Default Costs</th>
<th>Not Matching Prob. of Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quasi Market Leverage</td>
<td>34.980</td>
<td>34.949</td>
<td>35.662</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.301)</td>
<td>(0.011)</td>
</tr>
<tr>
<td>Probability of Default (5Y)</td>
<td>5.428</td>
<td>5.370</td>
<td>5.307</td>
</tr>
<tr>
<td></td>
<td>(0.018)</td>
<td>(0.078)</td>
<td>(50.007)</td>
</tr>
<tr>
<td>Operating ROA</td>
<td>12.178</td>
<td>12.198</td>
<td>12.303</td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.439)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>Variance of Equity</td>
<td>5.218</td>
<td>5.298</td>
<td>6.722</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(4.216)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>Equity Returns</td>
<td>9.570</td>
<td>7.562</td>
<td>9.154</td>
</tr>
<tr>
<td></td>
<td>(0.853)</td>
<td>(0.037)</td>
<td>(0.081)</td>
</tr>
<tr>
<td>Excess Returns</td>
<td>4.855</td>
<td>5.314</td>
<td>6.889</td>
</tr>
<tr>
<td></td>
<td>(0.045)</td>
<td>(0.876)</td>
<td>(0.71)</td>
</tr>
</tbody>
</table>
This table reports the structural parameters estimated via Simulated Method of Moments (SMM). Standard errors are in parentheses. The parameter $\gamma$ captures pre-default costs of financial distress (i.e. constant rate at which the firm loses value when it is in financial distress but has not defaulted yet), $\mu$ captures a firm fixed effect for the expected earnings’ growth rate, $\beta$ is the exposure of the firm’s cash flows to market risk, $\sigma_F$ is a firm fixed effect for the volatility of the expected earnings’ growth rate, $\alpha$ is the parameter that captures the expected loss given default. We estimate three different specifications of our model: *Model With Pre-Default Costs* includes both the parameter $\gamma$ to capture pre-default costs of financial distress and the parameter $\alpha$ that captures the expected loss given default, *Model Without Pre-Default Costs* exogenously sets $\gamma = 0$ so the costs of financial distress are experienced only at the time of default when the firm is expected to lose a portion $\alpha$ of its value, *Not Matching Prob. of Default* estimates the same parameters as the *Model Without Pre-Default Costs* but it excludes the Probability of Default moment (i.e. deviation from the empirical probability of default are not penalized in this specification). The *Not Matching Prob. of Default* allows for comparability with previous studies such as Glover (2016).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Model With Pre-Default Costs</th>
<th>Model Without Pre-Default Costs</th>
<th>Not Matching Prob. of Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma \times 100$</td>
<td>Pre-default costs</td>
<td>6.531</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.013)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha \times 100$</td>
<td>Loss at-default</td>
<td>22.444</td>
<td>55.440</td>
<td>49.818</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.289)</td>
<td>(0.697)</td>
<td>(2.319)</td>
</tr>
<tr>
<td>$\beta \times 100$</td>
<td>Exposure to market risk</td>
<td>89.729</td>
<td>66.309</td>
<td>80.773</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.23)</td>
<td>(0.184)</td>
<td>(23.801)</td>
</tr>
<tr>
<td>$\mu \times 100$</td>
<td>Idiosyncratic component</td>
<td>0.517</td>
<td>0.508</td>
<td>0.523</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>$\sigma_F \times 100$</td>
<td>Volatility of EBIT</td>
<td>14.818</td>
<td>20.251</td>
<td>18.231</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.016)</td>
<td>(0.017)</td>
<td>(2.802)</td>
</tr>
</tbody>
</table>
Table 7
Simulated Moments Estimation: Splits by Durability

This table describes the results for firms split by the durability of their output. Panel A shows the empirical moments, the simulated moments, and the \(t\)-statistics for the difference between empirical and simulated moments. A detailed description of the moments is provided in Table 2. Panel B reports the structural parameters estimated via Simulated Method of Moments (SMM). Standard errors are in parentheses. The parameter \(\gamma\) captures pre-default costs of financial distress (i.e. constant rate at which the firm loses value when it is in financial distress but has not defaulted yet), \(\mu\) captures the risk-neutral expected earnings’ growth rate, \(\beta\) is the exposure of the firm’s EBIT to market risk, \(\sigma_F\) is the idiosyncratic volatility of the expected earnings’ growth rate, \(\alpha\) is the parameter that captures the expected loss given default. Panel C compares the ex-ante value of the firm for the Model With Pre-Default Costs (Full Model) and two counterfactual scenarios: (i) when there are no pre-default costs of financial distress \(\gamma = 0\), and (ii) when there is no expected loss at the time of default \(\alpha = 0\). In column Full Model we standardize the value of the firm to $100.00. In columns \(\gamma = 0\) and \(\alpha = 0\) we report the increase in firm value that would have been if the parameters \(\gamma\) and \(\alpha\) are set to zero, respectively.

Panel A: Model Fit by Durability

<table>
<thead>
<tr>
<th></th>
<th>Quasi-Market Probability of Def. at 5 years</th>
<th>Operating ROA</th>
<th>Variance of Equity</th>
<th>Equity Returns</th>
<th>Excess Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model 33.781</td>
<td>4.635</td>
<td>11.177</td>
<td>5.479</td>
<td>8.960</td>
</tr>
<tr>
<td></td>
<td>(t)-stat 0.195</td>
<td>0.005</td>
<td>0.097</td>
<td>0.037</td>
<td>0.208</td>
</tr>
<tr>
<td>Non Durables</td>
<td>Data 37.118</td>
<td>5.333</td>
<td>13.660</td>
<td>4.586</td>
<td>10.119</td>
</tr>
<tr>
<td></td>
<td>Model 37.879</td>
<td>5.316</td>
<td>13.617</td>
<td>5.380</td>
<td>8.666</td>
</tr>
<tr>
<td></td>
<td>(t)-stat 0.374</td>
<td>0.002</td>
<td>0.051</td>
<td>1.176</td>
<td>0.447</td>
</tr>
</tbody>
</table>

Panel B: Parameter Estimates by Durability

<table>
<thead>
<tr>
<th></th>
<th>(\gamma \times 100)</th>
<th>(\alpha \times 100)</th>
<th>(\beta \times 100)</th>
<th>(\mu \times 100)</th>
<th>(\sigma_F \times 100)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Durables</td>
<td>6.585</td>
<td>13.375</td>
<td>109.304</td>
<td>0.664</td>
<td>16.053</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.727)</td>
<td>(0.121)</td>
<td>(0.001)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Non Durables</td>
<td>6.171</td>
<td>15.187</td>
<td>105.484</td>
<td>0.313</td>
<td>15.464</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.4876)</td>
<td>(0.0576)</td>
<td>(0.0002)</td>
<td>(0.0043)</td>
</tr>
</tbody>
</table>

Panel C: Counterfactuals

<table>
<thead>
<tr>
<th></th>
<th>Full Model</th>
<th>(\gamma = 0)</th>
<th>(\alpha = 0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Durables</td>
<td>Firm Value</td>
<td>100.00</td>
<td>106.61</td>
</tr>
<tr>
<td></td>
<td>% Change</td>
<td>6.61%</td>
<td>0.78%</td>
</tr>
<tr>
<td>Non Durables</td>
<td>Firm Value</td>
<td>100.00</td>
<td>105.77</td>
</tr>
<tr>
<td></td>
<td>% Change</td>
<td>5.77%</td>
<td>1.02%</td>
</tr>
</tbody>
</table>
Table 8
Estimation with Different Distress Boundaries

This table reports the structural parameters and the model fit for the estimation of the Model With Pre-Default Costs for different distress boundary. The Model With Pre-Default Costs includes both the parameter $\gamma$ to capture costs of financial distress experienced prior to bankruptcy and the parameter $\alpha$ that captures the expected costs of default. The distress boundary is denoted with $X_D$, and it represents the level of EBIT below which the firm experiences pre-default costs of financial distress. In this table we estimate four different versions of the Model With Pre-Default Costs with various distress boundaries: $X_D = k \times C$ for $k \in \{2.0, 1.5, 1.0, 0.5\}$. Panel A reports the estimated model parameters with standard errors in parentheses. The parameter $\gamma$ captures pre-default costs of financial distress (i.e. constant rate at which the firm loses value when it is in financial distress but has not defaulted yet), $\mu$ captures a firm fixed effect for the expected earnings’ growth rate, $\beta$ is the exposure of the firm’s cash flows to market risk, $\sigma_F$ is a firm fixed effect for the volatility of the expected earnings’ growth rate, $\alpha$ is the parameter that captures the expected loss given default. Panel B shows the empirical moments, the simulated moments, and the $t$-statistics for the difference between empirical and simulated moments. A detailed description of the moments is provided in Table 2.

Panel A: Parameter Estimates

<table>
<thead>
<tr>
<th>$X_D = 2.0 \times C$</th>
<th>$\gamma \times 100$</th>
<th>$\alpha \times 100$</th>
<th>$\beta \times 100$</th>
<th>$\mu \times 100$</th>
<th>$\sigma_F \times 100$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_D = 1.5 \times C$</td>
<td>3.718</td>
<td>25.511</td>
<td>83.785</td>
<td>0.455</td>
<td>15.948</td>
</tr>
<tr>
<td>$X_D = 1.0 \times C$</td>
<td>4.154</td>
<td>27.718</td>
<td>83.785</td>
<td>0.455</td>
<td>15.646</td>
</tr>
<tr>
<td>$X_D = 0.5 \times C$</td>
<td>6.531</td>
<td>22.434</td>
<td>89.729</td>
<td>0.517</td>
<td>14.818</td>
</tr>
</tbody>
</table>

Panel B: Model Fit

<table>
<thead>
<tr>
<th>$X_D = 2.0 \times C$</th>
<th>Quasi-Market Leverage</th>
<th>Prob. of Def. at 5 years</th>
<th>Operating ROA</th>
<th>Variance of Equity</th>
<th>Equity Returns</th>
<th>Excess Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>34.980</td>
<td>5.428</td>
<td>12.178</td>
<td>5.218</td>
<td>9.570</td>
<td>4.855</td>
</tr>
<tr>
<td>Model</td>
<td>36.589</td>
<td>5.499</td>
<td>12.654</td>
<td>5.883</td>
<td>7.322</td>
<td>5.071</td>
</tr>
<tr>
<td>$X_D = 1.5 \times C$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Data</td>
<td>34.980</td>
<td>5.428</td>
<td>12.178</td>
<td>5.218</td>
<td>9.570</td>
<td>4.855</td>
</tr>
<tr>
<td>Model</td>
<td>34.788</td>
<td>5.314</td>
<td>12.643</td>
<td>5.562</td>
<td>7.528</td>
<td>5.277</td>
</tr>
<tr>
<td>$X_D = 1.0 \times C$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Data</td>
<td>34.980</td>
<td>5.428</td>
<td>12.178</td>
<td>5.218</td>
<td>9.570</td>
<td>4.855</td>
</tr>
<tr>
<td>Model</td>
<td>34.949</td>
<td>5.370</td>
<td>12.198</td>
<td>5.298</td>
<td>7.562</td>
<td>5.314</td>
</tr>
<tr>
<td>$X_D = 0.5 \times C$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Data</td>
<td>34.980</td>
<td>5.428</td>
<td>12.178</td>
<td>5.218</td>
<td>9.570</td>
<td>4.855</td>
</tr>
<tr>
<td>Model</td>
<td>35.012</td>
<td>5.472</td>
<td>12.646</td>
<td>5.116</td>
<td>8.875</td>
<td>6.608</td>
</tr>
</tbody>
</table>

46
This table compares the ex-ante value of the firm for the Model With Pre-Default Costs and two counterfactual scenarios: (i) when there are no pre-default costs of financial distress $\gamma = 0$, and (ii) when there is no expected loss at the time of default $\alpha = 0$. In column Model With Pre-Default Costs we standardize the value of the firm to $100.00$. In columns $\gamma = 0$ and $\alpha = 0$ we report the increase in firm value that there would have been if the parameters $\gamma$ and $\alpha$ are set to zero, respectively. We present the results for different specifications of the distress boundary. The distress boundary is denoted with $X_D$, and it represents the level of EBIT below which the firm experiences pre-default costs of financial distress. We consider four different distress boundaries: $X_D = k \times C$ for $k \in \{2.0, 1.5, 1.0, 0.5\}$.

<table>
<thead>
<tr>
<th>$X_D = k \times C$</th>
<th>Firm Value</th>
<th>$\gamma = 0$</th>
<th>$\alpha = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2 \times C$</td>
<td>100.00</td>
<td>105.00</td>
<td>101.26</td>
</tr>
<tr>
<td>% Change</td>
<td>5.00%</td>
<td>1.26%</td>
<td></td>
</tr>
<tr>
<td>$1.5 \times C$</td>
<td>100.00</td>
<td>104.39</td>
<td>101.36</td>
</tr>
<tr>
<td>% Change</td>
<td>4.39%</td>
<td>1.36%</td>
<td></td>
</tr>
<tr>
<td>$1 \times C$</td>
<td>100.00</td>
<td>105.54</td>
<td>101.15</td>
</tr>
<tr>
<td>% Change</td>
<td>5.54%</td>
<td>1.15%</td>
<td></td>
</tr>
<tr>
<td>$0.5 \times C$</td>
<td>100.00</td>
<td>105.87</td>
<td>101.37</td>
</tr>
<tr>
<td>% Change</td>
<td>5.87%</td>
<td>1.37%</td>
<td></td>
</tr>
</tbody>
</table>
Appendix A  Pricing Kernel

We prove that under the risk neutral measure $\mathcal{Q}$, the firm’s EBIT process is governed by the expression in Equation (3). Recall that the firm’s EBIT under the physical probability space $\mathcal{P}$ follows the process defined in Equation (2). Let the (exogenous) pricing kernel be

$$\frac{d\xi_t}{\xi_t} = -rdt - \varphi dB_{t}^{A,\mathcal{P}}$$

(A.1)

where $\varphi = (\mu_A^P - r)/\sigma_A$ is the market Sharpe ratio, and $B_t^{A,\mathcal{P}}$ is a standard Brownian Motion under the physical probability space.

We define the density process for the risk-neutral measure as

$$\nu_t = \mathbb{E}_t \left[ \frac{d\mathcal{Q}}{d\mathcal{P}} \right]$$

Following Harrison and Kreps (1979), the density process and the pricing kernel are related as follows

$$\nu_t = \xi_t e^{\int_0^t rds} = \xi_t e^{rt}$$

Applying Ito’s lemma we have

$$d\nu_t = e^{rt}d\xi_t + \xi_t re^{rt}dt$$

(A.2)

Substituting Equation (A.1) in Equation (A.2) and recalling that $\xi_t = \nu_t/e^{rt}$

$$d\nu_t = -\varphi \xi_t e^{rt}dB_{t}^{A,\mathcal{P}} \implies \frac{d\nu_t}{\nu_t} = -\varphi dB_{t}^{A,\mathcal{P}}$$

Applying the First Fundamental Theorem of Asset Pricing, we have

$$dB_{t}^{A,\mathcal{Q}} = dB_{t}^{A,\mathcal{P}} + \varphi dt$$

(A.3)

Substituting Equation (A.3) in Equation (2) we obtain that the firm’s EBIT under the risk-neutral
\( Q \) measure is governed by the process

\[
\frac{dX_t}{X_t} = \begin{cases} 
\mu dt + \beta \sigma_A dB_t^{A,Q} + \sigma_F dB_t^F & \text{for } X_B < X_D < X_t \\
(\mu - \gamma)dt + \beta \sigma_A dB_t^{A,Q} + \sigma_F dB_t^F & \text{for } X_B < X_t < X_D 
\end{cases}
\]

(A.4)

The total firm volatility is

\[
\sigma_X = \sqrt{(\beta \sigma_A)^2 + \sigma_F^2}
\]

therefore we can re-write firm’s EBIT process under the risk-neutral measure \( Q \) more compactly as follows

\[
\frac{dX_t}{X_t} = \begin{cases} 
\mu dt + \sigma_X dB_t & \text{for } X_B < X_D \leq X_t \\
(\mu - \gamma)dt + \sigma_X dB_t & \text{for } X_B < X_t < X_D 
\end{cases}
\]

(A.5)

where

\[
dB_t = \frac{\beta \sigma_A}{\sigma_X} dB_t^{A,Q} + \frac{\sigma_F}{\sigma_X} dB_t^F
\]

is a Standard Brownian Motion under the risk-neutral measure. Equation (A.5) is exactly the firm’s EBIT process under the risk neutral measure \( Q \) described in Equation (3).

**Appendix B  Arrow-Debreu securities**

For ease of notation, let \( V(X) \equiv V \) where \( V(X) \) is the unlevered asset value defined in Equation (4). Denote the state when the firm is not distressed, \( X_B < X_D \leq X_t \leq X_U \), as \( ND \) (No Distress state) and denote the state when it is distressed, \( X_B < X_t < X_D < X_U \), as \( DS \) (Distressed State). Let \( p^U_{ND}(X) \) and \( p^U_{DS}(X) \) be the present value of $1 to be received at the time of restructuring, contingent on restructuring occurring before default when the state is \( ND \) and \( DS \), respectively.

Using the standard no-arbitrage argument, these claims must satisfy the following system of partial differential equations (PDEs)

\[
\begin{align*}
\frac{\sigma^2 U}{2} V^2 \frac{\partial^2 p^U_{ND}(\cdot)}{\partial V^2} + \mu V \frac{\partial p^U_{ND}(\cdot)}{\partial V} - r p^U_{ND}(\cdot) &= 0 & \text{for } X_{i,B} < X_{i,D} \leq X_{i,t} \\
\frac{\sigma^2 U}{2} V^2 \frac{\partial^2 p^U_{DS}(\cdot)}{\partial V^2} + (\mu - \gamma) V \frac{\partial p^U_{DS}(\cdot)}{\partial V} - r p^U_{DS}(\cdot) &= 0 & \text{for } X_{i,B} < X_{i,t} < X_{i,D}
\end{align*}
\]

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The general solution to this system of PDEs is

\[
\begin{align*}
\mathbf{p}_U^{ND}(X) &= H_{1,ND}V_1^{\beta_{1,ND}} + H_{2,ND}V_2^{\beta_{2,ND}} \\
\mathbf{p}_U^{DS}(X) &= H_{1,DS}V_1^{\beta_{1,DS}} + H_{2,DS}V_2^{\beta_{2,DS}}
\end{align*}
\]

The constants \( \beta_{1,ND}, \beta_{2,ND}, \beta_{1,DS}, \) and \( \beta_{2,DS} \) are

\[
\begin{align*}
\beta_{1,ND} &= \frac{1}{\sigma^2} \left[ -(\mu - 0.5\sigma^2) + \sqrt{(\mu - 0.5\sigma^2)^2 + 2r\sigma^2} \right] \\
\beta_{2,ND} &= -\frac{1}{\sigma^2} \left[ (\mu - 0.5\sigma^2) + \sqrt{(\mu - 0.5\sigma^2)^2 + 2r\sigma^2} \right] \\
\beta_{1,DS} &= \frac{1}{\sigma^2} \left[ -(\mu - \gamma - 0.5\sigma^2) + \sqrt{(\mu - \gamma - 0.5\sigma^2)^2 + 2r\sigma^2} \right] \\
\beta_{2,DS} &= -\frac{1}{\sigma^2} \left[ (\mu - \gamma - 0.5\sigma^2) + \sqrt{(\mu - \gamma - 0.5\sigma^2)^2 + 2r\sigma^2} \right]
\end{align*}
\]

The constants \( H_{1,ND}, H_{2,ND}, H_{1,DS}, \) and \( H_{2,DS} \) are solved by imposing the following boundary conditions

\[
\begin{align*}
\mathbf{p}_U^{ND}(X_U) &= 1 \\
\mathbf{p}_D^{DS}(X_D) &= \mathbf{p}_U^{DS}(X_B) = 0
\end{align*}
\]

Re-writing the above conditions in matrix form yields the following solution

\[
\begin{bmatrix}
H_{1,ND} \\
H_{2,ND} \\
H_{1,DS} \\
H_{2,DS}
\end{bmatrix} = \mathbf{M}^{-1}.
\]

where

\[
\mathbf{M} = \begin{bmatrix}
V_1^{\beta_{1,ND}} & V_2^{\beta_{2,ND}} & 0 & 0 \\
0 & 0 & V_1^{\beta_{1,DS}} & V_2^{\beta_{2,DS}} \\
V_1^{\beta_{1,ND}} & V_2^{\beta_{2,ND}} & 0 & 0 \\
\beta_{1,ND}V_1^{\beta_{1,ND}} & \beta_{2,ND}V_2^{\beta_{2,ND}} & -\beta_{1,DS}V_1^{\beta_{1,DS}} & -\beta_{2,DS}V_2^{\beta_{2,DS}}
\end{bmatrix}
\]
Let $p_{ND}^B(X)$ and $p_{DS}^B(X)$ be the present value of $1$ to be received at the time of default, contingent on default occurring before refinancing when the state is $ND$ and $DS$, respectively. Using the standard no-arbitrage argument, these claims must satisfy the following PDEs:

\[
\begin{align*}
\frac{\sigma^2}{2} V^2 \frac{\partial^2 p_{ND}^B(\cdot)}{\partial V^2} + \mu V \frac{\partial p_{ND}^B(\cdot)}{\partial V} - r p_{ND}^B(\cdot) &= 0 \quad \text{for } X_{it} \geq X_{i,D} > X_{i,B} \\
\frac{\sigma^2}{2} V^2 \frac{\partial^2 p_{DS}^B(\cdot)}{\partial V^2} + (\mu - \gamma) V \frac{\partial p_{DS}^B(\cdot)}{\partial V} - r p_{DS}^B(\cdot) &= 0 \quad \text{for } X_{i,D} > X_{it} > X_{i,B}
\end{align*}
\]

The general solution to this pair of PDEs is

\[
\begin{align*}
p_{ND}^B(X) &= J_{1,ND} V^{\beta_{1,ND}} + J_{2,ND} V^{\beta_{2,ND}} \\
p_{DS}^B(X) &= J_{1,DS} V^{\beta_{1,DS}} + J_{2,DS} V^{\beta_{2,DS}}
\end{align*}
\]

The constants $J_{1,ND}$, $J_{2,ND}$, $J_{1,DS}$, and $J_{2,DS}$ are solved by imposing the following boundary conditions:

\[
\begin{align*}
p_{ND}^B(X_U) &= 0 & p_{DS}^B(X_B) &= 1 \\
p_{ND}^B(X_D) &= p_{DS}^B(X_D) & \left. \frac{\partial p_{ND}^B(X)}{\partial X} \right|_{X=X_D} &= \left. \frac{\partial p_{DS}^B(X)}{\partial X} \right|_{X=X_D}
\end{align*}
\]

Re-writing the above conditions in matrix form yields the following solution:

\[
\begin{bmatrix}
J_{1,ND} \\
J_{2,ND} \\
J_{1,DS} \\
J_{2,DS}
\end{bmatrix} = \begin{bmatrix}
0 \\
1 \\
0 \\
0
\end{bmatrix} \cdot \begin{bmatrix}
1
\end{bmatrix}
\]

\[\text{(B.3)}\]

**Appendix C  Simulated Method of Moments**

For each firm $i$, we calculate a vector of empirical moments, $h(Y_i)$, using the empirical data $Y_i = [y_{i,1}, y_{i,2}, \cdots, y_{i,T_i}]$ where $T_i$ is the empirical sample length for firm $i$. We use six empirical moments: the operating ROA, the quasi-market leverage, the excess return of firm’s equity with respect to the risk-free rate, the probability of default at 5 years, the variance and returns of equity. We estimate the parameters of the model, $\theta = [\alpha, \gamma, \mu, \sigma_F, \beta]$, using the Simulated Method of Moments.
The SMM searches for the vector of parameters to “fit” the simulated moments to their empirical counterparts. More specifically, we search for the vector of parameters that minimizes the weighted distance between the simulated and empirical moments, $\Lambda(\theta)$:

$$\theta^* = \arg\min_{\theta} \Lambda(\theta)$$  \hspace{1cm} (C.1)

where

$$\Lambda(\theta) = g(\theta)'\hat{W}g(\theta)$$

$$g(\theta) = h(Y_i) - \frac{1}{S}\sum_{s=1}^{S} h(Y_s(\theta))$$

where $S$ is the number of simulations, $\theta$ is the vector of parameters, $Y_k(\theta)$ is the vector of the simulated data for the $k$-th simulation given parameters $\theta$, and $\hat{W}$ is a positive definite weighting matrix. This estimator is known to be asymptotically normal for fixed $S$; for $T_i \to \infty$ the estimator’s asymptotic distribution is

$$\sqrt{T_i}(\theta^* - \theta) \overset{d}{\to} N(0, \text{Var}(\theta^*))$$  \hspace{1cm} (C.2)

where

$$\text{Var}(\theta^*) = \left(1 + \frac{1}{S}\right) \left[D' \cdot \hat{W} \cdot D\right]^{-1}$$

$$D = \frac{\partial g(\theta)}{\partial \theta'} \bigg|_{\theta = \theta^*}$$

$\hat{W}$ is the optimal weighting matrix. The optimal weighting matrix is chosen as to place greater weights on more precisely estimated moments (i.e. moments with lower variance):

$$\hat{W} = \left[\text{Var}(h(Y_i))\right]^{-1}$$  \hspace{1cm} (C.3)

The estimated variance-covariance matrix, $\text{Var}(h(Y_i))$, is calculated using the influence function approach described in Erickson and Whited (2002) which has better finite sample properties as shown in Bazdresch, Kahn, and Whited (2017). This methodology ensures that $\text{Var}(h(Y_i)) \overset{p}{\to} \text{Var}(h(Y_i))$. 

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We conduct the estimation in three steps. First, we build a fine grid containing 20 equally spaced points for each parameter which implies evaluating the model on 3,200,000 points. Second, we use the minimum found in the previous step as the mid-point and we build another grid of 20 equally spaced points for each parameter. Third, we use the results from the previous step as the starting values of a local minimization algorithm (Nelder-Mead) to achieve a more precise minimum.

Appendix D  Subsamples Split by Industry

We finally estimate our model on splits by industry. We define an industry according to their 2-digit SIC code, which results in 8 different industries in our samples. We present the parameter estimates in Table D.1 and the model fit for each industry in Table D.2. Table D.1 shows that there is a large variation across industries. Industries such as Mining, Retail Trade and Services show considerably higher estimated values of pre-default costs compared to Construction, Manufacturing and Wholesale Trade.

Overall, the model is able to fit the various industries quite well. The simulated moments match the empirical moments as shown by $t$-statistics that are mostly below 2 with a few exceptions. There is one industry for which the model struggles: Construction. For this industry, even if the model is able to match the probability of default, the operating ROA and the variance of Equity, it struggles to simultaneously match the empirical leverage of 52.452% and the equity returns of 9.589%.

[Insert Table D.1 and Table D.2 here]

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25 Computations were performed on the Niagara supercomputer at the SciNet HPC Consortium (Ponce et al., 2019; Loken et al., 2010). SciNet is funded by: the Canada Foundation for Innovation; the Government of Ontario; Ontario Research Fund - Research Excellence; and the University of Toronto.
Table D.1
Parameter Estimates: Industry Splits

This table reports the structural parameters estimated via Simulated Method of Moments (SMM) for various industries. Standard errors are in parentheses. The parameter $\gamma$ captures pre-default costs of financial distress (i.e. constant rate at which the firm loses value when it is in financial distress but has not defaulted yet), $\mu$ captures a firm fixed effect for the expected earnings’ growth rate, $\beta$ is the exposure of the firm’s cash flows to market risk, $\sigma_F$ is a firm fixed effect for the volatility of the expected earnings’ growth rate, $\alpha$ is the parameter that captures the expected loss given default.

<table>
<thead>
<tr>
<th>Industry</th>
<th>$\gamma \times 100$</th>
<th>$\alpha \times 100$</th>
<th>$\beta \times 100$</th>
<th>$\mu \times 100$</th>
<th>$\sigma_F \times 100$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture, Forestry, &amp; Fishing</td>
<td>7.500</td>
<td>17.750</td>
<td>50.400</td>
<td>0.400</td>
<td>10.909</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(1.16)</td>
<td>(0.133)</td>
<td>(0.002)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>Construction</td>
<td>4.100</td>
<td>10.300</td>
<td>20.300</td>
<td>0.963</td>
<td>9.273</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.507)</td>
<td>(0.127)</td>
<td>(0.002)</td>
<td>(0.015)</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>4.625</td>
<td>35.600</td>
<td>110.200</td>
<td>0.600</td>
<td>19.273</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.783)</td>
<td>(0.202)</td>
<td>(0.002)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>Mining</td>
<td>8.120</td>
<td>36.500</td>
<td>115.100</td>
<td>0.275</td>
<td>17.545</td>
</tr>
<tr>
<td></td>
<td>(0.026)</td>
<td>(1.86)</td>
<td>(0.224)</td>
<td>(0.002)</td>
<td>(0.007)</td>
</tr>
<tr>
<td>Retail Trade</td>
<td>8.750</td>
<td>17.500</td>
<td>121.100</td>
<td>0.000</td>
<td>14.455</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.564)</td>
<td>(0.168)</td>
<td>(0.002)</td>
<td>(0.007)</td>
</tr>
<tr>
<td>Services</td>
<td>8.129</td>
<td>29.998</td>
<td>79.608</td>
<td>0.511</td>
<td>14.767</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.863)</td>
<td>(0.204)</td>
<td>(0.002)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>Transportation &amp; Public Utilities</td>
<td>5.625</td>
<td>30.625</td>
<td>75.600</td>
<td>0.600</td>
<td>15.727</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.928)</td>
<td>(0.19)</td>
<td>(0.002)</td>
<td>(0.009)</td>
</tr>
<tr>
<td>Wholesale Trade</td>
<td>3.750</td>
<td>21.500</td>
<td>57.500</td>
<td>0.625</td>
<td>13.909</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.656)</td>
<td>(0.192)</td>
<td>(0.002)</td>
<td>(0.021)</td>
</tr>
<tr>
<td>Average</td>
<td>6.325</td>
<td>24.972</td>
<td>78.726</td>
<td>0.497</td>
<td>14.482</td>
</tr>
</tbody>
</table>
Table D.2
Simulated Moments Estimation: Industry Splits

This table shows the empirical moments and the simulated moments for different industries. The $t$-statistics for the difference between empirical and simulated moments are also reported. A detailed description of the moments is provided in Table 2.

<table>
<thead>
<tr>
<th>Industry</th>
<th>Quasi-Market Leverage</th>
<th>Probability of Default at 5 years</th>
<th>Operating ROA</th>
<th>Variance of Equity</th>
<th>Equity Returns</th>
<th>Excess Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture, Forestry, &amp; Fishing</td>
<td>Data 40.287</td>
<td>6.555</td>
<td>12.274</td>
<td>3.903</td>
<td>1.185</td>
<td>-3.668</td>
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<tr>
<td></td>
<td>$t$-stat 0.119</td>
<td>0.028</td>
<td>15.182</td>
<td>0.136</td>
<td>1.208</td>
<td>5.233</td>
</tr>
<tr>
<td>Construction</td>
<td>Data 52.452</td>
<td>8.568</td>
<td>8.954</td>
<td>5.835</td>
<td>9.589</td>
<td>4.891</td>
</tr>
<tr>
<td></td>
<td>Model 45.676</td>
<td>8.954</td>
<td>9.104</td>
<td>5.534</td>
<td>2.996</td>
<td>0.760</td>
</tr>
<tr>
<td></td>
<td>$t$-stat 29.701</td>
<td>0.802</td>
<td>0.636</td>
<td>0.168</td>
<td>9.192</td>
<td>3.611</td>
</tr>
<tr>
<td></td>
<td>$t$-stat 4.711</td>
<td>0.006</td>
<td>0.223</td>
<td>2.045</td>
<td>0.195</td>
<td>0.502</td>
</tr>
<tr>
<td></td>
<td>$t$-stat 0.039</td>
<td>0.376</td>
<td>3.254</td>
<td>0.093</td>
<td>0.025</td>
<td>0.792</td>
</tr>
<tr>
<td>Retail Trade</td>
<td>Data 37.144</td>
<td>6.027</td>
<td>15.514</td>
<td>4.936</td>
<td>10.671</td>
<td>5.831</td>
</tr>
<tr>
<td></td>
<td>Model 36.790</td>
<td>5.843</td>
<td>15.785</td>
<td>5.302</td>
<td>8.689</td>
<td>6.435</td>
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<tr>
<td></td>
<td>$t$-stat 0.081</td>
<td>0.182</td>
<td>2.046</td>
<td>0.250</td>
<td>0.831</td>
<td>0.077</td>
</tr>
<tr>
<td>Services</td>
<td>Data 32.456</td>
<td>6.518</td>
<td>12.212</td>
<td>6.116</td>
<td>7.411</td>
<td>2.842</td>
</tr>
<tr>
<td></td>
<td>Model 33.142</td>
<td>6.502</td>
<td>12.244</td>
<td>6.050</td>
<td>6.527</td>
<td>4.278</td>
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<tr>
<td></td>
<td>$t$-stat 0.304</td>
<td>0.001</td>
<td>0.029</td>
<td>0.008</td>
<td>0.165</td>
<td>0.436</td>
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<tr>
<td>Transportation &amp; Public Utilities</td>
<td>Data 35.538</td>
<td>6.070</td>
<td>11.517</td>
<td>5.893</td>
<td>7.494</td>
<td>2.763</td>
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<td></td>
<td>Model 35.659</td>
<td>6.018</td>
<td>11.631</td>
<td>5.805</td>
<td>7.048</td>
<td>4.805</td>
</tr>
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<td></td>
<td>$t$-stat 0.009</td>
<td>0.015</td>
<td>0.364</td>
<td>0.015</td>
<td>0.042</td>
<td>0.882</td>
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<tr>
<td>Wholesale Trade</td>
<td>Data 42.821</td>
<td>5.561</td>
<td>11.468</td>
<td>4.630</td>
<td>8.497</td>
<td>3.687</td>
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<td>11.452</td>
<td>4.928</td>
<td>6.292</td>
<td>4.047</td>
</tr>
<tr>
<td></td>
<td>$t$-stat 0.966</td>
<td>0.064</td>
<td>0.007</td>
<td>0.165</td>
<td>1.028</td>
<td>0.027</td>
</tr>
</tbody>
</table>