Heterogeneous Beliefs and FOMC Announcements

Abstract

This paper studies the effect of FOMC announcements on the dynamics of investors’ beliefs. I document the open interest decreases significantly after FOMC, which implies investors unwind their positions due to less disagreement. To jointly explain the announcement premium and large trading volume dynamics, I develop a general equilibrium model with heterogeneous beliefs under learning. Upon announcements, investors disagree less on the underlying fundamentals and rebalance their portfolios, which leads to both large trading dynamics and asset prices fluctuations.
1 Introduction

The ultimate objectives of the Federal Open Market Committee (FOMC) announcements are expressed in terms of macroeconomic variables such as output, employment, and inflation. The monetary policy released in the announcements can not affect these macroeconomic variables directly, but it changes people’s beliefs of the future course of the macroeconomy. The dynamics of belief result in large trading behavior and asset prices fluctuations in the financial markets. The immediate impact on the financial market helps policymakers to achieve their ultimate objectives. Therefore, to explore the policy transmission mechanism and its welfare implications, it’s important to understand the effect of the FOMC announcements on the dynamics of people’s beliefs.

In this paper, I demonstrate that the large trading volume of stocks and announcement premium upon announcements provide asset-market-based evidence to capture the dynamics of heterogeneous beliefs. I address this effect in two steps. First, I empirically document the open interest of options and futures decrease significantly after the FOMC announcements, which states that the FOMC announcements reduce investor’s disagreement. Investors unwind their positions, which mainly contribute to the large trading behavior upon announcements. I then develop a heterogeneous beliefs model with learning and announcements under recursive utility. The FOMC announcements carry more precise information about the future course of the macroeconomy, which reduces investor’s disagreements. Investors trade a lot to rebalance their portfolio so that the asset prices react to the monetary policy immediately. Overall, using the asset-market-based evidence, the paper studies the dynamics of investor’s beliefs upon the FOMC announcements within a general equilibrium heterogeneous beliefs framework.

Empirically, the asset prices and the trading volume response to the FOMC announcements immediately. During the period of 1961-2014, about 55% of the market equity premium is realized on about 30 days per year with significant macroeconomic announcements. Using TAQ data, I compare the one-minute trading volume of SPX on days without FOMC announcements and with FOMC announcements. Before the announcements, the trading volume is similar as days without announcements. When the announcements are released, the trading volume increases around 4 times immediately and keeps at a higher level than that without announcements. However, whether
the disagreements become larger or smaller, both of them can lead to the large trading volume. If the disagreements become larger, the investors will invest in their previous positions more aggressively. If the disagreements become smaller, the investors will trade a lot to unwind their positions. Since the two explanations are totally opposite, which have different policy transmission mechanism and welfare implications, it’s important to find empirical support to distinguish between these two.

I motive my approach with the new fact on the responses of option and future to the FOMC announcements. Using data of open interest of SPX, I find that both the call and put open interest decrease around 4% at the first day after the announcements and 7% at the following day from 1996 to 2018. The pattern is stronger for the past five years (2014-2018): the open interest decreases around 13%. This result indicates the FOMC announcements are more informative during the recent years. I study the open interest of other financial assets (such as the 30-day federal funds futures) and find the same results hold. Therefore, the FOMC announcements reduce investor’s disagreements so that they unwind their positions and trade to reblance their portfolio.

To study the belief dynamics using asset-market-based evidence, I then develop a general equilibrium model with heterogeneous beliefs under learning and announcements. The agents differ in their beliefs about the growth rate of aggregate output. The agent’s share holding is determined by two elements: the agent’s portfolio share invested in the risky asset and the agent’s wealth share in the economy. When the agent becomes more optimistic comparing to other agents in the economy, he invests larger share of his wealth in the stock market since he expects the excess return of stock is higher, which is the \textit{portfolio share channel}. In the meantime, agent consumes much more so that the agent accumulates less wealth when the disagreement increases. I call this as the \textit{wealth accumulation channel}. The uncertainty, measured as the posterior variance of the expected growth rate also affects the share holding through the above two channels. The optimistic agent invests larger proportion of his wealth into the stock market when he is more certain about his optimistic belief. While in the meantime, he consumes more when his uncertainty becomes smaller, which lowers the wealth accumulation. Therefore, the model implies that the stock holding is not only determined by the investor’s disagreements, uncertainty but also their relative wealth.
On days without announcements, the agents learn from the output and update their beliefs. The FOMC announcements carry more precise information about the future course of the macroeconomy, which reduces investor’s disagreements significantly. Since the wealth share does not change upon the announcements, the huge decrease in the disagreements and uncertainty mainly affects the share holdings through the portfolio share channel. Even though the smaller uncertainty leads to a larger (smaller) share holding of the optimistic (pessimistic) agent, the smaller disagreements reduce (increases) the optimistic (pessimistic) agent’s share holding way significantly. Therefore, the agents trade to rebalance their portfolio upon the announcements.

Since the agent has recursive preference under the early resolution of uncertainty, macro announcements result in non-trivial reductions of uncertainty, and are associated with realizations of a substantial amount of equity premium. The smaller disagreements after the announcements also contribute to the large announcement premium. For the beliefs on days without announcements, I map and calibrate it to forecasts of real GDP from the Survey of Professional Forecasters (SPF). Therefore, combing with the asset-market-based evidence upon announcements, I can estimate the belief dynamics and study the welfare implications.

**Related literature**

This paper builds on the literature on heterogeneous beliefs. It is well known that deriving dynamic equilibrium models with heterogeneous agents is a major challenge. Basak (2000, 2005) analyzed a similar pure-exchange economy with agents having CRRA functions and heterogeneous beliefs about both fundamental and non-fundamental variables. Basak used the representative-agent approach with stochastic weights for the respective agents to derive the equilibrium. Borovička (2018) studies long run survival of agents with heterogeneous beliefs under Epstein-Zin preferences. He emphasizes the role of higher inter-temporal elasticity of substitution (IES) (relative to risk aversion) to guarantee that the agent with possibly incorrect beliefs can “save” their way out of immiseration in the long run. However, he assumes constant distorted beliefs and does not study the asset pricing implications, such as the equity premium and trading volume.

My paper contributes to the macro announcements literature both empirically and theoretically.
Savor and Wilson (2013) document that stock market returns and Sharpe ratios are significantly higher on days with macroeconomic news releases in the U.S. Lucca and Moench (2015) provide evidence for the FOMC announcement day premium a pre-FOMC announcement drift. From the theoretical perspective, Ai and Bansal (2018) take a revealed preference approach and establish the equivalence between the announcement premium and generalized risk sensitivity in a representative agent economy. However, all of them only focus on the asset prices and return upon the announcements. To explain the large trading volume, I document that the open interest decreases significantly after the FOMC announcements, which indicates the monetary policy released in the announcements reduces the investor’s disagreement. I also develop a model with heterogeneous agents that differ in their beliefs to reconcile the price and trading volume dynamics.

This paper is also related to literature that studies the effects of policy changes on asset price. Kuttner (2001) uses daily data on federal funds futures prices to identify target rate surprises from the change in the current-month futures rate. He finds that bond yields’ response to anticipated changes is essentially zero, while their response to unanticipated movements is large and highly significant. Bernanke and Kuttner (2005) use the same method to study the monetary policy implications for equity returns. They find that an unanticipated cut to the Fed fund rate results in large positive shocks to the stock market. None of them try to capture the dynamics of investor’s beliefs, which is the key to the asset prices fluctuations.

The rest of the paper is organized as follows. I document three stylized facts for the trading volume and open interest upon the FOMC announcements in Section 2. In Section 2, I present a general equilibrium model with heterogeneous beliefs under learning and announcements. Section 4 analyzes the social planner problem and captures the optimal allocations. In Section 5, I study how the disagreements and uncertainty affect the trading volume and the announcement premium. In sections 6 and 7, I discuss the calibration strategies and present the main quantitative results. Section 8 concludes.
2 Stylized facts

It has been well known in the literature that there is a large fraction of the market equity premium is realized on a relatively small number of trading days with pre-scheduled macroeconomic announcements (Savor and Wilson (2013), Ai, Bansal, Im, and Ying (2018)). In this section, I document three new facts to highlight other differences between days with FOMC announcements and days without FOMC announcements, which provides empirical support for the dynamics of heterogeneous beliefs and uncertainty upon announcements. I provide the details about the data construction in Appendix A.

1. The trading volume of the stock immediately increases a lot upon announcements.

![Figure 1: This figure shows the intraday trading volume for the SPY ETF on FOMC announcement days (blue line) and on non-FOMC announcement days (red line). The trading volume is calculated by the total traded share in one minute. The black line shows the time (14:15 pm) when the FOMC announcements usually come out.](image)

In Figure 1, I plot the one-minute trading volume of SPX on days without FOMC announce-
ments and with FOMC announcements. Most of the FOMC announcements before 2013 are released at 14:15 pm. I only keep these announcements for Figure 1 so that the response to the news is clear. Before the announcements, the trading volume is smaller as days without announcements. However, when the announcements are released, the trading volume increases around 4 times immediately and keeps at a higher level than that without announcements.

The huge increase in trading volume demonstrates that investor’s beliefs change a lot when the public information in the announcements are released. However, there are two possible opposite explanations for this phenomenon. One explanation is that, investors interpret the public news differently so that their disagreements after the announcements increase so that investors invest more aggressively in their previous positions. The second explanation is that, investors disagree less since they update their beliefs after the public news. They rebalance their portfolio, which leads to the large trading volume. The two explanations have different information dynamics and welfare implications of the macroeconomic announcements. Therefore, it’s very important to distinguish which one contributes more when the announcements come. Using the open interest of options and futures, I document that, the disagreements after announcements become smaller.

Figure 2: This figure shows the call open interest of SPX around FOMC announcements from 1996 to 2017 and from 2012 to 2017.
2. The open interest of options and futures is smaller after the FOMC announcements.

Figure 2 illustrates that the average call and put open interest of SPX before and after the FOMC announcements. Both the call and put open interest of SPX decreases a lot on the day after the announcements and it keeps decreasing. This facts shows that the disagreements become smaller after the announcements since the investors unwind their positions instead of investing more aggressively. In Table 1, I report that the open interest decreases around 4% at the first day after the announcements and around 7% at the following day from 1996 to 2018. This effect is more obvious if I only look at the past 5 years. The open interest decreases around 13% at the first day after the announcements, which means the FOMC announcements are more informative in the recent years so that the disagreements decrease more after the announcements.

Figure 3: This figure shows the open interest of 30 day federal funds futures around FOMC announcements from 1996 to 2018.

In the monetary policy literature, they use the 30-day federal funds futures to measure the monetary policy surprise since it’s very informative to the announcements (such as Kuttner
I document that the open interest of 30-day federal funds futures also decrease after the macro announcements as Figure 3 shows.

3 Economy with heterogeneous beliefs

I consider a continuous-time, pure-exchange security market economy with a infinite horizon \([0, \infty]\). There is a single consumption good which serves as the numeraire. Agents have identical preferences but differ in their beliefs about the endowment growth rate.

3.1 Information structure and heterogeneous beliefs

The stochastic structure of the economy is given by a filtered probability space \((\Omega, \mathcal{F}, \{\mathcal{F}_t\}, P)\) with an augmented filtration defined by a family of \(\sigma\)-algebras \(\{\mathcal{F}_t\}, t \geq 0\) generated by a univariate Brownian motion \(W = [B_{Y,t}, B_{\theta,t}]\). \(\{\mathcal{F}_W\}\) denotes the augmented filtration generated by \(W(t)\), and \(\mathcal{H}\) is a \(\sigma\)-field independent of \(\{\mathcal{F}_W\}\). The field \(\mathcal{H}\) whose role is to allow for heterogeneity in agents’ priors consists of all possible initial beliefs. The complete information filtration \(\{\mathcal{F}_t\}\) is the augmentation of the filtration \(\mathcal{H} \times \{\mathcal{F}_W\}\).

Two agents \((i = 1, 2)\) commonly observe the dynamics of aggregate endowment \(Y_t\) and the volatility \(\sigma_Y\), but the growth rate \(\theta_t\) is not observed by the investors and they have different expected long-run growth rate \(\bar{\theta}^i\). The aggregate endowment process \(Y\) satisfies

\[
\frac{dY_t}{Y_t} = \theta_t dt + \sigma_Y dB_{Y,t}, \quad Y_0 > 0
\]

with constant \(\sigma_Y\) and the growth rate \(\theta_t\) follows

\[
d\theta_t = \rho (\bar{\theta}^i - \theta_t) dt + \sigma_{\theta} dB_{\theta,t}
\]

where \(\rho > 0\) determines the mean reversion rate of the persistent shock and the standard Brownian motion \(B_{\theta,t}\) is independent of \(B_{Y,t}\).

The investors form optimal estimations of the growth rate by filtering the aggregate endowment \(Y_t\) through the incomplete information filtration \(\mathcal{F}_W \subset \mathcal{F}_t, t \geq 0\). Particularly, the prior beliefs
about \( \theta_t \) at time \( t = 0 \) are heterogeneous for each investor and, thus, is \( \mathcal{H} \)-measurable. The beliefs of the agents about \( \theta_t \) are updated in a Bayesian fashion, via \( m^i(t) = E^i[\theta_t | \mathcal{F}_t] \), where \( E^i[\cdot] \) denotes the expectation relative to the subjective probability measure \( P^i \), which is equivalent to the true measure \( P \) (which may disagree on \( \mathcal{H} \), so that investors have heterogeneous prior beliefs).

**Timing of information and Bayesian learning** The pre-scheduled announcements come every \( T \) days. At time 0, the agent’s prior belief about \( \theta_0 \) can be represented by a normal distribution with mean \( m^i(0) \) and variance \( Q^i(0) \). On the days without announcements \( t \in ((n-1)T, nT) \), investor \( i \) updates his beliefs based on the observed endowment process using the Kalman-Bucy filter (Liptser and Shiryaev (2001)):

\[
    dm^i(t) = \rho (\bar{\theta}^i - m^i(t)) dt + \frac{Q^i(t)}{\sigma_y} d\tilde{B}^i_{Y,t} \\
    = \rho (\bar{\theta}^i - m^i(t)) dt + \frac{Q^i(t)}{\sigma_y^2} \left[ \frac{dY_t}{Y_t} - m^i(t) dt \right], \quad i = 1, 2. \\
\]

and the posterior variance \( Q^i(t) = E^i[\theta_t - m^i(t)]^2 | \mathcal{F}_t] \) follows \(^1\)

\[
    dQ^i(t) = \left[ \sigma_y^2 - 2\rho Q^i(t) - \frac{1}{\sigma_y^2} Q^i(t)^2 \right] dt
\]

The innovation process of each agent is such that given his perceived growth rate, \( m^i(t) \), the observed aggregate endowment obeys

\[
    dY_t = Y_t \left[ m(t) dt + \sigma_y d\tilde{B}^i_{Y,t} \right] = Y_t \left[ m^i(t) dt + \sigma_y d\tilde{B}^i_{Y,t} \right], \quad i = 1, 2.
\]

and hence indeed “agrees” with the process he observes. Furthermore, the investors information and innovation filtration coincide. Equation (2) implies that that agent \( i \), views the evolution of the Brownian motion \( W \) as distorted by a drift component \( \bar{\mu}^i(t) \), i.e.

\[
    d\tilde{B}^i_{Y,t} = \bar{\mu}^i(t) dt + d\tilde{B}^i_{Y,t}, \quad \bar{\mu}^i(t) = \frac{m^i(t) - m(t)}{\sigma_y}
\]

\(^1\)It’s easy to show if both agents have the same posterior variance at period 0, i.e. \( Q^1(0) = Q^2(0) \), they will always have the same posterior variance along the path.
where $\tilde{B}_{Y,t}$ is a Brownian motion under $P^i$ by Girsanov’s theorem. Consequently, the aggregate endowment is perceived to contain an additional drift component $\tilde{\mu}^i(t) \sigma_y$, and $\tilde{\mu}^i(t)$ can be interpreted as a degree of optimism (if $\tilde{\mu}^i(t) > 0$) or pessimism (if $\tilde{\mu}^i(t) < 0$) about aggregate endowment. On the days with announcements, $t \in \{nT, n \geq 1\}$, additional signals about $x_t$ are revealed through announcements. For $n = 1, 2, 3, \cdots$, we denote $s_n$ as the signal observed at time $nT$ and assume

$$s_n = \theta_{nT} + \zeta_n$$

, where $\zeta_n$ is i.i.d. over time, and normally distributed with mean zero and variance $\sigma^2_S$. The agent updates his beliefs using Bayes’ rule:

$$m^i_{nT} = Q^i_{nT} \left[ \frac{1}{\sigma^2_S} s_n + \frac{1}{Q^i_{nT}} m^i_{nT} \right] ; \quad \frac{1}{Q^i_{nT}} = \frac{1}{\sigma^2_S} + \frac{1}{Q^i_{nT}} \tag{7}$$

where $m^i_{nT}$ and $Q^i_{nT}$ are the posterior mean and variance after announcements, and $m^i_{nT}$ and $Q^i_{nT}$ are the posterior mean and variance before announcements, respectively. A special case is that when $\sigma^2_S = 0$, the announcements can completely reveal the information about $x_t$ so that $m^i_{nT} = \theta_{nT}$.

**Proposition 1.** The disagreement is $m^1(t) - m^2(t)$ is deterministic with explicit solution when the two agents have the same prior variance: (i) during the days without announcements, the law of motion of the disagreement is\(^2\)

$$dm^1_t - dm^2_t = \left[ \frac{\rho (\tilde{\theta}^1 - \tilde{\theta}^2)}{\sigma^2_S} - \left( \rho + \frac{Q^i(t)}{\sigma^2_y} \right) (m^1_t - m^2_t) \right] dt \tag{8}$$

(ii) When the announcement comes, the disagreement becomes smaller and follows

$$m^1_{nT} - m^2_{nT} = \frac{Q^+_{nT}}{Q^-_{nT}} \left( m^1_{nT} - m^2_{nT} \right) < m^1_{nT} - m^2_{nT} \tag{9}$$

\(^2\)The disagreement is $m^1_t - m^2_t = (m^1_{t_0} - m^2_{t_0}) e^{-\int_0^t (\rho + \frac{Q^i(s)}{\sigma^2_y}) ds} + \rho (\tilde{\theta}^1 - \tilde{\theta}^2) \left[ 1 - e^{-\int_0^t (\rho + \frac{Q^i(s)}{\sigma^2_y}) ds} \right]$.
Theorem 1 shows that on the day without announcements, the disagreements have two components: the learning part will reduce the disagreements while the new add part will increase the disagreements. On the days with announcements, the public news will reduce the disagreements a lot since the agents observe the same signal and update their beliefs using Bayes’ rule.

3.2 The Asset Market

The assets are modeled as in the Lucas (1978) tree economy. I normalize the total supply of risky assets to be one unit. The riskless bond is in zero net supply. I denote the processes \( \{ P_t \} \) and \( \{ r_t \} \) as the risky asset price and interest rate processes, respectively.

When \( t \in \left( (n-1)T, nT \right) \), the total return on the risky asset is

\[
dR_t = \frac{Y_t dt + dP_t}{P_t} = \mu_{R,t} dt + \sigma_{R,t} dY_t
\]

\[
= \mu_{R,t}^i dt + \sigma_{R,t}^i dY_t^i, \quad (10)
\]

where \( \mu_{R,t}^i \) and \( \sigma_{R,t}^i \) are endogenously determined in the equilibrium, which represents the risky security dynamics as perceived by investor \( i \). At announcements \( t = nT \),

\[
\frac{R_t^i}{R_t} = \frac{P_t^i}{P_t}
\]

which is the same for both agents given the agreements on the price. The next theorem captures the relationship of returns between the two agents.

**Proposition 2.** The price-agreement across investors imply that on days without announcements:

(i) the agents have the same perceived volatility of returns, which means

\[
\sigma_{R,t} \equiv \sigma_{R,t}^i, \quad t \in \left( (n-1)T, nT \right)
\]

(ii) the perceived return mean follows:

\[
\mu_{R,t}^i = \mu_{R,t} + \bar{\mu}^i(t) \sigma_{R,t}, \quad t \in \left( (n-1)T, nT \right)
\]

The difference of the perceived return is captured by the disagreement \( \bar{\mu}^i(t) \), which is given by differences of opinion about the expected endowment growth rate.
3.3 Agent’s preference and optimization

I assume that both agents are endowed with a Kreps-Porteus preference with risk aversion \( \gamma \) and intertemporal elasticity of substitution \( \psi \). In continuous time, the preference is represented by a stochastic differential utility, which can be specified by a pair of aggregators \( (f, \varphi) \) such that in the interior of \((nT, (n+1)T)\),

\[
dV_t = [-f(C_t, V_t) - \frac{1}{2} \varphi'(V_t) ||\sigma_V(t)||^2]dt + \sigma_V(t)dB_t
\]

We adopt the convenient normalization \( \varphi(v) = 0 \), and denote \( \tilde{f} \) the normalized aggregator. Under this normalization, \( \tilde{f}(C,V) \) is:

\[
\tilde{f}(C,V) = \frac{\beta}{1 - \frac{1}{\psi}} \frac{C^{1 - 1/\psi} - ((1 - \gamma)V)^{1 - 1/\psi}}{((1 - \gamma)V)^{1 - 1/\psi} - 1}.
\]

The case of \( \psi = 1 \) is obtained as the limit of (12) with \( \psi \to 1 \):

\[
\tilde{f}(C,V) = \beta V [(1 - \gamma)\ln C - \ln [(1 - \gamma)V]].
\]

Because announcements typically result in discrete jumps in the posterior belief about \( \theta_t \), the value function is typically not continuous at announcements. For \( t = nT \), the pre-announcement utility and post-announcement utility are related by

\[
V_{i,-}^t = E_{i,-}^t \left[V_{i,+}^t | \varphi_{i,-}^t \right]
\]

where \( E_{i,-}^t \) represents agent \( i \)'s expectation with respect to the pre-announcement information at time \( t \).

Both agents choose his consumption rate and the portfolio decision to solve

\[
V^i = \max_{\{C^i_t, \pi^i_t\}} E^i \left[ \int_0^\infty \tilde{f}(C^i_s, V^i_s)ds \right]
\]

s.t. \( \frac{dW^i_t}{W^i_t} = \left[ r_t + \pi^i_t (\mu_{R,t} - r_t) - \frac{C^i_t}{W^i_t} \right] dt + \pi^i_t \sigma_{R,t} dB^i_{Y,t} \)

where \( \pi^i_t \) is the ratio of the risky asset holdings of agent \( i \) to its total wealth \( W^i_t \).

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3.4 The equilibrium

Definition. An equilibrium is a set of price processes \( \{P_t\} \) and \( \{r_t\} \), and decisions \( \{C_{1,t}, C_{2,t}, \pi^1_t, \pi^2_t\} \) such that

1. Given the price processes, decisions solve the consumption-savings problems of both agents (14).

2. When the announcement come, the boundary conditions

\[
V^i_\tau^- = E_t^i \left[ V^i_\tau^+ | \mathcal{F}_t^- \right]
\]

(15)

hold for both agents.

3. The risky asset market clears

\[
\pi^1_t W_{1,t} + \pi^2_t W_{2,t} = P_t
\]

(16)

4. The goods market clears:

\[
C_{1,t} + C_{2,t} = Y_t
\]

(17)

Given market clearing in risky asset and goods markets, the bond market clears by Walras’ law. Finally, an equilibrium relation that proves useful when deriving the solution is that

\[
W_{1,t} + W_{2,t} = P_t
\]

(18)

That is, since bonds are in zero net supply, the wealth of both agents must sum to the value of the risky asset.

4 Planner’s problem and optimal allocations

Since the welfare theorems hold in the economy, in this section, I show how I solve optimal allocations from the social planner’s problem and characterize the conditions for the existence of nondegenerate long-run equilibria.
4.1 The planner’s problem

I utilize a characterization based on the more general variational utility approach studied by Geoffard (1996) and Dumas, Uppal, and Wang (2000). They show that recursive preferences for each agent $i$ can be represented as a solution to the maximization problem

$$\lambda_i^t V_i^t (C^i) = \sup_{\{v^i_s\}_{s \geq t}} E_t \left[ \int_t^\infty \lambda_s^i F (C^i_s, v^i_s) \, ds \right]$$

(19)

subject to

$$d\lambda^i_t = -v^i_t \lambda^i_t \, dt, \quad \lambda^i_0 > 0$$

(20)

where $v^i$ is the discount rate process, and $\lambda^i$ is the discount factor process. For the case of the Duffie–Epstein–Zin preferences, the felicity function $F (C, v)$ is given by

$$F (C, v) = \beta \frac{C^{1-\gamma}}{1-\gamma} \left( \frac{1 - \frac{1-\gamma}{1-\psi} \frac{v}{1-\gamma} \frac{1-\gamma}{1-\psi}}{1 - \frac{1-\gamma}{1-\psi}} \right)^{1-\frac{1-\gamma}{1-\psi}}$$

I follow Dumas et al. (2000) and Borovička (2018) and introduce a fictitious planner who maximizes a weighted average of the continuation values of the two agents. The planner’s value function is the solution to the problem

$$J_0 (\lambda^1_0, \lambda^2_0, Y_0, m_0) = \sup_{(C^1, C^2)} \lambda^1_0 V^1_0 (C^1) + \lambda^2_0 V^2_0 (C^2)$$

(21)

$$= \sup_{(C^1, C^2, v^1, v^2)} \left\{ E_0^2 \left[ \int_0^\infty \lambda_s^1 F (C^1_s, v^1_s, t) \, ds \right] + E_0^2 \left[ \int_0^\infty \lambda_s^2 F (C^2_s, v^2_s, t) \, ds \right] \right\}$$

subject to

$$d\lambda^2_t = -v^2_t \lambda^2_t \, dt, \quad \lambda^2_0 > 0$$

$$d\lambda^1_t = \lambda^1_t \left[ -v^1_t \, dt + (\mu^1 (t) - \bar{\mu}^2 (t)) \, dB^2_t \right], \quad \lambda^1_0 > 0 \text{ given } \mu^1 (0), \bar{\mu}^2 (0)$$
\[
\frac{dY_t}{Y_t} = m(t)\, dt + \sigma_y dB^2_{Y,t} \\
\frac{dm(t)}{m(t)} = \rho (\bar{\theta} - m(t))\, dt + \frac{Q(t)}{\sigma_y} d\tilde{B}^2_{Y,t} \\
C_t^1 + C_t^2 = Y_t
\]

where I write the planner’s objective function under the pessimist’s probability measure, without loss of generality. For \( t = nT \), the pre-announcement utility and post-announcement utility are related by

\[
J_t^- = E_t^{2^+} [J_t^+ | \mathcal{F}_t^-]
\]

### 4.2 The HJB equation

Due to the homogeneity of the value function,

\[
J(\lambda_1, \lambda_2, Y, m, t) = Y^{1-\gamma} (\lambda_1 + \lambda_2) \tilde{J} \left( \frac{\lambda_1}{\lambda_1 + \lambda_2}, m, t \right) = Y^{1-\gamma} \theta_2 \tilde{J} (\theta^1, m, t)
\]

where \( \theta^1 = \frac{\lambda_1}{\lambda_1 + \lambda_2} \) and \( \theta^2 = \lambda_1 + \lambda_2 \). \( \theta^1 \) represents the Pareto share of agent 1, which is obviously bounded between zero and one.

**Proposition 3.** \( \tilde{J}(\theta^1, m, t) \) is the solution of the nonlinear partial differential equation

\[
0 = \theta^1 \frac{\beta}{1 - \frac{1}{\psi}} (\xi^1)^{\frac{1}{1 - \psi}} ((1 - \gamma) \tilde{J}^1)^{\frac{1}{1 - \psi}} + (1 - \theta^1) \frac{\beta}{1 - \frac{1}{\psi}} (1 - \xi^1)^{\frac{1}{1 - \psi}} ((1 - \gamma) \tilde{J}^2)^{\frac{1}{1 - \psi}} + \tilde{J}_t + \left( -\frac{\beta (1 - \gamma)}{1 - \frac{1}{\psi}} + m (1 - \gamma) + \theta^1 (\mu^1(t) - \mu^2(t)) (1 - \gamma) \sigma_y - \frac{1}{2} \gamma (1 - \gamma) \sigma_y^2 \right) \tilde{J} \\
+ \theta^1 (1 - \theta^1) (\mu^1(t) - \mu^2(t)) (1 - \gamma) \sigma_y \tilde{J}_{\theta^1} + \frac{1}{2} (1 - \theta^1)^2 (\theta^1)^2 (\mu^1(t) - \mu^2(t))^2 \tilde{J}_{\theta^1 \theta^1} \\
+ \left[ \rho (\bar{\theta} - m) + (1 - \gamma) Q(t) + \theta^1 (\bar{\mu}^1(t) - \bar{\mu}^2(t)) \frac{Q(t)}{\sigma_y} \right] \tilde{J}_m + \theta^1 (1 - \theta^1) (\bar{\mu}^1(t) - \bar{\mu}^2(t)) \frac{Q(t)}{\sigma_y} \tilde{J}_{\theta^1 m} + \frac{1}{2} \left( \frac{Q(t)}{\sigma_y} \right)^2 \tilde{J}_{mm}
\]

with the following boundary conditions
(i) \( \tilde{J}(0,m,t) = H^2(m,t) \) and \( \tilde{J}(1,m,t) = H^1(m,t) \), where \( H^n(m,t) \) are the solution to the following equation

\[
0 = \frac{\beta}{1-\frac{1}{\psi}} \left[ (1-\gamma) H \right]^{1-\frac{1}{\psi}} + H_t + \left[ -\frac{\beta (1-\gamma)}{1-\psi} + (1-\gamma) (m + \sigma_i \Pi_j (t)) - \frac{1}{2} \gamma (1-\gamma) \sigma^2_{\gamma} \right] H \\
+ \left[ \rho (\bar{\theta} - m) + (1-\gamma) Q(t) + \Pi_j (t) \frac{Q(t)}{\sigma_{\gamma}} \right] H_m + \frac{1}{2} \frac{Q^2(t)}{\sigma^2_{\gamma}} H_{mm}
\]

(23)

where \( \bar{\Pi}_1 (t) = \bar{\mu}_1 (t) - \bar{\mu}_2 (t) \) and \( \bar{\Pi}_2 (t) = 0 \).

(ii) For all \( n = 1,2,... \)

\[
J(\theta^1, m^-, nT) = E^{2-}_{nT} \left[ J(\theta^1, m^+, nT) | \mathcal{F}_{nT} \right]
\]

The functions \( \tilde{J}^n(\theta^1, m, t) \) are the continuation values of two agents scaled by \( Y^{1-\gamma} \),

\[
\tilde{J}^1(\theta^1, m, t) = \tilde{J}(\theta^1, m, t) + (1-\theta^1) \tilde{J}^1(\theta^1, m, t) \\
\tilde{J}^2(\theta^1, m, t) = \tilde{J}(\theta^1, m, t) - \theta^1 \tilde{J}^1(\theta^1, m, t)
\]

and the consumption share \( \zeta^1 \) is given

\[
\zeta^1(\theta^1, m, t) = \frac{(\theta^1)^{\psi} \left[ (1-\gamma) \tilde{J}^1(\theta^1, m, t) \right]^{\frac{1-\psi}{1-\gamma}}}{(\theta^1)^{\psi} \left[ (1-\gamma) \tilde{J}^1(\theta^1, m, t) \right]^{\frac{1-\psi}{1-\gamma}} + (1-\theta^1)^{\psi} \left[ (1-\gamma) \tilde{J}^2(\theta^1, m, t) \right]^{\frac{1-\psi}{1-\gamma}}}
\]

and agent 2’s consumption share \( \zeta^2(\theta^1, m, t) = 1 - \zeta^1(\theta^1, m, t) \).

Figure 4 shows the value function \( \tilde{J}(\theta^1, m, t|t = 15) \) and policy function - agent 1’s consumption share \( \zeta^1(\theta^1, m, t|t = 15) \) for different current growth rate \( m \). The limit of the value function and policy function converges to the homogeneous economy where there is only one agent. The agent1’s consumption share is an increasing function in the Pareto share.
4.3 The long-run equilibrium and survival

In this section, I characterize the sufficient condition so that both agents can survive in the long run. The intuition is similar as Borovicka (2018) and I extends his results under non-constant belief distortions.

The law of motion of the Pareto share of agent 1 $\theta^1$ follows

$$d\theta^1 = \theta^1 (1 - \theta^1) \left[ \lambda_1^2 - \lambda_1^1 - \theta^1 (\bar{\mu}^1 (t) - \bar{\mu}^2 (t))^2 \right] dt + \theta^1 (1 - \theta^1) (\bar{\mu}^1 (t) - \bar{\mu}^2 (t)) dB^2_{Y,t}$$

$$\approx \mu_{\theta^1} (\theta^1, m, t) dt + \sigma_{\theta^1} (\theta^1, m, t) dB^2_{Y,t}$$

(24)

Proposition 4. Both agents will survive in the long run if one of the following two conditions holds

(i) $\lim_{\theta^1 \searrow 0} \left[ \min_{t} \mu_{\theta^1} (\theta^1, m, t \mid m = \bar{\theta}) \right] < 0$ and $\lim_{\theta^1 \nearrow 1} \left[ \max_{t} \mu_{\theta^1} (\theta^1, m, t \mid m = \bar{\theta}) \right] > 0$ (25)

(ii) $\lim_{\theta^1 \searrow 0} \left[ \max_{t} \mu_{\theta^1} (\theta^1, m, t \mid m = \bar{\theta}) \right] > 0$ and $\lim_{\theta^1 \nearrow 1} \left[ \min_{t} \mu_{\theta^1} (\theta^1, m, t \mid m = \bar{\theta}) \right] < 0$ (26)

The intuition of the proof is very simple, as long as the drift of $\theta^1$ is not always positive or negative, for strictly positive initial weights, the boundaries are unattainable, so that $\theta^1$ evolves on
the open interval \((0,1)\). Therefore, both agents can survive in the long run. Figure 1 illustrates that the long-run distribution at a given \(t\) days after announcements in my baseline calibration, which I will specify the details later in section 7. The left panel shows that the drift of \(\theta^1\) satisfies the condition (26) so that both agents survive in the long run. The right panel shows the evolution of the distribution of agent 1’s consumption share \(\phi \left( \zeta^1(\theta, t) \right) = \int \Phi \left( \xi^1(\theta, m, nT + t) \right) dm\) over time. When \(n \to \infty\), the distribution converges to the stationary distribution.

Figure 5: This figure shows that the evolution of the distribution of agent 1’s consumption share and the survival results in my baseline calibration, specified in section 7. The simulation starts from \(m_0 = \bar{\theta}\) and \(\zeta^1_0 = 0.5\) for all people. Here I focus on the distribution on the 15 days after announcements, i.e. \(t = 15\).

5 The asset pricing implications

In this section, I derive the dynamics of trading volume and equity premium in disagreements, especially when the announcements come.
5.1 The trading volume of the stock

The stock has endogenous price \( P_t \), which pays a dividend stream equal to aggregate endowment \( Y_t \):

\[
P_t = E_t^2 \left[ \int_t^\infty \frac{\pi_t^2}{\sigma_t^2} Y_u du \right] = W_t^1 + W_t^2
\]

(27)

, where the second equation is implied by the market clearing condition. Since utility is homogeneous of degree \( 1 - \gamma \), for each agent \( i \), time \(-t\) aggregate wealth in units of time \(-t\) consumption must be

\[
W_i^t = \frac{(1 - \gamma) V_i^t}{D_{Cf}(C_i^t, V_i^t)} = \frac{1}{\beta} Y_t \left[ (1 - \gamma) \tilde{f}(\theta^1, m, t) \right] \frac{1}{\gamma - 1} \left[ (1 - \gamma) \frac{\xi^i}{\theta^1}(\theta^1, m, t) \right] \frac{1}{\gamma - 1}
\]

(28)

By applying Ito’s Lemma on equation (27), we can capture the dynamics of the stock return as the following theorem states.

**Proposition 5.** When \( t \in ((n - 1) T, nT) \), the dynamics of the stock return follows

\[
dR_t = \frac{Y_t dt + dP_t}{P_t} = \mu_R^2(\theta^1, m, t) dt + \sigma_R(\theta^1, m, t) d\tilde{B}^2_{Y,t}
\]

\[
= \mu_R^1(\theta^1, m, t) dt + \sigma_R(\theta^1, m, t) d\tilde{B}^1_{Y,t}
\]

The proof is in the appendix. Recall that the dynamics of agent 1’s wealth follows:

\[
\frac{dW_t^1}{W_t^1} = \left[ r_t + \pi_t^1(\mu_t^1 - r_t) - \frac{C_t^1}{W_t^1} \right] dt + \pi_t^1(\theta^1, m, t) \sigma_t(\theta^1, m, t) d\tilde{B}_{Y,t}^1
\]

\[
= \mu_{W^1}(\theta^1, m, t) dt + \sigma_{W^1}(\theta^1, m, t) d\tilde{B}_{Y,t}^1
\]

where \( \pi_t^1(\theta^1, m, t) \) is the portfolio share invested in the risky asset at period \( t \). Therefore, \( \pi_t^1(\theta^1, m, t) \), the ratio of the risky asset holdings of agent \( i \) to its total wealth \( W_t^i \), is determined by

\[
\pi_t^1(\theta^1, m, t) = \frac{\sigma_{W^1}(\theta^1, m, t)}{\sigma_R(\theta^1, m, t)}
\]

The absolute value of the change in agent 1’s position (scaled by the asset’s value),

\[
n_{1,t} = \frac{\pi_t^1 W_t^1}{P_t}
\]

(29)

determines the trading volume \( TV_t \) at time \( t \):

\[
TV_t = \left| \int n_{1,t} d\Phi_t - \int n_{1,t-\delta t} d\Phi_{t-\delta t} \right| = \left| \int \frac{\pi_t^1 W_t^1}{P_t} d\Phi_t - \int \frac{\pi_{t-\delta t}^1 W_{t-\delta t}^1}{P_{t-\delta t}} d\Phi_{t-\delta t} \right|
\]
where \( \Phi_t \) captures the distribution of agent 1 at time \( t \). Given agent 2’s stock holdings is \( 1 - n_{1,t} \), the measure of trading activity does not depend on which agent I follow.

In the full economy with learning and announcements, the dynamics of disagreement and uncertainty evolve over time, which leads to the trading between the two agents. To understand the mechanism of how it works, it’s useful to start by focusing on how the constant disagreements and uncertainty affects the share holdings of the stock separately.

**The share holdings and disagreements**

To illuminate the mechanism how disagreements affect the trading volume, I assume the disagreements and posterior variance are constant over time, i.e. \( \tilde{\mu}^1(t) - \tilde{\mu}^2(t) \equiv \bar{\mu}^1 - \bar{\mu}^2 \) and \( Q(t) \equiv Q^* \). From equation (29) and (18), I can rewrite the share holdings of stock by agent 1 in the long run distribution as

\[
n_1 = \frac{\pi^1 W^1}{P} = \pi^1 \underbrace{\frac{W^1}{W^1 + W^2}}_{\text{the portfolio share channel}} \underbrace{\frac{W^1}{W^1 + W^2}}_{\text{the wealth accumulation channel}}
\]

(30)

, which means that agent 1’s share holdings is determined by two elements: (i) the agent 1’s portfolio share invested in the risky asset \( \pi^1 \); (ii) the agent 1’s wealth share in the long run \( \frac{W^1}{W^1 + W^2} \).

![Figure 6: Stationary distributions for agent 1’s portfolio share \( \pi^1 \) and wealth share \( \frac{W^1}{W^1 + W^2} \) with constant disagreements and posterior variance \( Q(t) \equiv Q^* \). The parameters are the same as the baseline calibration, except that I vary the disagreements.](image)

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Figure 2 illustrates that, when the disagreements between the two agents increase, the long-run distribution of share holdings of agent 1 $π^1$ shifts to the right, while the long-run wealth share of agent 1 shifts to the left. The intuition is very direct from agent 1’s wealth dynamics:

$$\frac{dW^1}{W^1} = \left[ r + π^1 (μ^R_t - r) - \frac{C^1_t}{W^1} \right] dt + π^1 σ_R d\tilde{B}^1_{Y,t}$$

When agent 1 become more optimistic comparing to agent 2, he invests larger share of his wealth in the stock market since he expects the excess return of stock $μ^R_t - r_t$ is higher, which is the portfolio share channel. But the true return is not as high as he thinks, therefore the true wealth he actually accumulate is lower than he wants. In the meantime, agent 1 consumes much more so that the long-run wealth share of agent 1 shifts to the left when the disagreement increases. I call this as the wealth accumulation channel.

Therefore, as Figure 3 shows, the sharing holdings of stock by agent 1 $n^1$ is not a monotone function in disagreements as what the literature shows, such as Buraschi and Jiltsov (2006) and Xiong and Yan (2009). All of them can only study finite periods due to the survival problem. Therefore, they can not consider how the distorted beliefs affect the wealth distribution in the long run, which is a key element to determining the share holdings in the real world. When the disagreement is low, the portfolio channel will dominate since the stocks earn a higher expected return. However, when the disagreement is high enough, the wealth accumulation channel dominates so that the share holding starts to decrease.
Figure 7: Stationary distributions and the average of agent 1’s share holdings of stock \( n^1 \) with constant disagreements and posterior variance \( Q(t) \equiv Q_{nT}^+ \). The parameters are the same as the baseline calibration, except that I vary the disagreements.

The share holdings and uncertainty

The posterior variance \( Q(t) \) also plays an important role in the share holdings, which is a measure of uncertainty. The less the \( Q(t) \) is, agent 1 (2) is more sure about his optimistic (pessimistic) belief. Similar as last subsection, to eliminate other effects, I assume the disagreements and the posterior uncertainty are constant over time. The left panel of Figure 4 illustrates the portfolio share channel, that agent 1 invests larger proportion of his wealth \( \pi^1 \) into the stock market when he is more sure about his optimistic belief, i.e. \( Q^+ \) is smaller. While in the meantime, he consumes more when his uncertainty becomes smaller, which lowers the wealth accumulation.
Figure 8: Stationary distributions for agent 1’s portfolio share $\pi^1$ and wealth share $\frac{W^1}{W^1 + W^2}$ with constant disagreements $m^1 - m^2 = 0.67\%$ and posterior variance. The parameters are the same as the baseline calibration, except that I vary the posterior variance.

The left panel of Figure 5 shows that the relationship between the share holdings $n^1$ and uncertainty. The distribution is more concentrated when the uncertainty is smaller though does not shift too much. The right panel of Figure 5 demonstrates that, comparing to the disagreements, the wealth accumulation channel dominates so that the share holdings $n^1$ is increasing in the uncertainty.

Figure 9: Stationary distributions and the average of agent 1’s share holdings of stock $n^1$ with constant disagreements $m^1 - m^2 = 0.67\%$ and posterior variance. The parameters are the same as the baseline calibration, except that I vary the posterior variance.
The trading volume under learning and macro announcements

In the full model with learning and macro announcements, the dynamics of disagreement and uncertainty affects the long-run distribution of wealth. However, when I focus on the trading volume in one day or immediately after the macro announcements, the long-run wealth distribution almost does not move. Therefore, the portfolio sharing channel always dominates and the change of disagreement has a larger effect on the share holdings as the left panel of Figure 2 and 4 shows.

Given the empirical facts in section 2, after announcements, both the disagreement and uncertainty become smaller. Though smaller uncertainty makes agent 1 more willing to invest in the stock market, he becomes less optimistic which reduces the portfolio share significantly. Therefore, after the macro announcements, the portfolio share $\pi_1$ decreases a lot, which leads to a way smaller share holdings of stock $n_1$ and a larger trading volume $TV$ comparing to the days without announcements.

5.2 The risk premium

From the definition of valuational utility (19) and the envelope theorem, the state price density under agent 2’s belief is given by

$$\Lambda^2 (\theta_1, m, t) = \exp \left( - \int_0^t \lambda_s^2 ds \right) D_C F (C_t^2, \lambda_s^2)$$

$$= \beta \exp \left( - \int_0^t \lambda_s^2 ds \right) Y_t^{-\gamma} \left[ \xi^2 (\theta_1, m, t) \right]^{-\frac{1}{\psi}} \left[ (1 - \gamma) F^2 (\theta_1, m, t) \right]^{\frac{\psi - \gamma}{\psi - 1}}$$

The state price density at time $t$ depends on the fundamental risk in $Y_t$ and the Pareto weight $\theta_1$. As equation (31) shows, the disagreements affect the state price density through two channels: the consumption allocations between the two agents and the continuation utility.

**Proposition 6.** When $t \in ((n - 1) T, nT)$, the pricing kernel $\Lambda^2$ is a continuous diffusion process with the law of motion

$$\frac{d\Lambda^2}{\Lambda^2} = -r (\theta_1, m, t) dt - \sigma_\Lambda^2 (\theta_1, m, t) dB_{Y,t}^2$$
where

\[ r(\theta^1, m, t) = \Lambda_t^2 + \gamma m - \frac{1}{2} \gamma (\gamma + 1) \sigma_y^2 \]

\[ - \frac{\Lambda_{\theta_1}^2}{\Lambda^2} (\mu_\theta - \gamma \sigma_\theta \sigma_y) - \frac{\Lambda_m^2}{\Lambda^2} (\mu_m - \gamma \sigma_m \sigma_y) - \frac{1}{2} \frac{\Lambda_{\theta_1}^2}{\Lambda^2} \sigma_\theta^2 - \frac{1}{2} \frac{\Lambda_m^2}{\Lambda^2} \sigma_m^2 - \frac{\Lambda_{\theta_1 m}^2}{\Lambda^2} \sigma_m \sigma_\theta \]  

(32)

is the risk-free interest rate and

\[
\sigma^2_\Lambda(\theta^1, m, t) = \gamma \sigma_y - \frac{1}{\psi} \left[ \frac{\xi^2_{\theta_1^1}(\theta^1, m, t)}{\xi^2(\theta^1, m, t)} \sigma_{\theta_1^1} + \frac{\xi^2_m(\theta^1, m, t)}{\xi^2(\theta^1, m, t)} \sigma_m \right] 
- \frac{1}{1 - \gamma} \left[ \tilde{f}_{\theta_1}^2(\theta^1, m, t) \sigma_{\theta_1^1} + \tilde{f}_m^2(\theta^1, m, t) \sigma_m \right] 
\]

(33)

is the market price of risk perceived by agent 2.

The state price density under agent 1’s belief is given by

\[ \frac{d\Lambda_t^1}{\Lambda_t^1} = -r(\theta^1, m, t) \, dt - \sigma^1_\Lambda(\theta^1, m, t) \, d\tilde{B}_t^1 \]

where

\[ \sigma^1_\Lambda(\theta^1, m, t) = \sigma^2_\Lambda(\theta^1, m, t) + \bar{\mu}^1(t) - \bar{\mu}^2(t) \]  

(34)

The proof is in the appendix. Both agents have the same risk free interest rate, which is determined in equation (32). For the first line in (32), the first term is agent 2’s endogenous discount rate process, which is affected by the dynamics of disagreements and Pareto share. The second term is the wealth effect associated with expected growth rate of aggregate endowment in the absence of Brownian shocks, and the third term is the precautionary savings effect associated with Brownian shocks to output. The second line of Equation (32) incorporates the direct impact of disagreement on the state price density. In particular, the first two terms capture an additional wealth effect associated with expected growth of \( \theta^1 \) and \( m \) in the absence of Brownian shocks. The last three terms is the precautionary savings effect associated with Brownian shocks and learning.

Equation (33) illustrates the market price of risk as perceived by agent 2. The first term is the market price of Brownian aggregate endowment risk. The second term is the standard market price of speculative risk, and the third term captures the impact of recursive preferences on the market.
price of Brownian risk. The state price density that perceived by the two agents are separated by their disagreements of the fundamental, and the optimist perceives a higher market price of risk than the pessimist., as equation (34) shows.

**Proposition 7.** When \( t \in ((n-1)T, nT) \), the dynamics of the stock return follows

\[
dR_t = \frac{Y_t dt + dP_t}{P_t} = \mu_R^2(\theta^1, m, t) dt + \sigma_R(\theta^1, m, t) dB^2_{Y,t}
\]

\[
= \mu_R^1(\theta^1, m, t) dt + \sigma_R(\theta^1, m, t) dB^1_{Y,t}
\]

and the risk premium of the risky asset perceived by agent \( i \in \{1, 2\} \) is

\[
\mu_R^i(\theta^1, m, t) - r(\theta^1, m, t) = \sigma_R(\theta^1, m, t) \sigma^i(\theta^1, m, t)
\]

. Therefore, the difference of the risk premium perceived by the two agents is determined by

\[
\mu_R^1(\theta^1, m, t) - \mu_R^2(\theta^1, m, t) = \sigma_R(\theta^1, m, t) \left[ \sigma^1(\theta^1, m, t) - \sigma^2(\theta^1, m, t) \right].
\]

The risk premium of agent \( i \) is determined by the exposure to the risk of the stock\( \sigma_R(\theta^1, m, t) \), and the equilibrium price of this risk given by \( \sigma^i(\theta^1, m, t) \). The equity premium perceived by the optimistic agent is always higher, given equations (34) and (36).

![Figure 10: The risk free interest rate and equity premium perceived by each agent.](image)

Figure 10 illustrates how the risk free interest rate and equity premium change as a function of agent 1’s Pareto share \( \theta^1 \). The risk free rate is increasing in agent 1’s pareto share while the
equity premium perceived by both agents are decreasing. The higher risk free rate is because agent 1 expects the economy grow faster. Since the volatility of the return almost does not change when $\theta^1$ change, the decrease in risk premium can only come from the decrease in risk prices.

**Proposition 8.** At announcements, $t = nT$, $x^i$ is discontinuous, and the announcement stochastic discount factor (A-SDF) under agent $i$’s belief is given by

$$x_{nT}^{i,*} = \frac{\Lambda_{nT}^{i,+}}{\Lambda_{nT}^{i,-}} = \left[\frac{\xi^i(\theta^1,m^+,nT)}{\xi^i(\theta^1,m^-,nT)}\right]^{-\frac{1}{\Psi}} \left[\frac{(1 - \gamma) \tilde{J}^i(\theta^1,m^+,nT)}{(1 - \gamma) \tilde{J}^i(\theta^1,m^-,nT)}\right]^{\frac{1}{\Psi} - \gamma}$$

(37)

, and the announcement premium is determined by

$$-\text{Cov}_i^t \left( x_{nT}^{i,*}, \frac{P(\theta^1,m^+,0)}{P(\theta^1,m^-,nT)} \right)$$

In my model, there are two channels to generate the announcement premium as in the data. The first channel is similar as Ai and Bansal (2018), Ai et al. (2018), the effect of generalized risk sensitivity on announcement premium. Since the agent has preference with early resolution of uncertainty $(\gamma > \frac{1}{\Psi})$, macro announcements result in non-trivial reductions of uncertainty, and are associated with realizations of a substantial amount of equity premium.

In the meanwhile, the disagreements become smaller. Agent 1 becomes less optimistic so that he invests a smaller portfolio share $\pi^1$ of his wealth into the stock market. There has to be a higher equity premium compensate him to bear the same risk using less wealth. Even though agent 2 does the opposite decision, agent 1 who is optimistic usually has a larger wealth share. Thus, the endogenous portfolio choice generates a larger announcement premium comparing to the representative agent model as in Ai and Bansal (2018) and Ai et al. (2018).

5.3 Short Interest

Comparing to the literature which studies the heterogeneous beliefs with short-sales constraints (e.g., Scheinkman and Xiong (2003)), I can study the short interest in the financial market, which
is significantly positive and increasing over time\(^3\). The short interest rate in the mode is defined as

\[
\text{short interest ratio} = \int \frac{|n_2| \mathbb{1}_{n_2 < 0}}{n_1 + n_2} d\Phi = \int \frac{(n_1 - 1) \mathbb{1}_{n_1 > 1}}{n_1 + n_2} d\Phi
\]

\[(38)\]

\[
= \int (n_1 - 1) \mathbb{1}_{n_1 > 1} d\Phi
\]

\[(39)\]

\[
= \int \left( \pi^1 \frac{W^1}{W^1 + W^2} - 1 \right) \mathbb{1}_{n_1 > 1} d\Phi
\]

\[(40)\]

Similarly as the share holdings, the short interest ratio is also determined by two elements: (i) the agent 1’s portfolio share invested in the risky asset \(\pi^1\); (ii) the agent 1’s wealth share in the long run \(\frac{W^1}{W^1 + W^2}\).

## 6 Quantitative Analysis

From the above section, I show that the trading volume and risk premium on non-announcement days and announcement days are mainly determined by the dynamics of disagreements \(m^1(t) - m^2(t)\), uncertainty \(Q(t)\) and their wealth share. Therefore, I divide the model parameters into three sets: (i) parameters related to disagreements and uncertainty, (ii) parameters determines the wealth share, and (iii) other parameters, such as preference and aggregate endowment process.

### 6.1 The disagreements and uncertainty with learning and macro announcements

I calibrate the disagreements to the forecasts of real GDP from the Survey of Professional Forecasters (SPF) as the representative distorted beliefs among people\(^4\). I follow Bordalo, Gennaioli, Ma, and Shleifer (2018) to construct the forecast of annual real GDP growth by the professionals from 1996 to 2017. The real GDP growth from end of quarter \(t - 1\) to end of quarter \(t + 3\) for each professional \(i\) is defined as

\[
G^i_{t-1 \rightarrow t+3} = \frac{F_t^i x^i_{t+3}}{x^i_{t-1}}
\]

\(^3\)Rapach, Ringgenberg, and Zhou (2016) finds that the short interest is the strongest known predictor of aggregate stock returns and the short interest ratio increases from 0.31% during 1973-1982 to 4.98% during 2003-2014.

\(^4\)The forecasts of real GDP from other surveys, such as Blue Chip Economic Indicators generate smaller results.
where \( t \) is the quarter of forecast and \( x \) is the level of real GDP is a given quarter; \( x_{t-1} \) uses the initial release of actual value in quarter \( t - 1 \), which is available by the time of the forecast in quarter \( t \). \( F_t x_{t+3}^i \) is the reported forecast of real GDP growth at the end of quarter \( t + 3 \). I define the distorted beliefs as

\[
G_{t-1 \rightarrow t+3}^i - E \left( G_{t-1 \rightarrow t+3}^i \right)
\]

where the \( E \left( G_{t-1 \rightarrow t+3}^i \right) \) is the consensus forecasts. I take the mean and the standard deviation of all positive (negative) distorted beliefs, as the proxy for the optimistic (pessimistic) agent’s belief distribution. Figure 11 shows the time series of the mean of the distorted beliefs for both group of agents.

![Figure 11: The time series of the distorted beliefs for the expected annual real GDP growth rate from 1996 to 2017. Data source: Survey of Professional Forecasters (SPF).](image)

6.2 The wealth share

Section 5.1 talks about that the wealth accumulation channel is crucial in determining the trading volume and macro announcements, which is always ignored in the heterogeneous beliefs litera-
ture. However, there is no data available which surveys the relative wealth between the optimistic agents and pessimistic agents. From the definition of short interest ratio in equation (40), given the disagreements estimated in the last section, the short interest rate reflects the relative wealth share \( \frac{W_1}{W_1 + W_2} \) between the two agents.

I construct the short interest ratio as Rapach et al. (2016), which is consistent with the definition in my model. The raw short interest numbers from Compustat are reported as the number of shares that are held short in a given firm. They normalize these numbers by dividing the level of short interest by each firm’s shares outstanding from the Center for Research in Security Prices (CRSP). The data cover a variety of asset classes, including common equities, American Depositary Receipts (ADRs), Exchange Traded Funds (ETFs), and Real Estate Investment Trusts (REITs). Each month, I calculate aggregate short interest as the equal-weighted mean of all asset-level short interest data. The average aggregate short interest ratio from 1996 to 2014 is 3.7%. The short interest ratio is highly correlated with the the disagreements of real GDP growth rate in Figure 10. For example, when the disagreements reach the highest in year 2008, the short interest ratio increases to 8.93% in July of 2008.
Figure 12: The time series of equal-weighted mean of all asset-level short interest (%), which includes common equities, ADRs, ETFs, and REITs from 1996 to 2014.

From the wealth accumulation dynamics, given the disagreements, consumption-to-wealth ratio is the key to the stationary distribution of wealth share. The consumption-to-wealth ratio is mainly determined by the intertemporal elasticity of substitution $\psi$. Therefore, I calibrate $\psi = 1.4$ to match the short interest ratio in the data, which implies the wealth share of the optimistic agent is $\frac{W_1}{W_1 + W_2} = 79.1\%$.

### 6.3 Other parameters

Parameters $\{\sigma_y, \sigma_\theta, \rho, \bar{\theta}\}$ are estimated to match the mean and standard deviation of both the real GDP and the growth rate of real GDP. I choose $\beta$ and $\gamma$ to match the risk free rate and market equity premium.
7 Results

This section presents results from solving the model, beginning by showing how the dynamics of disagreements and uncertainty, and then showing the trading volume and equity premium on days without and with announcements from simulating the model. The mathematical Appendix describes details on the simulation method. Later on, I will explore the welfare implications and some counterfactual analyses. Besides, I will add the link between the monetary policy surprises and the trading volume. Empirically, I find the monetary policy surprise is not correlated with the trading volume, which is consistent with my model’s implications.

7.1 The dynamics of disagreements and uncertainty

![Figure 13: The dynamics of disagreements and uncertainty.](image)

Figure 13 shows the dynamics of the disagreements and uncertainty on days without and with announcements. Agent 1 is more optimistic than agent 2 and the difference is varying across time so that we can match the distribution of beliefs in the Survey of Professional Forecasts. When announcements come, they learn from the new information and their disagreements become smaller, as indicated by the circles. In the meantime, since the announcements reveal more information about the expected growth rate, the uncertainty becomes smaller. After announcement, information from the output slowly arrives and as a result, the posterior variance gradually increases over
time before the next announcement.

7.2 The trading volume and the announcement premium

The left panel of Figure 14 shows the share holdings of agent 1. The dynamics of share holdings are inline with the disagreements. Since the fluctuations of disagreements are small on days without announcements, the trading volume is small. When the announcements come, the disagreements reduce a lot so that the agents rebalance their portfolios. Therefore, the trading volume upon announcements increases a lot as indicated by the circles in the right panel of the figure.

![Figure 14: The dynamics of share holdings of stock and the trading volume. The gray shade area represents the 95% confidence interval.](image)

Figure 15 illustrates how the agents rebalance their portfolio upon the announcements. When the announcements come, agent 1 become less optimistic so that he wants to sell the stock. Agent 2 wants to hold more stock since he becomes less pessimistic. Therefore, they rebalance the portfolio.

As pointed in section 5.2, the announcement premium is determined by two elements: (i) the informativeness of the new signal. (ii) the dynamics of disagreements upon announcements. Since the macro announcements reduce uncertainty, there is a high announcement premium under preferences with early resolution of uncertainty. The more informative the new signal, the larger the announcement premium. Besides, it reduces the disagreements therefore the agent 1 invests
less wealth into the stock market, which increases the announcements premium even more. Table 1 and 2 summarizes the parameters and the corresponding asset pricing implications.

![Stationary distributions agent 1’s share holdings of stock $n^1$ before and after announcements.](image)

**8 Conclusion**

In this paper, I demonstrate that the large trading volume of stocks and announcement premium provide asset-market-based evidence to capture the dynamics of heterogeneous beliefs upon announcements. I find the monetary policy released in the FOMC announcements reduce investor’s disagreements. Therefore, investors unwind their positions and rebalance their portfolio, which mainly contributes to the large trading volume upon announcements. Motivated by these financial market responses upon the announcements, I build a general equilibrium model with heterogeneous beliefs under learning and announcements. The model reconciles the price and trading volume dynamics. The reduction of disagreements upon announcements supports the positive welfare gain of monetary policy in the long run.
9 Appendix

9.1 Data Description

FOMC announcements  There are a total of eight pre-scheduled FOMC meetings each calendar year, and the dates of FOMC meetings are taken from the Federal Reserve’s web site. The pre-scheduled FOMC statements began in 1996, when the Committee started announcing its decision to the markets by releasing a statement at the end of each meeting. For meetings lasting two calendar days, we consider the second day (the day the statement is released) as the event date.

High-frequency trading volume  My primary data is comprised of intraday transaction prices and trading volume for the S&P 500 index ETF (ticker: SPY). All of the data are obtained from the TAQ database. The sample covers all regular trading days from January 1996 through December 2014. The raw data are cleaned following the procedures detailed in Brownlees and Gallo (2006) and Barndor-Nielsen et al. (2009). Further, in order to mitigate the effect of market micro-structure noise, we follow standard practice in the literature to sparsely sample the data at a one-minute sampling interval. The exact times at which the announcements are released are reported by Bloomberg. Most of the announcements are released at 14:15 pm eastern time before 2013. For the trading volume figure, I only focus on these announcements.

Open interest  The daily open interest data of SPX comes from Option Metrics, 1996 – 2016. To identify option contracts associated with S&P500 index, I follow WRDS introduction, namely I use those whose secid equals to 108105. Then I separate the contract to call and put options. I calculate the sum of open interest with respect to all available call (put) contracts on a single day, including different strike prices, days-to-expiration, and moneyness. The data from 2016 to 2018 is from CBOE. I get the daily open interest of the 30-days federal funds future from Bloomberg.

9.2 Proof of Proposition 1

In this section, to prepare for the proofs for Propositions 1, I first state a lemma that establishes that dynamics of beliefs without the new disagreements.

Lemma 1. When the two agents have the same prior variance, the disagreements $m^1(t) - m^2(t)$ will vanish in the long run.

Proof. $Q(t)$ is a deterministic function of time and obeys a Riccati equation:
\[ dQ^i(t) = \left[ \sigma_y^2 - 2\rho Q^i(t) - \frac{1}{\sigma_y^2} [Q^i(t)]^2 \right] dt \]

One can easily show that \( Q(t) \) has a closed-form solution. In general, I have

\[ Q^i(t) = \frac{\sigma_y^2 (1 - e^{-2\hat{\rho}(t+t^*)})}{(\hat{\rho} - \rho) e^{-2\hat{\rho}(t+t^*)} + \rho + \hat{\rho}} \]  \hspace{1cm} (41)

where \( t^* \) is defined as:

\[ t^* = \frac{1}{2\hat{\rho}} \ln \frac{\sigma_y^2 + (\hat{\rho} - \rho) Q^i(0)}{\sigma_y^2 - (\hat{\rho} + \rho) Q^i(0)} \]

From equation (41), it’s obvious that \( Q^1(t) \equiv Q^2(t) \overset{\Delta}{=} Q(t) \) for any \( t > 0 \) if \( Q^1(0) = Q^2(0) \), which means the two agents have the same posterior variance along the path \( \{t\}_{t \geq 0} \).

Therefore, from equation (3), the disagreement dynamics follow

\[ m_1^t - m_2^t = [m_1^{t-1} - m_2^{t-1}] \exp \left[ \int_{t-1}^{t} \left( \rho + \frac{Q(s)}{\sigma_y^2} \right) ds \right] \]

which is deterministic. Since \( \rho + \frac{Q(s)}{\sigma_y^2} > 0 \),

\[ \lim_{t \to \infty} m_1^t - m_2^t = 0. \]

Therefore, the disagreements will vanish in the long run even there are no announcements. Since agents update their beliefs using Bayes’ rule upon announcements, announcements will speed up the convergence of the beliefs.

I now prove Proposition 1.

\[ ^5 \text{With } Q(0) = 0, \text{ I have} \]

\[ Q(t) = \frac{\sigma_y^2 (1 - e^{-2\hat{\rho}t})}{(\hat{\rho} - \rho) e^{-2\hat{\rho}t} + \rho + \hat{\rho}} \]

where \( \hat{\rho} \) is defined as:

\[ \hat{\rho} = \sqrt{\rho^2 + \sigma_y^2 / \sigma_y^2} \]

and \( Q(t) \to Q^* = \frac{\sigma_y^2}{\rho + \hat{\rho}} \text{ as } t \to \infty. \)
9.3 Proof of Proposition 2

From the definition of the return
\[ dR_t = \frac{Y_t dt + dP_t}{P_t} = \mu_{R,t} dt + \sigma_{R,t} d\tilde{B}_{Y,t} \]
and the equivalence of the measures (6),
\[ \sigma_{R,t} \equiv \sigma_{R,t}^j, \quad \mu_{R,t} = \mu_{R,t}^j + \bar{\mu}^i (t) \sigma_{R,t} \quad t \in ((n-1)T, nT) \]

Thus, the agents have the same perceived volatility of returns and their perceived return is captured by their differences of opinion about the expected endowment growth rate.

9.4 Proof of Proposition 3

Applying Ito’s lemma to \( \lambda_t^1 \) leads to a maximization problem under the agent 2’s probability measure,
\[ \lambda^1_t V^1_t (C^1) = \sup_{\{V^1_s\}_s \geq t} E_t^2 \left[ \int_t^\infty \lambda^1_s F (C^1_s, V^1_s) ds \right] \] (42)
subject to
\[ d\lambda^1_t = \lambda^1_t \left[ -V^1_t dt + \left( \bar{\mu}^1 (t) - \bar{\mu}^2 (t) \right) d\tilde{B}_{Y,t} \right] , \quad \lambda^1_0 > 0 \] (43)

Therefore, I can write the social planner under agent 2’s measure, as equation (21) states.

Since \( J_0 (\lambda^1_0, \lambda^2_0, Y_0, m_0) = \sup_{(C^1, C^2, V^1, V^2)} \{ E_0^2 \left[ \int_0^\infty \lambda^1_s F (C^1_s, V^1_s, t) ds \right] + E_0^2 \left[ \int_0^\infty \lambda^2_s F (C^2_s, V^2_s, t) ds \right] \} \),
the corresponding HJB equation is
\[
\sup_{(C^1, C^2, V^1, V^2)} \lambda^1_t F (C^1_t, V^1_t, t) + \lambda^2_t F (C^2_t, V^2_t, t) + E_t^2 \left\{ J_t dt + J_{\lambda^1} d\lambda^1 + J_{\lambda^2} d\lambda^2 + J_Y dY + J_m dm \right\} \\
+ \frac{1}{2} E_t^2 \left\{ J_{\lambda^1 \lambda^1} (d\lambda^1)^2 + J_{\lambda^2 \lambda^2} (d\lambda^2)^2 + J_{YY} dY^2 + J_{mm} dm^2 \right\} \\
+ E_t^2 \left\{ J_{\lambda^1 \lambda^2} d\lambda^1 d\lambda^2 + J_{\lambda^1 Y} d\lambda^1 dY + J_{\lambda^1 m} d\lambda^1 dm + J_{\lambda^2 Y} d\lambda^2 dY + J_{\lambda^2 m} d\lambda^2 dm + J_{Y m} dY dm \right\} \] (44)
By the law of motion of the state variables, we can simply the above HJB equation,

\[
\sup_{(C^i, C^2, v^1, v^2)} \lambda^i F (C^i, v^1, t) + \lambda^2 F (C^2, v^2, t) + J_t - v^1 \lambda^i J_{\lambda^1} - v^2 \lambda^2 J_{\lambda^2} + m_t Y_t J_t + \rho \left( \tilde{\theta} - m(t) \right) J_m \\
+ \frac{1}{2} J_{\lambda^i} (v^1 \lambda^i)^2 + J_{\lambda^2} (v^2 \lambda^2)^2 + J_{Y Y} \sigma^2 Y_t^2 + J_{m m} \frac{Q^2(t)}{\sigma_y} + \\
+ J_{\lambda^1 Y} (\tilde{\mu}^1(t) - \tilde{\mu}^2(t)) \lambda^1 \sigma_t Y_t + J_{\lambda^2 Y} (\tilde{\mu}^1(t) - \tilde{\mu}^2(t)) \lambda^2 \frac{Q(t)}{\sigma_y} + J_{Y m} Q(t)
\]

(45)

The maximization over \((v^1, v^2)\) of the HJB equation can be solved separately since \((v^1, v^2)\) only appears in the first term for both agents. Define

\[
f \left( C^i, J_{\lambda^i}, t \right) \triangleq \sup_{v^i} F \left( C^i, v^1, t \right) - v^1 J_{\lambda^i} = \frac{\beta}{1 - \frac{1}{\psi}} \left\{ (C^i)^{1 - \frac{1}{\psi}} [(1 - \gamma) J_{\lambda^i}]^{1 - \frac{1}{\psi} - \frac{1}{\psi(1 - \gamma)}} - (1 - \gamma) J_{\lambda^1} \right\}
\]

(46)

I denote the Lagrange multiplier on the market clearing as \(\xi_{MC}\). The first order condition with respect to \(C^i\) is

\[
\lambda^i \beta \left( C^i \right)^{\frac{1}{\psi}} [(1 - \gamma) J_{\lambda^i}]^{1 - \frac{1}{\psi} - \frac{1}{\psi(1 - \gamma)}} + \xi_{MC} = 0, \quad i = 1, 2
\]

which means

\[
\lambda^i \left( C^i \right)^{\frac{1}{\psi}} [(1 - \gamma) J_{\lambda^i}]^{1 - \frac{1}{\psi} - \frac{1}{\psi(1 - \gamma)}} = \lambda^2 \left( C^2 \right)^{\frac{1}{\psi}} [(1 - \gamma) J_{\lambda^2}]^{1 - \frac{1}{\psi} - \frac{1}{\psi(1 - \gamma)}}
\]

Therefore, we can derive the consumption share \(\zeta^i = \frac{C^i}{Y}\) for each agent \(i\) as,

\[
\zeta^i = \frac{\lambda^i \left( C^i \right)^{\frac{1}{\psi}} [(1 - \gamma) J_{\lambda^i}]^{1 - \frac{1}{\psi} - \frac{1}{\psi(1 - \gamma)}}}{\lambda^1 \left( C^1 \right)^{\frac{1}{\psi}} [(1 - \gamma) J_{\lambda^1}]^{1 - \frac{1}{\psi} - \frac{1}{\psi(1 - \gamma)}} + \lambda^2 \left( C^2 \right)^{\frac{1}{\psi}} [(1 - \gamma) J_{\lambda^2}]^{1 - \frac{1}{\psi} - \frac{1}{\psi(1 - \gamma)}}}
\]

(47)

Due to the homogeneity of the value function,

\[
J(\lambda_1, \lambda_2, Y, m, t) = Y^{1 - \gamma} (\lambda_1 + \lambda_2) \tilde{J} \left( \frac{\lambda_1}{\lambda_1 + \lambda_2}, m, t \right) = Y^{1 - \gamma} \theta_2 \tilde{J} \left( \theta^1, m, t \right)
\]

where \(\theta^1 = \frac{\lambda_1}{\lambda_1 + \lambda_2}\) and \(\theta^2 = \lambda_1 + \lambda_2\). \(\theta^1\) represents the Pareto share of agent 1, which is obviously bounded between zero and one.
I apply Ito’s Lemma to $J(\lambda_1, \lambda_2, Y, m, t)$ to derive all the elements in equation (45). Combining with equation (46), I can simply the HJB equation as

$$
0 = \theta^1 \frac{\beta}{1 - \frac{1}{\psi}} \left( \zeta^1 \right)^{\frac{1-\frac{1}{\psi}}{1-\frac{1}{\psi}}} \left( (1 - \gamma) J^1 \right)^{\frac{1-\frac{1}{\psi}}{1-\frac{1}{\psi}}} + (1 - \theta^1) \frac{\beta}{1 - \frac{1}{\psi}} \left( (1 - \zeta^1) \right)^{\frac{1-\frac{1}{\psi}}{1-\frac{1}{\psi}}} \left( (1 - \gamma) J^2 \right)^{\frac{1-\frac{1}{\psi}}{1-\frac{1}{\psi}}}
$$

$$
+ \bar{J}_t + \left( -\frac{\beta (1 - \gamma)}{1 - \frac{1}{\psi}} + m (1 - \gamma) + \theta^1 \left( \bar{\mu}^1 (t) - \bar{\mu}^2 (t) \right) (1 - \gamma) \sigma_y - \frac{1}{2} \gamma (1 - \gamma) \sigma^2_y \right) \bar{J}_t
$$

$$
+ \theta^1 (1 - \theta^1) \left( \bar{\mu}^1 (t) - \bar{\mu}^2 (t) \right) (1 - \gamma) \sigma_y J_{\theta^1} + \frac{1}{2} (1 - \theta^1)^2 \bar{J}_m
$$

The functions $\tilde{J}^n (\theta^1, m, t)$ are the continuation values of two agents scaled by $Y^{1-\gamma}$,

$$
\begin{align*}
\tilde{J}^1 (\theta^1, m, t) & = \bar{J} (\theta^1, m, t) + (1 - \theta^1) \bar{J}_{\theta^1} (\theta^1, m, t) \\
\tilde{J}^2 (\theta^1, m, t) & = \bar{J} (\theta^1, m, t) - \theta^1 \bar{J}_{\theta^1} (\theta^1, m, t)
\end{align*}
$$

and the consumption share $\zeta^1$ is given

$$
\zeta^1 (\theta^1, m, t) = \frac{(\theta^1)^{\psi} \left[ (1 - \gamma) \tilde{J}^1 (\theta^1, m, t) \right]^{\frac{1-\psi}{1-\psi}}}{(\theta^1)^{\psi} \left[ (1 - \gamma) \tilde{J}^1 (\theta^1, m, t) \right]^{\frac{1-\psi}{1-\psi}} + (1 - \theta^1) \left[ (1 - \gamma) \tilde{J}^2 (\theta^1, m, t) \right]^{\frac{1-\psi}{1-\psi}}}
$$

and agent 2’s consumption share $\zeta^2 (\theta^1, m, t) = 1 - \zeta^1 (\theta^1, m, t)$.

Now the boundary condition. Let $\theta^1 = 0$ or 1, we can derive the following boundary condition of $\theta^1$: $\bar{J}(0, m, t) = H^2 (m, t)$ and $\bar{J}(1, m, t) = H^1 (m, t)$, where $H^n (m, t)$ are the solution to the following equation

$$
0 = \frac{\beta}{1 - \frac{1}{\psi}} \left[ (1 - \gamma) H \right]^{\frac{1-\frac{1}{\psi}}{1-\frac{1}{\psi}}} + H_t + \left[ -\frac{\beta (1 - \gamma)}{1 - \frac{1}{\psi}} + (1 - \gamma) \left( m + \sigma_y \bar{\Pi}^1 (t) \right) - \frac{1}{2} \gamma (1 - \gamma) \sigma^2_y \right] H
$$

$$
+ \left[ \rho (\bar{\theta} - m) + (1 - \gamma) Q (t) + \bar{\Pi}^1 (t) \frac{Q (t)}{\sigma_y} \right] H_m + \frac{1}{2} \frac{Q^2 (t)}{\sigma^2_y} H_{mm}
$$

(49)

where $\bar{\Pi}^1 (t) = \bar{\mu}^1 (t) - \bar{\mu}^2 (t)$ and $\bar{\Pi}^2 (t) = 0$. 

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9.5 Proof of Proposition 4

Applying the Ito’s Lemma of the Pareto share of agent 1 \( \theta^1 = \frac{\lambda_1}{\lambda_1 + \lambda_2} \), the law of motion of the Pareto share of agent 1 \( \theta^1 \) follows

\[
d\theta^1 = \frac{\lambda_2}{(\lambda_1 + \lambda_2)^2} d\lambda_1 - \frac{\lambda_1}{(\lambda_1 + \lambda_2)^2} d\lambda_2 - \frac{\lambda_2}{(\lambda_1 + \lambda_2)^3} d\lambda_1^2 + \frac{\lambda_1}{(\lambda_1 + \lambda_2)^3} d\lambda_1^2 + \frac{2(\lambda_1 - \lambda_2)}{(\lambda_1 + \lambda_2)^3} d\lambda_1 d\lambda_2
\]

\[
= \theta^1 (1 - \theta^1) \left[ \lambda_1^2 - \lambda_2^2 - \theta^1 (\mu^1 (t) - \bar{\mu}^2 (t)) \right] dt + \theta^1 (1 - \theta^1) (\mu^1 (t) - \bar{\mu}^2 (t)) d\tilde{B}^2_{Y,t}
\]

As long as the drift of \( \theta^1 \) is not always positive or negative, for strictly positive initial weights, the boundaries are unattainable, so that \( \theta^1 \) evolves on the open interval \((0, 1)\). Therefore, both agents can survive in the long run. The following conditions are sufficient for the long run survive:

\[
\begin{align*}
(i) \quad & \lim_{\theta^1 \searrow 0} \min_{t} \mu_{\theta^1} (\theta^1, m, t | m = \bar{\theta}) < 0 \quad \text{and} \quad \lim_{\theta^1 \nearrow 1} \max_{t} \mu_{\theta^1} (\theta^1, m, t | m = \bar{\theta}) > 0 \\
(ii) \quad & \lim_{\theta^1 \searrow 0} \max_{t} \mu_{\theta^1} (\theta^1, m, t | m = \bar{\theta}) > 0 \quad \text{and} \quad \lim_{\theta^1 \nearrow 1} \min_{t} \mu_{\theta^1} (\theta^1, m, t | m = \bar{\theta}) < 0
\end{align*}
\]

9.6 Proof of Proposition 5

Proof. First, since bonds are in zero net supply, the wealth of the two agents must sum to the value of the risky asset, that is

\[
W_t = W^1_t + W^2_t = P_t
\]

Due to the homogeneity of the recursive preference,

\[
W^1_t = \frac{(1 - \gamma) V^1_t}{D_{Cf} (C^1_t, V^1_t)} = \frac{1}{\beta} Y^1_t [\zeta^1 (\theta^1, m)]^{\frac{1}{\psi}} [(1 - \gamma) \tilde{f}^1 (\theta^1, m)]^{-\frac{1}{1-\gamma}}
\]

41
Define \( H(\theta^1, m) = \left[ \zeta^1 (\theta^1, m) \right]^{\frac{1}{\psi}} \left[ (1 - \gamma) \hat{f}^1 (\theta^1, m) \right]^{\frac{1}{1 - \psi}} \) and apply the Ito’s Lemma to the above equation, we can derive the law of motion

\[
\frac{dW^1_t}{W^1_t} = \frac{dY_t}{Y_t} + \frac{H_1 (\theta^1, m)}{H (\theta^1, m)} d\theta^1 + \frac{1}{2} \frac{H_{11} (\theta^1, m)}{H (\theta^1, m)} (d\theta^1)^2 + \frac{H_2 (\theta^1, m)}{H (\theta^1, m)} dm \\
+ \frac{1}{2} \frac{H_{22} (\theta^1, m)}{H (\theta^1, m)} (dm)^2 + \frac{H_{12} (\theta^1, m)}{H (\theta^1, m)} d\theta^1 dm + \frac{H_1 (\theta^1, m)}{H (\theta^1, m)} dY_t + \frac{H_2 (\theta^1, m)}{H (\theta^1, m)} dm dY_t
\]

\[
= \left[ m_t + \frac{H_1}{H} \mu_\theta + \frac{H_2}{H} \mu_m + \frac{1}{2} \frac{H_{11}}{H} \sigma_\theta^2 + \frac{1}{2} \frac{H_{22}}{H} \sigma_m^2 + \frac{H_{12}}{H} \sigma_\sigma_m + \frac{H_1}{H} \sigma_\theta \sigma_m + \frac{H_2}{H} \sigma_m \sigma_y \right] dt \\
+ \left[ \sigma_y + \frac{H_1}{H} \sigma_\theta + \frac{H_2}{H} \sigma_m \right] dB^2_{Y,t}
\]

Similarly, we can define \( G(\theta^1, m) = \left[ \zeta^2 (\theta^1, m) \right]^{\frac{1}{\psi}} \left[ (1 - \gamma) \hat{f}^2 (\theta^1, m) \right]^{\frac{1}{1 - \psi}} \) and drive the law of motion of \( W^2_t \),

\[
\frac{dW^2_t}{W^2_t} = \left[ m_t + \frac{G_1}{G} (\mu_\theta + \sigma_\theta \sigma_y) + \frac{G_2}{G} (\mu_m + \sigma_m \sigma_y) + \frac{1}{2} \frac{G_{11}}{G} \sigma_\theta^2 + \frac{1}{2} \frac{G_{22}}{G} \sigma_m^2 + \frac{G_{12}}{G} \sigma_\sigma_m \right] dt \\
+ \left[ \sigma_y + \frac{G_1}{G} \sigma_\theta + \frac{G_2}{G} \sigma_m \right] dB^2_{Y,t}
\]

\[
= \left[ m_t + (\bar{\mu}^1 (t) - \bar{\mu}^2 (t)) \sigma_w (\theta^1, m) \right] dt + \sigma_w (\theta^1, m) dB^1_{Y,t}
\]
Therefore,
\[
    dR_t = \frac{Y_t dt + dW_t}{W_t} = \frac{Y_t dt + dW_t}{W_t} = \frac{Y_t}{W_t + W_t^2} dt + \frac{W_t^1}{W_t + W_t^2} dW_t^1 + \frac{W_t^2}{W_t + W_t^2} dW_t^2
\]
\[
= \beta + \left[ \zeta_1 \right] \psi \left[ (1 - \gamma)f^1 \right] \frac{1 - \psi}{1 - \psi} (\mu_{W_1} + \bar{\mu}_1(t) \sigma_{W_1}) + \left[ \zeta_2 \right] \psi \left[ (1 - \gamma)f^2 \right] \frac{1 - \psi}{1 - \psi} (\mu_{W_2} + \bar{\mu}_1(t) \sigma_{W_2}) dt
\]
\[
+ \left[ \zeta_1 \right] \psi \left[ (1 - \gamma)f^1 \right] \frac{1 - \psi}{1 - \psi} \sigma_{W_1} + \left[ \zeta_2 \right] \psi \left[ (1 - \gamma)f^2 \right] \frac{1 - \psi}{1 - \psi} \sigma_{W_2} d\bar{B}_{Y,t}
\]
\[
= \mu_R (\theta^1, m) dt + \sigma_R (\theta^1, m) d\bar{B}_{Y,t},
\]
\[
\text{(53)}
\]
\[
= \left[ \mu_R (\theta^1, m) + (\bar{\mu}_1(t) - \bar{\mu}_2(t)) \sigma_R (\theta^1, m) \right] dt + \sigma_R (\theta^1, m) d\bar{B}_{Y,t},
\]
\[
\text{(54)}
\]

9.7 Proof of Proposition 6, 7 and 8

Proof. Apply Ito’s Lemma to (31), we can get
\[
\frac{d\Lambda^2}{\Lambda^2} = -\lambda_t^2 dt - \gamma dY_t + \frac{1}{2} \gamma (\gamma + 1) \left( \frac{dY_t}{Y_t} \right)^2 + \frac{\Lambda_1^2 (\theta^1, m, t)}{\Lambda^2 (\theta^1, m, t)} d\theta^1 + \frac{1}{2} \frac{\Lambda_{11}^2 (\theta^1, m, t)}{\Lambda^2 (\theta^1, m, t)} (d\theta^1)^2 + \frac{\Lambda_2^2 (\theta^1, m, t)}{\Lambda^2 (\theta^1, m, t)} d\theta^2 + \frac{1}{2} \frac{\Lambda_{22}^2 (\theta^1, m, t)}{\Lambda^2 (\theta^1, m, t)} (d\theta^2)^2
\]
\[
+ \frac{1}{2} \frac{\Lambda_{12}^2 (\theta^1, m, t)}{\Lambda^2 (\theta^1, m, t)} d\theta^1 d\theta^2 + \frac{1}{2} \frac{\Lambda_{12} (\theta^1, m, t)}{\Lambda (\theta^1, m, t)} d\theta^1 dY_t
\]
\[
= \left[ -\lambda_t^2 - \gamma m + \frac{1}{2} \gamma (\gamma + 1) \sigma_y^2 + \frac{\Lambda_1^2 (\theta^1, m, t)}{\Lambda^2 (\theta^1, m, t)} (\mu_\theta - \theta \sigma_\theta \sigma_y) + \frac{\Lambda_2^2 (\theta^1, m, t)}{\Lambda^2 (\theta^1, m, t)} (\mu_m - \gamma \sigma_m \sigma_y) + \frac{1}{2} \frac{\Lambda_{11}^2 (\theta^1, m, t)}{\Lambda^2 (\theta^1, m, t)} \sigma_\theta^2 + \frac{1}{2} \frac{\Lambda_{22}^2 (\theta^1, m, t)}{\Lambda^2 (\theta^1, m, t)} \sigma_m^2 + \frac{\Lambda_{12}^2 (\theta^1, m, t)}{\Lambda^2 (\theta^1, m, t)} \sigma_\theta \sigma_m \right] dt +
\]
\[
\left[ -\gamma \sigma_y + \frac{\Lambda_1 (\theta^1, m, t)}{\Lambda^2 (\theta^1, m, t)} \sigma_\theta + \frac{\Lambda_2 (\theta^1, m, t)}{\Lambda^2 (\theta^1, m, t)} \sigma_m \right] d\bar{B}_{Y,t},
\]
\[
= -r (\theta^1, m, t) dt - \sigma^2 (\theta^1, m, t) d\bar{B}_{Y,t},
\]
\[
\text{(55)}
\]
Therefore, the risk free rate and the perceived risk by agent 2 is
\[
    r (\theta^1, m, t) = \lambda_t^2 + \gamma m - \frac{1}{2} \gamma (\gamma + 1) \sigma_y^2
\]
\[
    - \frac{\Lambda_1^2 (\theta^1, m, t)}{\Lambda^2 (\theta^1, m, t)} (\mu_\theta - \theta \sigma_\theta \sigma_y) - \frac{\Lambda_2^2 (\theta^1, m, t)}{\Lambda^2 (\theta^1, m, t)} (\mu_m - \gamma \sigma_m \sigma_y) - \frac{1}{2} \frac{\Lambda_{11}^2 (\theta^1, m, t)}{\Lambda^2 (\theta^1, m, t)} \sigma_\theta^2 - \frac{1}{2} \frac{\Lambda_{22}^2 (\theta^1, m, t)}{\Lambda^2 (\theta^1, m, t)} \sigma_m^2 - \frac{\Lambda_{12}^2 (\theta^1, m, t)}{\Lambda^2 (\theta^1, m, t)} \sigma_\theta \sigma_m
\]
43
\[
\sigma^2_{\Lambda}(\theta^1, m, t) = \gamma \sigma_y - \frac{1}{\psi} \left[ \frac{\sigma^2_{\theta^1}(\theta^1, m, t)}{\xi^2(\theta^1, m, t)} \sigma_{\theta^1} + \frac{\sigma^2_m(\theta^1, m, t)}{\xi^2(\theta^1, m, t)} \sigma_m \right]
\]

\[
\frac{1}{\psi} - \gamma \left[ \frac{\tilde{f}^2_{\theta^1}(\theta^1, m, t)}{\tilde{f}^2(\theta^1, m, t)} \sigma_{\theta^1} + \frac{\tilde{f}^2_m(\theta^1, m, t)}{\tilde{f}^2(\theta^1, m, t)} \sigma_m \right]
\]

(56)

We can use the same method to drive the law of motion of SDF perceived by agent 1: The state price density under agent 1’s belief is given by

\[
\frac{d\Lambda^1_t}{\Lambda^1_t} = -r(\theta^1, m, t) dt - \sigma^1_{\Lambda}(\theta^1, m, t) d\tilde{B}^1_{Y,t}
\]

where

\[
\sigma^1_{\Lambda}(\theta^1, m, t) = \sigma^2_{\Lambda}(\theta^1, m, t) + \tilde{\mu}^1(t) - \tilde{\mu}^2(t)
\]

(57)

By the law of motion of the stock return, the risk premium of the risky asset perceived by agent \(i \in \{1, 2\}\) is

\[
\mu^i_R(\theta^1, m, t) - r(\theta^1, m, t) = \sigma_R(\theta^1, m, t) \sigma^i_{\Lambda}(\theta^1, m, t)
\]

(58)

and the difference of the risk premium perceived by the two agents is determined by

\[
\mu^1_R(\theta^1, m, t) - \mu^2_R(\theta^1, m, t) = \sigma_R(\theta^1, m, t) [\sigma^1_{\Lambda}(\theta^1, m, t) - \sigma^2_{\Lambda}(\theta^1, m, t)]
\]

(59)
References


Table 1: The change of open interests of SPX comparing to that before announcement

<table>
<thead>
<tr>
<th></th>
<th>the 1st day change</th>
<th>the 2nd day change</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>call</td>
<td>put</td>
</tr>
<tr>
<td>1996-2018</td>
<td>-3.71%</td>
<td>-3.52%</td>
</tr>
<tr>
<td>2014-2018</td>
<td>-12.85%</td>
<td>-11.05%</td>
</tr>
</tbody>
</table>
Table 2: Key parameters

I calibrate the model at annually frequency. We assume that announcements are made at the monthly frequency, that is, $T = \frac{1}{12}$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>symbol</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>aggregate output</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>long run output growth rate</td>
<td>$\bar{\theta}$</td>
<td>1.50%</td>
</tr>
<tr>
<td>volatility of output</td>
<td>$\sigma_y$</td>
<td>3.20%</td>
</tr>
<tr>
<td>persistence of the AR(1) process</td>
<td>$\rho$</td>
<td>5%</td>
</tr>
<tr>
<td>volatility of the AR(1) process</td>
<td>$\sigma_\theta$</td>
<td>0.10%</td>
</tr>
<tr>
<td><strong>disagreements and uncertainty</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>agent 1’s new added disagreement</td>
<td>$\varepsilon_1$</td>
<td>0.013</td>
</tr>
<tr>
<td>agent 2’s new added disagreement</td>
<td>$\varepsilon_2$</td>
<td>-0.011</td>
</tr>
<tr>
<td>probability of the new added disagreement</td>
<td>$p$</td>
<td>$\frac{1}{3}$</td>
</tr>
<tr>
<td>variance of the signal upon announcements</td>
<td>$\sigma_\delta^2$</td>
<td>$10^{-8}$</td>
</tr>
<tr>
<td><strong>preference</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>risk aversion</td>
<td>$\gamma$</td>
<td>5</td>
</tr>
<tr>
<td>Elasticity of intertemporal substitution</td>
<td>$\psi$</td>
<td>1.4</td>
</tr>
<tr>
<td>subjective discount factor</td>
<td>$\beta$</td>
<td>0.05</td>
</tr>
</tbody>
</table>
Table 3: Aggregate Quantities and Prices

This table presents annualized macroeconomic and asset pricing moments from the data and the benchmark model. Panel A reports the distribution of the heterogeneous beliefs of both agents. Panel B reports the asset prices and return where I impose the operation leverage of 4. The first three excess returns are calculated under the objective measure. Panel C reports the trading volume and short interest.

<table>
<thead>
<tr>
<th>Moments</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Heterogeneous beliefs</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The mean of the optimistic forecast comparing to consensus</td>
<td>3.50%</td>
<td>3.50%</td>
</tr>
<tr>
<td>The variance of the optimistic forecast comparing to consensus</td>
<td>1.50%</td>
<td>1.30%</td>
</tr>
<tr>
<td>The mean of the pessimistic forecast comparing to consensus</td>
<td>-3.10%</td>
<td>-3.10%</td>
</tr>
<tr>
<td>The variance of the pessimistic forecast comparing to consensus</td>
<td>1.30%</td>
<td>1.10%</td>
</tr>
<tr>
<td><strong>Panel B: Asset prices and returns</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>risk free rate</td>
<td>2.50%</td>
<td>2.70%</td>
</tr>
<tr>
<td>volatility of risk free rate</td>
<td>1.30%</td>
<td>1.64%</td>
</tr>
<tr>
<td>market equity premium (per year)</td>
<td>6.19%</td>
<td>5.53%</td>
</tr>
<tr>
<td>Announcement premium (per day)</td>
<td>11.27 bps</td>
<td>4.80 bps</td>
</tr>
<tr>
<td>Non Announcement premium (per day)</td>
<td>1.27 bps</td>
<td>1.24 bps</td>
</tr>
<tr>
<td>Agent 1’s Announcement premium (per day)</td>
<td></td>
<td>4.91 bps</td>
</tr>
<tr>
<td>Agent 1’s Non Announcement premium (per day)</td>
<td></td>
<td>1.42 bps</td>
</tr>
<tr>
<td>Agent 2’s Announcement premium (per day)</td>
<td></td>
<td>4.74 bps</td>
</tr>
<tr>
<td>Agent 2’s Non Announcement premium (per day)</td>
<td></td>
<td>0.48 bps</td>
</tr>
<tr>
<td><strong>Panel C: Trading volume and short interest</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>trading volume per hour after announcements</td>
<td>4.5</td>
<td>5.5</td>
</tr>
<tr>
<td>trading volume per hour before announcements</td>
<td></td>
<td></td>
</tr>
<tr>
<td>short interest</td>
<td>3.70%</td>
<td>3.41%</td>
</tr>
</tbody>
</table>