Is There A Shortfall in Public Investment?
An Asset Pricing Appraisal

Abstract

Is investment in public sector capital inadequate in the U.S.? The answer is yes from investors’ perspective. This paper takes an asset pricing approach to evaluating the overall (in)adequacy of public sector investment. I propose a two-sector general equilibrium model that demonstrates how the share of public sector capital may enter the pricing kernel. From this theory I derive a factor pricing model with shocks to the public sector investment share (“PUB shocks”) as a risk factor. I confront the factor model with a variety of test assets and find that PUB shocks are priced and carry a consistently positive price of risk. I find further support from the analysis of a sample of U.S. government contractors: I postulate that the extent to which a firm depends on government customers for revenue is a relevant proxy for its exposure to public sector investment. I find that high-dependency firms (that is, firms with greater sales to government relative to their total sales) provide a 7.4% higher average return annually compared to low-dependency firms. A subsample analysis reveals that this return spread is widening as the public sector investment share declines; it implies a growing shortfall in public sector investment in recent years.
Public investment is consequential to the economy, and it has received renewed interest since the 2016 United States presidential election. As stories of crumbling infrastructure abound, policymakers have been discussing potential increases in government expenditures on public sector capital. At the center of these discussions is the appropriate level of public sector investment, the evidence on which, however, is limited and often controversial. Thus when it comes to the question of whether more resources should be allocated to augment the stock of public sector capital, there is little consensus.

In this paper, I take an asset pricing approach to evaluating the overall (in)adequacy of public sector investment. The basic idea is as follows. If investors perceive a shortfall in public sector capital, then positive shocks to the share of public sector investment (henceforth, “PUB shocks”) would be considered good news (in the sense that they coincide with favorable shifts in investors’ welfare) and thus should carry a positive price of risk (meaning, ceteris paribus, assets with more positive exposure to such shocks should provide higher risk premiums). I operationalize this idea and employ standard asset pricing methodology to determine the price of risk for PUB shocks. I first propose a theory as to how the share of public sector capital may enter the pricing kernel in general equilibrium. Based on this theory, I derive a factor pricing model with PUB shocks as a risk factor and confront the model with a variety of test assets. The results indicate that exposure to PUB shocks is priced and has a robustly positive price of risk. I find further support from the analysis of a sample of U.S. government contractors, which reveals that firms with heavier reliance on government as a customer are more sensitive to variations in public investment and provide higher stock returns on average. I also find that the spread in average returns between firms with high and low government dependency has widened in recent years, pointing to a growing shortfall in public sector capital.

For starters, I briefly review the evolution of public sector investment in the U.S., comparing it with that of private sector (nonresidential) investment. On average, national investment (private plus public sector investments) represents about 12% of gross domestic product (GDP) in the postwar U.S. economy, of which roughly one third is public sector investment. The latter ratio, which I refer to as the public sector investment share, alternatively, one can use GDP as the denominator when defining the public sector investment share, the behavior of which turns out to be very similar.
has witnessed significant variations: as shown in panel (a) of Figure 1, it increased in the
1950s, peaked in the early 1960s, and has since been trending downward. The most recent
reading shows a new record low of less than 15%, meaning that the size of public sector
investment is merely one sixth of that of private sector investment. Besides the secular
trend, the public sector investment share has seen sizable fluctuations at business-cycle
frequencies as well. Panel (b) of Figure 1 displays the cyclical component of this ratio,
alongside a macro uncertainty index known to be strongly countercyclical ([Jurado, Lud-
vigson, and Ng, 2015] henceforth, JLN). One can clearly see that a higher public sector
investment share and greater macro uncertainty accompany most contractions; the oppo-
site is true for most expansions.  

Figure 1: Public sector investment share. The solid line in panel (a) represents the public
sector investment share, that is, the ratio of public sector (nondefense) investment to the sum
of public and private sector (nonresidential) investments. The cyclical component of this ratio,
which is derived by applying the Hamilton (2018) regression filter, is shown in panel (b), together
with a macroeconomic uncertainty index constructed by Jurado, Ludvigson, and Ng (2015). The
magnitudes of the cyclical component and the uncertainty index are indicated on the left and
right vertical axes, respectively. Shaded areas indicate U.S. recessions identified by NBER. Related
variables are more precisely defined in Section 3.

These facts raise questions about the appropriate level of public sector investment.
From a macro perspective, what is the optimal allocation of capital between the private
and public sectors? Does the declining share of public sector investment represent less
crowding-out of more productive private sector capital and thus better welfare? Or does

4The downward trend of the public sector investment share reflects the slower growth of public sector
investment (0.97% quarterly) relative to that of private sector investment (1.07%). Its countercyclicality
mirrors the fact that public sector investment is much less procyclical than private sector investment: the
growth of private sector investment has correlations of around 60% with GDP growth and -52% with the
JLN uncertainty index, whereas the analogous magnitudes for public sector investment are 17% and -16%.
it indicate a growing shortfall in public sector capital that actually impairs the economy? Should there be an expansion in public sector investment (a mooted plan in the U.S.), what would the net effect likely be?

Investors may provide informative answers to these questions, and their views are reflected in asset prices. In standard asset pricing theory, state variables with pervasive influence on investors’ welfare are priced, meaning assets with different exposure to such variables should carry different risk premiums. I hypothesize that the share of public sector capital is a priced state variable, for which the price of risk reflects investors’ opinion of its proper level. I begin by providing theoretical support for this hypothesis.

To theoretically link the share of public sector capital to investors’ welfare, I develop a parsimonious two-sector general equilibrium (GE) model. There are three key ingredients in the model. First, I postulate a neoclassical aggregate production function that features private-public capital complementarity (Baxter and King, 1993). This feature implies that augmenting the stock of public sector capital increases the marginal product of private sector capital, and vice versa. (Technically, it guarantees a nontrivial stationary equilibrium with both sectors accounting for nonzero fractions of capital in the long run.) Second, I incorporate time-varying uncertainty as a driver of business cycles per Bloom et al. (2018). This feature captures the notion that business conditions are uncertain, and, more importantly, the level of uncertainty varies countercyclically. Third, I posit the public sector as a “safe” sector, similar in spirit to the stylized model of Rodrik (1998). This feature renders public investment a risk-reducing instrument. 5

As a result of these features, an expansion in the share of public sector investment has two effects: an ambiguous productivity effect per Aschauer (1989a) whereby higher public sector investment crowds out private capital but raises its marginal product; and a positive risk-mitigating effect whereby higher public investment contributes to lower fundamental volatility. So from the perspective of a representative agent, unless the marginal product of private sector capital exceeds that of public sector capital to the extent that the productivity effect becomes negative and dominates the risk-mitigating effect, expanding public sector investment would be perceived as beneficial (which corresponds to a positive price of risk for PUB shocks).

Admittedly, there are other mechanisms as to how public sector investment may affect the economy. 6 But I focus on the productivity effect and the risk-mitigating effect

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5Intuitively, under the assumption that the public sector is the safe sector, increasing public sector investment would allocate more capital to the safe sector and thus reduce the overall volatility.

6For example, on Keynesian grounds, higher government investment may boost demand and help close the output gap; but it may add to the fiscal deficit that, if spiralling out of control, can cause dreadful consequences. Sympathetic to this view, [The Economist] (2019) warns that “disregarding deficits” runs the risks of
for good reason (besides parsimony). I consider the productivity effect for its practical relevance. As pointed out by Blanchard (2016), “U.S. government borrowing costs are very low... the relevant opportunity cost of public investment would not be the rate on government bonds but the marginal product of the private capital that would be crowded out.” I incorporate the risk-mitigating effect to match the stylized fact that larger governments are associated with less volatile economies (Gali, 1994; Fatas and Mihov, 2001). It also enables the model to endogenously produce different cyclicalities for private and public sector investments (a pattern that underlies Figure 1), which is illustrated next.

To bring more discipline to the model, I calibrate it reasonably and then examine its implications. The calibrated model fares well in replicating a selection of salient business-cycle regularities. In particular, it can rationalize the different cyclical properties of private and public sector investments with a risk-based mechanism. As an illustration, consider a hypothetical recession episode that is caused by a combination of negative productivity shocks and positive uncertainty shocks. On the one hand, elevated uncertainty prompts a reallocation of capital from the private sector to the public sector. This is referred to as a real-flight-to-safety effect. On the other hand, output maximization requires allocating capital to whichever sector with the highest marginal product. This is referred to as a balancing effect. When uncertainty rises, the first effect dominates, stimulating public sector investment. As uncertainty recedes, the second effect prevails, restoring investment in the private sector. Costly capital adjustment prolongs this process.

The workings of the real-flight-to-safety effect can be understood as follows. There are two primary channels through which uncertainty influences private and public sector investments. Greater uncertainty working through the precautionary savings channel lowers the risk-free rate, exerting a positive influence on investments in both sectors. Meanwhile, higher uncertainty raises the risk premium on private sector capital (that is, the risk premium channel) to an extent that more than offsets the decrease in the risk-free rate. Together this amounts to an increase in the cost of private sector capital, leading to a fall in capital value relative to dividends. The depressed capital value imposes a negative in-
fluence on private sector investment via Tobin’s $q$. As a result, private sector investment plummets because of enormous increases in the risk premium, whereas public sector investment is buttressed by the strengthened precautionary savings motive.\footnote{It is worth noting that the real-flight-to-safety effect is the key to generating the different cyclicalities for private and public sector investments, which would otherwise move in lockstep. Because of this effect, agents willingly choose a relatively higher rate of investment in public sector capital during recessions even though its marginal product can be lower than that of private sector capital.}{\footnote{In the extant literature (Ramey 2011), a typical explanation of the contrasting cyclical properties of private and public sector investments is derived from the Keynesian argument that government spending should cushion the economy from downturns by boosting demand. The key mechanism in the Keynesian theory is the fiscal multiplier effect, and its magnitude is a primary point of contention (that is, whether it is large enough to outweigh the potential costs). To the extent that the extra demand brought by public sector investment is worth the potential costs, the government should let public sector investment, if not increase, at least fall less relative to private sector investment during recessions. Compared with this demand-based explanation, my model features an alternative rationale that emphasizes a risk-mitigating effect of public sector investment.}}

The balancing effect simply stems from the assumption of private-public capital complementarity. This assumption implies that a low capital stock in either sector entails a high marginal product of capital in that sector, which, ceteris paribus, warrants a high investment rate. In particular, in the absence of sector heterogeneity, the balancing effect would equalize the marginal products of private and public sector capital.

The calibrated model makes several predictions that are consistent with data. First, the model predicts a positive relationship between the public sector investment rate and the risk premium on private sector capital. This prediction is in line with the empirical evidence documented by\footnote{He, Kelly, and Manela (2017) also use the market excess return as a proxy for the Total-Factor-Productivity-style persistent technology shock.} Belo and Yu (2013); they find that a higher public sector investment rate tends to precede higher excess returns on stocks at both the aggregate and the firm level. Also, the model predicts that greater uncertainty leads to a higher public sector investment share while a lower risk-free interest rate; but controlling for uncertainty, these two variables are positively related. I find supporting evidence for this prediction.

Most important, the model generates a pricing kernel that is determined by the share of public sector capital alongside economic uncertainty and the aggregate stock of productive capital. Based on this pricing kernel, I derive a three-factor model with PUB shocks, uncertainty shocks, and the market excess return as risk factors. PUB shocks influence the share of public sector capital; they may stem from, for example, unforeseen fiscal developments. Uncertainty shocks represent news that alter the variability of business conditions. The market excess return captures the standard productivity shocks that affect the efficiency with which physical capital is used (that is, shocks to the stock of productive capital). This factor model underpins the first part of my empirical investi-
The GE model only provides a theoretical argument for why agents may care about the share of public sector capital. In practice, does it really concern investors to the extent that they might demand hedges against unfavorable changes in the public sector investment share? If yes, what changes are considered unfavorable, increase or decrease? To answer these questions, I empirically estimate the price of risk for PUB shocks. Equipped with the factor model derived from the GE theory, I perform standard two-pass asset pricing tests using a variety of well-known equity portfolios. My main finding is that assets’ exposure to PUB shocks possess significant explanatory power for cross-sectional differences in average asset returns, and that the estimated price of risk for PUB shocks is consistently positive. This finding is robust to a wide range of test assets including portfolios formed on size, book-to-market (BM) ratio, momentum, investment, and profitability. It implies that investors’ marginal utility moves up when the share of public sector investment declines, and that assets paying off in this case are valuable hedges.

To seek additional support, I examine a sample of U.S. government contractors. I postulate that the extent to which a firm depends on government for revenue is a relevant proxy for its exposure to public investment. I form stock portfolios based on firms’ government dependency, which is measured by the average fraction of sales to government over the past three years. I find that high-dependency firms are more sensitive to variations in public investment and provide higher stock returns on average compared to low-dependency firms. A zero-investment portfolio that is long stocks in the highest dependency quintile and short stocks in the lowest dependency quintile provides an average return of 7.4% annually. I confirm that this return spread is not driven by differential loadings on classic risk factors. Lastly, I conduct a subsample analysis and find that the spread in average returns between high- and low-dependency firms was minor in the 1980s and 1990s, but it has widened considerably in recent years and looks to continue. Together these findings suggest that there is a shortfall in public investment, and that it seems to be worsening lately.

Related literature. This paper contributes to a substantial literature studying the economic effects of public sector capital. Since the seminal work by Aschauer (1989a,b), a lot of research has been dedicated to understanding the mechanisms by which public sector investment influences the economy. Some studies examine public sector investment at

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13It is worth emphasizing that this factor model is actually more general than the GE framework presented here. One may come up with alternative frameworks in which the equilibrium pricing kernels are determined by the same set of state variables.
the aggregate level, while others focus on specific types of investments. In any case, the common goal of these studies is to estimate the value of public sector capital, which together with the information on its potential costs (e.g., higher deficits and taxes, displacement of private sector investment, etc.) help to answer the normative question of whether government should increase or decrease public sector investment. Compared with existing studies, I take a novel approach to this question, inferring investors’ opinion on this matter from asset prices. I demonstrate that shocks to the share of public sector investment are a source of risk that is priced in the cross section of expected returns and carries a positive price of risk. It suggests that investors’ welfare declines when public sector investment dwindles, and assets that pay off in this case are considered as valuable hedges and hence deliver lower average returns.

My work also relates to Belo and Yu (2013), who made the first attempt to link public sector capital to the stock market. I extend their work and demonstrate how public sector capital may enter the pricing kernel in general equilibrium. The model in this paper stems from a strand of macro-finance literature that studies the joint dynamics of macro quantities and asset prices in a GE framework. Pioneering work by Jermann (1998) and Tallarini (2000) examines time-inseparable preferences (habit formation preferences and recursive preferences, respectively) in this framework and has achieved some success in reconciling business-cycle regularities with asset pricing facts. Their models are extended in various ways to address many issues, among which Eberly and Wang (2011)’s two-sector model is the most similar to mine. Our main difference is that they study sector heterogeneity in the form of asymmetric adjustment costs, whereas I consider two sectors with asymmetric risk profiles.

Outline. The remainder of this paper is structured as follows. Section 1 introduces a two-sector general equilibrium model that demonstrates the asset pricing role of public sector investment in theory. Section 2 discusses the model implications. To investigate whether and how PUB shocks are priced in practice, Section 3 takes to data a factor pricing model derived from this GE theory, and Section 4 conducts a portfolio analysis using a sample of U.S. government contractors. Section 5 concludes. Appendix A and B provide...
supplementary details and results.

1 Model

In this section, I develop a two-sector general equilibrium model to study the asset pricing role of public sector investment. I start by explaining the model ingredients in the context of a centralized economy. Then I solve the central planning problem and substitute the optimal policy into a set of equilibrium conditions derived from a corresponding decentralized economy. The welfare theorems hold in this setting, guaranteeing that the optimal policy obtained from the central planning problem constitutes a competitive equilibrium in the decentralized economy. As a result, I attain equilibrium quantities and prices expressed as functions of the state of the economy.

1.1 Setup

Consider a two-sector production economy cast in continuous time with an infinite horizon. An infinitely lived representative agent with recursive preferences presides over this economy, whose objective is to maximize her expected lifetime utility. The private and public sectors accumulate capital separately with different technologies. A single type of good is produced via an aggregate production technology with capital from both sectors as inputs. This produced good can be either consumed right away or transformed into capital and installed in either sector. Figure 2 provides a schematic representation of the basic model structure. Details on each element are provided next.

**Capital accumulation.** The private and public (governmental) sectors—denoted by \( p \) and \( g \), respectively—accumulate capital independently. The stocks of capital in these two sectors are denoted by \( K_p^t \) and \( K_g^t \), respectively, which evolve according to

\[
\frac{dK_p^t}{K_p^t} = [\phi(i^p_t) - \delta]dt + \sigma dZ_t + \zeta_t dW_t \\
\frac{dK_g^t}{K_g^t} = [\phi(i^g_t) - \delta]dt + \sigma dZ_t,
\]

where \( i^p_t \equiv I^p_t / K_p^t \) and \( i^g_t \equiv I^g_t / K_g^t \) are investment-capital ratios, and \( \delta \) is the depreciation rate.\(^{16}\) As is standard in the literature, I assume that capital investment incurs adjustment costs: investing in sector \( i \in \{ p, g \} \) at a rate of \( i_t K_i^t \) per unit of time can sustain a capital

\(^{16}\)I use the same depreciation rate for private and public sector capital because data are generally unavailable to produce a comprehensive measure of government inventory depreciation (U.S. Bureau of Economic Analysis 2019). Besides, this parameter has little impact on my results.
growth rate of $\phi(\frac{\lambda}{e})$ before depreciation. Function $\phi(\cdot)$, which satisfies $\phi'(\cdot) > 0$ and $\phi''(\cdot) < 0$, represents a classic investment technology with adjustment costs. Intuitively, investment activities, such as constructing new buildings and installing new equipment, incur extra costs on top of the purchase prices. For public sector investment, the adjustment costs may also come from the legislative process associated with fiscal planning.

It is worth noting that, in this specification, the quantity of capital is measured in “productivity” (or “efficiency”) units, as opposed to physical units. To understand the nuance, one can think of each capital stock defined above as a product of the raw quantity of capital measured in physical units and a productivity index (that is, $K_t \equiv A_t K_t$, where $K_t$ is the raw quantity of capital, and $A_t$ indexes productivity). The capital accumulation measured in physical units is deterministic and endogenously determined (e.g., $\frac{dK_t}{K_t} = [\phi(\lambda_t) - \delta] dt$), whereas the corresponding productivity is subject to exogenous shocks (e.g., $\frac{dA_t}{A_t} = \sigma dZ_t$). With the investment rate $\lambda_t$ and the adjustment costs function $\phi(\cdot)$ accordingly defined, one can apply Ito’s lemma to $K_t$ and obtain a stochastic process of the presented form.\(^{17}\)

I consider two mutually independent Wiener processes, $Z$ and $W$, as sources of exogenous shocks. Without loss of generality, I assume that both sectors have the same exposure ($\sigma$) to the process $Z$, while the private sector additionally has a time-varying exposure ($\zeta_t$) to the process $W$. Hence $\zeta_t$ is effectively the volatility spread between the

\[^{17}\text{Brunnermeier and Sannikov (2014) provide another illustration.}\]
private and public sectors, which I assume follows

\[ d\zeta_t = \kappa(\zeta - \zeta_t)dt - \nu \sqrt{\zeta_t} dZ_t, \]

where \( \kappa \) controls the speed of mean-reversion, \( \zeta \) is the long-run mean, and \( \nu \) governs the exposure to innovations in the process \( Z \). I assume \( \nu > 0 \) in all cases, so one may interpret a negative shock to \( Z \) as a “recession” shock that reduces the productivity of both sectors and elevates uncertainty in the private sector. This specification is in accordance with Bloom et al. (2018), who argue that recessions are best modeled as a combination of negative first-moment shocks and positive second-moment shocks.

Aggregate production. I denote by \( K_t \equiv (K^p_t + K^g_t) \) the aggregate stock of capital, and by \( \chi_t \equiv \frac{K^g_t}{K_t} \) the fraction accounted for by the public sector. Applying Ito’s lemma to \( K_t \) and \( \chi_t \), I obtain

\[
\frac{dK_t}{K_t} = \left[ (1 - \chi_t)\phi(i^p_t) + \chi_t\phi(i^g_t) - \delta \right] dt + \sigma dZ_t + (1 - \chi_t)\zeta_t dW_t,
\]

\[
d\chi_t = \chi_t(1 - \chi_t)\left[ \phi(i^g_t) - \phi(i^p_t) + (1 - \chi_t)\zeta_t^2 \right] dt - \chi_t(1 - \chi_t)\zeta_t dW_t.
\]

An aggregate production technology employs capital from both sectors to produce a final good at a rate of \( M(\chi_t)K_t \) per unit of time. \( M(\chi_t) \) is defined as a constant elasticity of substitution (CES) function

\[
M(\chi_t) = m \left[ a(1 - \chi_t)^{\frac{s-1}{s}} + (1 - a)\chi_t^{\frac{s-1}{s}} \right]^{\frac{s}{s-1}}
\]

that has an interior maximum (that is, \( \exists \chi^* \) such that \( M(\chi^*) \geq M(\chi) \) for \( \forall \chi \in (0, 1) \)). The parameter \( a \) determines the output-maximizing allocation of capital between the private and public sectors, \( m \) the scale, and \( s \) the elasticity of substitution. Function \( M(\chi_t) \)

18 I could allow uncertainty shocks to impact the public sector to a lesser extent. That does not change the main results.

19 Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2018) distinguish between two types of uncertainty, that is, macroeconomic and microeconomic uncertainty. The former concerns the variability in productivity of all firms as a whole, while the latter measures the dispersion in productivity across firms. Because the private sector is considered as a whole in my model, uncertainty \( \zeta_t \) corresponds to “macroeconomic uncertainty” in this definition.

20 To clarify, the output rate \( M(\chi_t)K_t \) is derived from \( m \left[ a(K^p_t)^{s-1} + (1 - a)(K^g_t)^{s-1} \right]^{s/(s-1)} \), which is a standard CES production function with two inputs, \( K^p_t \) and \( K^g_t \). If \( s \rightarrow 0 \), private and public sector capital become perfect complements. If \( s \rightarrow 1 \), this function converges to the popular Cobb-Douglas function. If
has a natural interpretation as a “misallocation discount” because, for a given amount of aggregate capital $K_t$, any deviation of its allocation from the output-maximizing level ($\chi^*$) reduces $M(\chi_t)$ and, in turn, the aggregate output. The causes of misallocation here are the heterogeneous risk profiles of the private and public sectors as well as the capital adjustment costs, without which $\chi_t$ would remain at the output-maximizing level.  

Under this setting, the process $Z$ and $W$ each play two different roles. Shocks to the process $Z$ influence the economy by driving the productivity and uncertainty as already mentioned. Shocks to the process $W$ affect the aggregate output via two channels: the productivity channel and the (mis)allocation channel. To illustrate, consider a positive shock to the process $W$, which increases the aggregate stock of capital ($K_t$) but reduces the capital share of the public sector ($\chi_t$). The former effect unambiguously bolsters output, whereas the latter depends on the state of the economy. In particular, if there is an inadequate stock of capital in the public sector, the latter effect would diminish output, working against the former one. This clarification is critical for understanding the risk prices for the process $Z$ and $W$, as will be obvious later.

Another point worth mentioning is that, to model government’s contribution to production, existing studies consider either the current flow of government spending (e.g., Barro [1990]) or the accumulated stock of public sector capital (e.g., Baxter and King [1993]) as an additional input into the production function. Because the government input considered here is intended to represent productive capital such as infrastructure, I adopt the accumulated stock approach.

**Preferences and resource constraint.** The representative agent has recursive preferences with the time discount $\beta$, the elasticity of intertemporal substitution (EIS) $\psi$, and the relative risk aversion (RRA) $\gamma$:

$$V_t = E_t \int_t^{\infty} u(C_\tau, V_\tau) d\tau \quad \text{with} \quad u(C, V) = \beta(1 - \gamma) V \frac{C^{1-1/\psi}}{1-1/\psi} \left\{ \frac{C^{1-1/\psi}}{[(1-\gamma) V]^{1-1/\psi}} - 1 \right\}, \quad (5)$$

where $E_t$ is an expectation operator conditional on time-$t$ information. As is well known, recursive preferences allow a separation between the EIS and the RRA. The agent’s objec-

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21 Because the term “misallocation” carries different meanings in different context, it is worth emphasizing that its interpretation here follows that in the economic growth literature (Jones [2016]). Capital is considered to be misallocated when its distribution between the private and public sectors is not at the output-maximizing level.

22 In practice, this may correspond to the case in which an underdeveloped infrastructure harms the economy by lowering the marginal products of private inputs.
tive is to maximize utility while subject to the resource constraint

$$C_t + I_t^P + I_t^S = M(\chi_t)K_t, \quad (6)$$

which implies that she faces a twofold decision: (1) dividing output between consumption and investment, and (2) allocating investment between the private and public sectors.

**Discussion.** Two critical assumptions of the model merit further discussion. First, I assume that augmenting the stock of public sector capital raises the marginal product of private sector capital, and vice versa. This assumption, which underpins the CES production function (4), stems from a substantial literature on the productivity effects of public investment. In particular, the seminal work by [Aschauer (1989a,b)] finds that public sector capital has nontrivial influence on aggregate productivity: increasing the stock of public sector capital contributes to the marginal product of private sector capital. His finding has gained traction in the literature, and subsequent studies generally come to the same conclusion (despite some disputes on the magnitude of the effects). [23][24] Second, I postulate the public sector as a "safe" sector with less volatile risk dynamics. Under this assumption, an expansion in the share of public sector capital ($\chi_t$) reduces the aggregate volatility, $\sqrt{\sigma^2 + (1 - \chi_t)^2 \varsigma_t^2}$. (At the extreme with all capital installed in the public sector ($\chi_t = 1$), the aggregate volatility would become constant at $\sigma_r$.) This risk-mitigating assumption is motivated by the literature on government size and macroeconomic stability ([Gali, 1994]; [Fatas and Mihov, 2001]). In particular, [Fatas and Mihov, 2001] document a strong negative correlation between government size and macroeconomic variability; the results hold regardless of the measures and are robust both for OECD countries and across states in the U.S. [25] With these two assumptions, the model captures two impor-

[24] Anecdotal evidence suggests that the productive role of public sector capital continues to be relevant. For example, [Gopalswamy and Rathinam, 2018] propose a new approach to autonomous driving that involves upgrading the road infrastructure. They argue that, by taking some responsibility off the shoulders of car manufacturers, this approach can "accelerate the deployment of autonomous driving and correspondingly reap its benefits."
[25] Admittedly, in my model, the risk-mitigating role of public investment arises simply because of a composition effect: insofar as the public sector is the safe sector, expanding its capital share lowers the aggregate volatility. But the literature has argued for other mechanisms as well. For example, [Fatas and Mihov, 2001] suggest the Keynesian "automatic stabilizers" like taxes and transfers as the main drivers. [Andres, Domenech, and Fatas, 2008] examine this idea via a theoretical approach: they demonstrate how to rationalize the stabilizing role of government size in a RBC model augmented with Keynesian features. More recently, [Goldman, 2019] documents a stabilizing effect of government purchases on government contractors as well as their neighboring firms during the Great Recession. I am sympathetic to these alternative mechanisms but prefer the current setup, which is similar in spirit to [Rodrik, 1998], for its simplicity. Exploring other sophisticated stabilization mechanisms within this framework is left for future research.
tant considerations—that is, the influence on productivity and stability—in determining the appropriate level of public sector investment.

1.2  Solution

I solve the model in two steps. First, I obtain the optimal investment policy by working out the central planning problem. Then I decentralize the model and derive equilibrium conditions that connect macro quantities and prices. Finally, substituting the optimal investment policy into the equilibrium conditions enables me to express all quantities and prices as functions of the state variables. The following summarizes the key solution steps; omitted details and proofs are given in Appendix.

Central planning.  In this model, the state of the economy can be summarized by three variables: the aggregate stock of capital $K_t$, the share of public sector capital $\chi_t$, and the level of economic uncertainty $\varsigma_t$.\footnote{One feature of the model is that the two state variables—$\chi_t$ and $\varsigma_t$—are separately driven by two independent Wiener processes, $W$ and $Z$. Hence, shocks to the process $W$ and $Z$ can also be interpreted as “allocation shocks” and “uncertainty shocks”, respectively.} The first variable merely controls the scale of the economy, while the last two are the effective state variables that determine economic prospects (or, equivalently, investment opportunities). Providing the current state of the economy, the representative agent chooses the consumption-investment policy to maximize her expected lifetime utility

$$V(\chi_t, \varsigma_t, K_t) = \max_{\iota^c, \iota^g} \mathbb{E}_t \int_t^\infty u(C_\tau, V_\tau) d\tau$$

subject to (2) and (3). The model is homogeneous in scale, so I conjecture that the representative agent’s value function takes the form of

$$V(\chi_t, \varsigma_t, K_t) = \frac{(\xi_t K_t)^{1-\gamma}}{1-\gamma},$$

where $\xi_t \equiv \xi(\chi_t, \varsigma_t)$ is a function to be determined. I interpret $\xi_t$ as a welfare multiplier that gauges the influence of future economic prospects on the ex ante lifetime utility. Good economic prospects—that is, a near-optimal allocation of capital and low economic uncertainty—contribute to a large $\xi_t$, meaning that the agent expects to derive a higher lifetime utility given the current stock of capital. I demonstrate the property of $\xi_t$ in Figure

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6 The process followed by $\xi_t$ can be obtained using Ito’s lemma:

$$\frac{d\xi_t}{\xi_t} = \mu_{\xi,t} dt + \sigma_{\xi,t} dZ_t + \zeta_{\xi,t} dW_t,$$  \hspace{1cm} (8)

where $\{\mu_{\xi,t}, \sigma_{\xi,t}, \zeta_{\xi,t}\}$ are determined in equilibrium. The HJB equation associated with the central planning problem is given by

$$\frac{\beta}{1 - 1/\psi} = \max_{\iota', \iota} \frac{\beta}{1 - 1/\psi} \left( \frac{c_{\iota'}}{\xi_{\iota'}} \right)^{1-1/\psi} + \mu_{\iota',t} + \mu_{\xi,t} - \frac{\gamma}{2} \left[ \sigma^{2} + (1 - \chi_t)^{2} \xi_{\iota'}^{2} + \sigma_{\xi,t}^{2} + \zeta_{\xi,t}^{2} \right] + (1 - \gamma) [\sigma \sigma_{\xi,t} + (1 - \chi_t) \chi_t \xi_{\iota, t}],$$  \hspace{1cm} (9)

where I define $c_t \equiv [M(\chi_t) - \iota^p_t (1 - \chi_t) - \iota^s_t \chi_t]$ as the consumption-capital ratio. The optimal investment policy is pinned down by the first-order conditions,

$$\left( \frac{c_t}{\xi_t} \right)^{1/\psi} = \frac{\beta}{\phi'(\iota^p_t) \xi_t - \chi_t \partial \chi_t} \frac{1}{c_t}, \hspace{1cm} \left( \frac{c_t}{\xi_t} \right)^{1/\psi} = \frac{\beta}{\phi'(\iota^s_t) \xi_t + (1 - \chi_t) \partial \chi_t} \frac{1}{c_t} \hspace{1cm} \text{[27]}$$  \hspace{1cm} (10)

Combining (9) and (10) gives a system of partial differential equations on $\xi(\chi_t, \xi_t)$ that is solved using an iterative method. The details of this procedure are given in Appendix B. With the solution for $\xi(\chi_t, \xi_t)$, the optimal investment policy $\{\iota^p(\chi_t, \xi_t), \iota^s(\chi_t, \xi_t)\}$ can be obtained (see Figure 4). Lastly, the equilibrium pricing kernel is a function of the state variables (its exact expression is in Appendix A)

$$\Lambda_t \equiv \Lambda(\chi_t, \xi_t, K_t),$$  \hspace{1cm} (11)

and its law of motion is given by

$$\frac{d\Lambda_t}{\Lambda_t} = -r_idt - \eta_idZ_t - \theta_idW_t,$$  \hspace{1cm} (12)

where $r_i$ is the risk-free interest rate and $\{\eta_i, \theta_i\}$ represent the risk prices for the process $Z$ and $W$. Their expressions are given in equation (20) and (21), as they enter my analysis.

**Decentralization.** To study the asset pricing implications, I specify a decentralized economy that mirrors the centralized one defined above. As before, the private and public sectors independently accumulate capital. The aggregate production employs capital from both sectors and produces a single type of good via the technology specified in (4). With the good price normalized to be one, the marginal products of private and public sector

\footnote{The partial derivative $\frac{\partial Y}{\partial X_1 X_2 \ldots X_n}$ is denoted by $\partial X_1 X_2 \ldots X_n Y$.}
capital are equal to
\[ r^p_t = -\chi_t M'(\chi_t) + M(\chi_t) \]
\[ r^g_t = (1 - \chi_t) M'(\chi_t) + M(\chi_t) \tag{13} \]

The private sector is populated by an all-equity financed firm that is owned by a unit mass of identical and infinitely lived agents who share the same recursive preferences. This representative firm manages the aggregate production and uses the output to pay a lump-sum tax, fund its investment, and distribute dividends to shareholders. Its market value is given by the conditional expectation of total discounted payouts:
\[ V^p_t = \mathbb{E}_t \int_t^{\infty} \frac{\Lambda}{\Lambda_t} D^p_t d\tau \tag{14} \]

where \( V^p_t \) represents the firm value and \( D^p_t = M(\chi_t)K_t - T_t - I^p_t \) is the payout rate. The government is benevolent and sets the lump-sum tax based on the marginal product of public sector capital, that is, \( T_t = r^g_t K^g_t \). It uses the tax revenue to fund public sector investment and transfers any remaining to agents. Hence, the flow fiscal budget constraint is \( \Pi_t + I^g_t = T_t \), where \( T_t \) is the lump-sum tax, and \( \Pi_t \) represents the transfer payments. Consumption thus equals the sum of the total payouts and the transfer payments (\( C_t = D^p_t + \Pi_t \)). I assume that the government chooses the Pareto-optimal investment policy obtained above—by setting \( I^g_t \) to \( I^g(\chi_t, \varsigma_t)K^g_t \)—to maximize welfare. Admittedly, my assumptions on government behavior are somewhat unrealistic, but they allow me to focus on the asset pricing role of public sector capital.

The private firm chooses an investment policy to maximize its value subject to the law of motion of capital in (1). The scale-invariant property of the problem implies that the firm’s value can be expressed as \( V^p_t = q_t K^p_t \), where \( q_t \) can be construed as the market price of private sector capital, which I assume follows
\[ \frac{dq_t}{q_t} = \mu_{q,t} dt + \sigma_{q,t} dZ_t + \varsigma_{q,t} dW_t \tag{15} \]

Suppose the amount of private sector capital increases by \( \epsilon \), and then the aggregate output would become \( M(\frac{K^g_t}{K_t+\epsilon})(K_t + \epsilon) \). Taking derivative w.r.t. \( \epsilon \) and evaluating at \( \epsilon = 0 \), I obtain
\[ \left. \frac{\partial M(\frac{K^g_t}{K_t+\epsilon})(K_t + \epsilon)}{\partial \epsilon} \right|_{\epsilon=0} = -\chi_t M'(\chi_t) + M(\chi_t) \]
which is \( r^p_t \). A similar calculation gives \( r^g_t \). Notice that \( M(\chi_t)K_t - r^p_t K^p_t - r^g_t K^g_t = 0 \).

Ricardian equivalence holds in this setting. Therefore, alternative taxation and financing schemes would give the same results. This setting is similar to a version of Baxter and King (1993)’s model without distortionary taxation.
with \( \{ \mu_{q,t}, \sigma_{q,t}, \varsigma_{q,t} \} \) to be determined in equilibrium. After some algebra, I obtain the HJB equation associated with the firm’s problem as

\[
    r_t - (\sigma_t + \sigma_{q,t}) \eta_t + (\zeta_t + \zeta_{q,t}) \theta_t = \max_{i_t^p} \left\{ \frac{r_t - i_t^p}{q_t} + \phi(i_t^p) - \delta_t + \mu_{q,t} + \sigma_{q,t} + \zeta_t \varsigma_{q,t} \right\}.
\] (16)

The first-order condition from the HJB equation establishes the link between the price of private sector capital and the corresponding investment rate:

\[
    q_t = \frac{1}{\phi'(i_t^p)}.
\] (17)

Substituting in the optimal private sector investment policy \( i_t^p(\chi_t, \varsigma_t) \) attained above, I get the equilibrium price of private sector capital as a function of the state variables, \( q(\chi_t, \varsigma_t) \).

The dynamics of the return to private sector capital can then be easily derived

\[
    dR_t^p = \left[ \frac{r_t^p - i_t^p(\chi_t, \varsigma_t)}{q(\chi_t, \varsigma_t)} \right] dt + \left[ \frac{q(\chi_t, \varsigma_t)K_t^p}{q(\chi_t, \varsigma_t)K_t^p} \right] d\Lambda_t,
\] (18)

and, in light of the equilibrium pricing kernel (11), the expected excess return to (or, “risk premium” on) private sector capital can be broken down into three components:

\[
    \mathbb{E}_t[dR_t^p - r_t dt] = -\mathbb{E}_t \left[ dR_t^p \cdot \frac{d\Lambda_t}{\Lambda_t} \right] = \mathbb{E}_t \left[ dR_t^p \cdot \frac{d\chi_t}{\chi_t} \right] - \frac{\partial \chi_t}{\Lambda_t} v^2 \varsigma_t^2 \chi_t dt + \frac{\mathbb{E}_t[dR_t^p \cdot d\varsigma_t]}{v^2 \varsigma_t dt} - \frac{\partial \varsigma_t}{\Lambda_t} v^2 \varsigma_t dt + \frac{\mathbb{E}_t[dR_t^p \cdot dK_t]}{\mathbb{E}_t[(dK_t/K_t)^2]} - \frac{\partial K_t}{\Lambda_t} v^2 \varsigma_t dt \]

EE The first component captures the PUB risk premium that stems from the welfare impact of over- or under-supplied public sector capital. It is a product of \( \beta_{Pub} \), the exposure to \( \chi \) risk (or "PUB risk"), and \( -\frac{\partial \chi_t}{\Lambda_t} \varsigma_t^2 \chi_t \), the price of PUB risk. Intuitively, if public sector capital is under-supplied, then shocks that expand its stock would relieve misallocation
and bolster output, thereby decreasing agents’ marginal utility (that is, $\partial_{\chi} \Lambda_t < 0$). In this case, an asset with more positive exposure to such shocks (or, bigger $\beta_{Pub}$) would be perceived as risky and thus have to deliver a higher risk premium as compensation. The second and third components capture the uncertainty and productivity risk premiums, respectively. They arise because both economic uncertainty and productivity are drivers of agents’ utility.\footnote{The productivity risk represents the standard technology shock to economic growth. It corresponds to the endowment risk in consumption-based asset pricing models.} They have a parallel structure (that is, product of beta and risk price) and can be similarly interpreted; I leave that to readers.\footnote{I demonstrate the properties of $-\partial_{\chi} \Lambda_t \Lambda_t \varsigma_{t} \chi_{t-1}$, $-\partial_{\chi} \Lambda_t \Lambda_t \nu_{t}$, and $-\partial_{\chi} \Lambda_t \Lambda_t \nu_{t} [dK_t/K_t]^2$ in Figure 8 and the properties of betas in Figure 9.}

The remaining equilibrium conditions can be derived from utility maximization and market clearing, but they are unnecessary for my analysis and thus omitted for brevity.

2 Model Implications

In this section, I analyze the equilibrium behavior of the model. After numerically solving the model, I plot the key equilibrium quantities as functions of the state variables. When viewed together with the steady state distribution (Figure 3), these plots constitute a good characterization of the underlying economics. They also demonstrate the model implications and predictions that are discussed throughout this section. The model calibration is detailed in Appendix A, where I explain my baseline parameter choices and also provide a sensitivity analysis.

2.1 Steady state distribution

The first and foremost thing is to check stationarity. The stationarity of this model hinges on the equilibrium behavior of the public sector capital share ($\chi$).\footnote{The stochastic process for another state variable $\xi_t$ is exogenously given in equation (2), which is a CIR process and thus stationary by design.} Figure 3 presents the steady state distribution of $\chi$ along with its drift and diffusion coefficients. The unconditional distribution displayed in panel (a) is center-peaked and well-behaved (with no clusters or washouts). Keeping in mind this steady state distribution, one can get better sense about other equilibrium quantities plotted against $\chi$. In panel (b), the drift coefficient $\mu_{\chi}$ exhibits a sine-like pattern, reflecting the stationarity of $\chi$. Specifically, when $\chi$ is small, $\mu_{\chi}$ is bigger than 0, meaning that $\chi$ is expected to increase as time elapses; when $\chi$ is large, the opposite applies. As a result, $\chi$ swings around its steady state value.
Figure 3: **Public sector capital share.** Panel (a) shows the steady state distribution of $\chi$ ($\equiv \frac{K^g}{K_t}$), the public sector capital share. Panel (b) and (c) plot against $\chi$ the drift ($\mu_\chi$) and diffusion ($\varsigma_\chi$) coefficients in the stochastic process for $\chi$, which is defined in equation (3), while holding the level of uncertainty $\varsigma$ at 0.01 (dotted line), $\bar{\varsigma}$ (solid line), and 0.1 (dashed line). The solid vertical lines indicate the unconditional mean of $\chi$.

Underlying this pattern is the balancing effect, which I shall explain later. Panel (c) plots the diffusion coefficient $\varsigma_\chi$, which is hump-shaped with respect to $\chi$. It attains maximum when capital is equally distributed between the private and public sectors.
2.2 Macroeconomic and asset pricing implications

After confirming the stationarity, I proceed to other solution plots. I start with equilibrium quantities concerning investment, consumption, and welfare. These are the central quantities from which all the other variables are calculated.

**Investment.** Figure 4 plots the private and public sector investment rates as functions of the public sector capital share ($\chi$) under different levels of uncertainty ($\varsigma$). It shows that with a greater share of public sector capital comes a higher (lower) investment rate in the private (public) sector. This pattern is driven by a balancing effect that stems from the assumption of private-public capital complementarity. A primary implication of this assumption is that augmenting the stock of public (private) sector capital raises the marginal product of private (public) sector capital. The behavior of the marginal products are closely mirrored in that of the investment rates, as capital flows to its most productive use. The balancing effect, which helps to keep both sectors alive in the long run, is the key ingredient to guarantee a nontrivial stationary equilibrium. Another notable obser-

![Graph](image.png)

Figure 4: **Private and public sector investment rates.** Panel (a) and (b) plot against the public sector capital share ($\chi$) the private and public sector investment rates, respectively, while holding the level of uncertainty ($\varsigma$) at 0.01 (dotted line), $\bar{\varsigma}$ (solid line), and 0.1 (dashed line). The solid vertical lines indicate the unconditional mean of $\chi$.

The observation from Figure 4 is that, holding constant the public sector capital share, increases in uncertainty lead to a lower private sector investment rate, but nudge up the public sector investment rate. This pattern is driven by a real-flight-to-safety effect that features a risk-based mechanism. It is the core mechanism of this model and thus warrants a detailed discussion; I elaborate on it in Figure 10.
Consumption and economic growth. Figure 5 plots against each state variable $c_t$, the consumption-capital ratio, and $\mu_{K,t}$, the growth rate of aggregate capital (while holding the other state variable at its steady state value). Panel (a) shows that both the consumption-capital ratio and the aggregate capital growth exhibit hump-shaped relations with the public sector capital share. This is again driven by the balancing effect. Intuitively, when the relative stock of public sector capital is low, its marginal product is high. So an expansion in public sector capital would elevate productivity and thus increase consumption and economic growth. But after a certain point, the situation reverses: further expansions in public sector capital would crowd out private sector capital (with a higher marginal product), diminishing consumption and growth as a result.

![Graph of consumption-capital ratio and economic growth](image)

Figure 5: Consumption-capital ratio and economic growth. Panel (a) plots the consumption-capital ratio ($\frac{c}{K}$) and the growth rate of aggregate capital ($\mu_K$) against the public sector capital share ($\chi$) while holding the level of uncertainty ($\varsigma$) at its steady state value. Panel (b) plots these two variables against $\zeta$ while holding $\chi$ at its steady state value. The consumption-capital ratio is represented by solid lines with its magnitude indicated on the left vertical axes. The growth rate of aggregate capital is represented by dashed lines with its magnitude indicated on the right vertical axes. The solid vertical lines indicate the unconditional means of $\chi$ and $\varsigma$.

Panel (b) reveals the contrasting influence of economic uncertainty on the consumption-capital ratio and the aggregate capital growth: greater uncertainty leads to a slower capital growth yet a higher consumption-capital ratio. This pattern is the result of two competing effects: the wealth effect and the intertemporal substitution effect. Specifically, rising uncertainty impairs economic prospects and diminishes aggregate wealth. On the one hand, the wealth effect imposes a negative influence on consumption, bringing it down in proportion to wealth, if not more. On the other hand, with worse economic prospects
discouraging investment, the intertemporal substitution effect exerts a positive influence on consumption. In the baseline calibration, the EIS ($\psi$) is bigger than 1, resulting in a stronger intertemporal substitution effect relative to the wealth effect.\[33\]

**Welfare.** Equation (7) shows that $\xi$, the welfare multiplier, determines agents’ expected lifetime utility when the aggregate stock of capital is given. Figure 6 plots $\xi$ as a function of the public sector capital share under different levels of uncertainty. It demonstrates that $\xi$ is a hump-shaped function of the public sector capital share, and it is decreasing in uncertainty. Thereby, shocks to uncertainty and capital allocation can affect agents’ utility and drive the pricing kernel.

![Figure 6: Welfare multiplier.](image)

Next, I examine equilibrium quantities concerning the risk-free interest rate, the risk prices, and the risk premium. These are asset pricing quantities that determine the rela-

\[33\] Had I set the EIS smaller than 1, one would see the opposite pattern: greater uncertainty leads to a lower consumption-capital ratio but a higher growth rate of aggregate capital.
The relationship between risk and return.

\[ r_t = \beta + \gamma [\mu_{K,t} + \mu_{c,t} + \sigma_{c,t} \sigma + \zeta_{c,t} (1 - \chi_t) \xi_t] \]
\[ \quad + (\gamma - 1/\psi) \left\{ \frac{\beta}{1 - 1/\psi} \left[ \left( \frac{\xi_t}{\xi_t} \right)^{1-1/\psi} - 1 \right] - (\mu_{c,t} - \mu_{\xi,t}) \right\} \]
\[ \quad - \frac{1}{2} \gamma (\gamma + 1) [\sigma^2 + (1 - \chi_t)^2 \xi_t^2] - \frac{1}{2} (1/\psi)(1/\psi + 1)(\sigma_{c,t}^2 + \zeta_{c,t}^2) \]
\[ \quad - \gamma (1/\psi + 1) [\sigma_{c,t} \sigma + \zeta_{c,t} (1 - \chi_t) \xi_t] \]
\[ \quad - \gamma (\gamma - 1/\psi) [\sigma_{\xi,t} \sigma + \zeta_{\xi,t} (1 - \chi_t) \xi_t] - \frac{1}{2} (\gamma - 1/\psi)(\gamma - 1/\psi + 1)(\sigma_{\xi,t}^2 + \zeta_{\xi,t}^2) \]
\[ \quad - (1/\psi)(\gamma - 1/\psi) (\sigma_{c,t} \sigma_{\xi,t} + \zeta_{c,t} \zeta_{\xi,t}) \]

**Risk-free interest rate.** Equation (20) provides the expression for the risk-free interest rate, which can be divided into three elements. The first term (\( \beta \)) represents agents’ impatience. The second and third terms capture the effect of intertemporal substitution. The remaining terms reflect agents’ precautionary savings motive. Panel (a) in Figure 7 plots the risk-free interest rate as a function of the public sector capital share under different levels of uncertainty. It shows that expanding the share of public sector capital raises the risk-free rate, whereas increasing uncertainty lowers the risk-free rate. The latter is simply because rising uncertainty strengthens agents’ precautionary savings motive, thereby pushing down the risk-free rate. The former is due to the risk-mitigating role played by the public sector. Specifically, a bigger public sector reduces the economy’s exposure to risks stemming from the private sector, thereby weakening the precautionary savings motive and pushing up the risk-free rate. Clearly, the behavior of the risk-free rate in this model is primarily driven by the precautionary savings motive, consistent with the empirical evidence documented by Pflueger, Siriwardane, and Sunderam (2018).

**Prices of risk.** Equation (21) provides the expressions for \( \eta \) and \( \theta \), the risk prices associated with the process \( Z \) and \( W \), respectively. They are plotted against the public sector capital share under different levels of uncertainty in panel (b) and (c) of Figure 7. Evidently, both \( \eta \) and \( \theta \) increase in the share of public sector capital, but decrease in uncertainty. This is because an expansion in public sector capital diminishes risks to agents’

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The intuition behind these three elements is as follows. (1) Less patient agents would demand a higher interest rate on savings. (2) If there is already a high degree of intertemporal substitution in place, agents would expect to become much “richer” going forward and hence have no incentive to save unless the interest rate is high. (3) Nonetheless, if there is great risk to the intertemporal substitution technology, agents would take precaution and favor safe savings even with a lower interest rate.
Figure 7: Pricing kernel. Panel (a), (b), and (c) plot against the public sector capital share ($\chi$) the risk-free rate and the risk prices associated with the process $Z$ and $W$, respectively, while holding the level of uncertainty ($\varepsilon$) at 0.01 (dotted line), $\bar{\varepsilon}$ (solid line), and 0.1 (dashed line). The solid vertical lines indicate the unconditional mean of $\chi$.

Marginal utility, whereas an increase in uncertainty elevates risks.

$$
\eta_t = \begin{cases} 
\frac{-\partial \Lambda_t}{\Lambda_t} (\gamma - 1/\psi) \frac{\partial \xi_t}{\xi_t} + 1/\psi \frac{\partial c_t}{c_t} & \text{if } \chi_t < \bar{\varepsilon} \\
\gamma \sigma & \text{if } \chi_t = \bar{\varepsilon} \\
\frac{-\partial \Lambda_t}{\Lambda_t} (\gamma - 1/\psi) \frac{\partial \xi_t}{\xi_t} + 1/\psi \frac{\partial c_t}{c_t} & \text{if } \chi_t > \bar{\varepsilon}
\end{cases} $$

$$
\theta_t = \begin{cases} 
\frac{-\partial \Lambda_t}{\Lambda_t} (\gamma - 1/\psi) \frac{\partial c_t}{c_t} + 1/\psi \frac{\partial c_t}{c_t} & \text{if } \chi_t < \bar{\varepsilon} \\
\gamma \sigma & \text{if } \chi_t = \bar{\varepsilon} \\
\frac{-\partial \Lambda_t}{\Lambda_t} (\gamma - 1/\psi) \frac{\partial c_t}{c_t} + 1/\psi \frac{\partial c_t}{c_t} & \text{if } \chi_t > \bar{\varepsilon}
\end{cases} $$

In light of equation (19), it is actually more interesting to examine the behavior of $-\frac{-\partial \Lambda_t}{\Lambda_t} \varepsilon^2 \chi_t$, the price of PUB risk, and $-\frac{-\partial \Lambda_t}{\Lambda_t} \sigma^2 \varepsilon_t$, the price of uncertainty risk, as well as $-\frac{-\partial \Lambda_t}{\Lambda_t} \varepsilon^2 \chi_t$, the price of productivity risk. Their exact expressions can be derived from $\eta$ and $\theta$, as revealed in equation (21). To understand, recall that the process $Z$ and $W$ each play two roles: shocks to $Z$ drive uncertainty, and shocks to $W$ drive the distribution of capital; besides, they both affect capital productivity. As a result, the price of risk associated with $Z$ is the sum of two components: the first component corresponds to its role as negative uncertainty risk, and the second component corresponds to its role as productivity risk. Symmetrically, the price of risk associated with $W$ also has two components: the first component reflects its influence on the share of public sector capital, and the second component reflects its influence on capital productivity. Based on this dissection, I was able to obtain the prices of PUB risk, uncertainty risk, and productivity risk from $\eta$ and $\theta$.

\[35\] See equation (2) and (3).
\[36\] This is because a positive shock to the process $Z$ drives down uncertainty, not up.
Figure 8: Prices of risk. Panel (a), (b), and (c) plot against the public sector capital share ($\chi$) the price of PUB risk, $-\frac{\partial_0 \Lambda_t}{\Lambda_t} \xi \chi_t$, the price of uncertainty risk, $-\frac{\partial_0 \Lambda_t}{\Lambda_t} \nu \zeta_t$, and the price of productivity risk, $-\frac{K_t \partial K_t}{K_t} E_t[(dK_t/K_t)^2]$, respectively, while holding the level of uncertainty ($\zeta$) at 0.01 (dotted line), $\bar{\zeta}$ (solid line), and 0.1 (dashed line). The solid vertical lines indicate the unconditional mean of $\chi$.

Figure 8 plots against the public sector capital share the prices of PUB risk, uncertainty risk, and productivity risk. Unambiguously, panel (b) shows that the price of uncertainty risk is negative, meaning greater uncertainty is associated with higher marginal utility. It implies that assets with more positive exposure to uncertainty shocks are valuable hedges for agents, thereby offering less compensation in the form of average returns. This implication is consistent with the empirical findings in Ang et al. (2006), among others. Panel (c) shows that the price of productivity risk is positive, suggesting that a higher stock of productive capital is associated with lower marginal utility.

The price of PUB risk shown in panel (a) is more subtle, as its sign changes with the share of public sector capital. Specifically, when the share of public sector capital is low, the price of PUB risk is positive (meaning expanding public sector capital reduces marginal utility). But when the share of public sector capital is high, the price of PUB risk turns negative (meaning expanding public sector capital raises marginal utility). In the former case, assets that pay off when public sector capital expands are perceived as risky and thus have to provide higher average returns. In the latter case, such assets are con-
sidered as hedges against misallocation and hence may provide lower average returns.

\[ \mathbb{E}_t[\frac{dR_t^P}{dt} - r_t] = (\sigma + \sigma_q, t)\eta_t + (\zeta_t + \zeta_q, t)\theta_t \]

\[ = -\frac{\zeta_t + \zeta_q, t}{\zeta, \lambda, t} \left[ (\gamma - 1/\psi) \frac{\partial \zeta, \lambda, t}{\zeta_t} + 1/\psi \frac{\partial \zeta, t}{c_t} \right] \zeta^2, \lambda, t \]

\[ \beta_{Pub} \]

\[ -\frac{\sigma + \sigma_q, t}{v\sqrt{\xi_t}} \left[ (\gamma - 1/\psi) \frac{\partial \xi, t}{\xi_t} + 1/\psi \frac{\partial c, t}{c_t} \right] v^2 \xi_t \]

\[ \beta_{Unc} \]

\[ + \frac{(\sigma + \sigma_q, t)\sigma + (\zeta_t + \zeta_q, t)(1 - \chi_t)\zeta_t}{\sigma^2 + (1 - \chi_t)^2\xi_t^2} \frac{\gamma}{-K \partial K, \Lambda, t/\Lambda_t} \left[ \sigma^2 + (1 - \chi_t)^2\xi_t^2 \right] \]

\[ \beta_{Prod} \]

Risk premium on private sector capital. Having seen the property of the pricing kernel, I proceed to examine the return to private sector capital. Equation (22) provides the expression for the risk premium on private sector capital, which I divide into three components respecting equation (19). According to its betas plotted in Figure 9, the return to private sector capital is negatively exposed to PUB risk and uncertainty risk while positively exposed to productivity risk. These betas, combined with the information on risk prices, enable me to compute the value of each risk premium component. The PUB risk premium is negative (positive) when the public sector capital share is low (high). The uncertainty risk premium is positive because high uncertainty is associated with low returns to private sector capital, as evidenced by negative \( \beta_{Unc} \). The productivity risk premium is positive and accounts for the biggest chunk of the total risk premium. Together these three components constitute the risk premium on private sector capital, which, as shown in Figure 9, is decreasing in the public sector capital share but increasing in uncertainty.

A particularly interesting observation is that the expected return to private sector capital—that is, the sum of the risk premium and the risk-free rate—is predominantly driven by changes in the risk-free rate as \( \chi \) varies, but is predominantly driven by changes in the risk premium as \( \zeta \) varies. Specifically, as the share of public sector capital expands, the risk premium falls, yet the risk-free rate rises; with the latter dominating, the expected return to private sector capital goes up. In contrast, as uncertainty increases, the risk pre-
mium rises, whereas the risk-free rate declines; with the former dominating instead, the expected return to private sector capital also goes up. This observation accords with the stylized fact that the correlations between the risk-free rate and risky assets’ returns are often weak. And my model suggests that this can be caused by changes in the risk premium playing the dominant role sometimes, echoing Barsky’s (1989) argument.

2.3 Additional implications

So far, I have examined the macro and asset pricing quantities independently. Next, I will put them together and see what new implications they can provide.

Real flight to safety. I start by elaborating on an important observation from Figure 4, that is, greater uncertainty reduces the private sector investment rate but increases the public sector investment rate. This pattern is driven by a real-flight-to-safety effect that warrants a thorough examination.

Panel (a) in Figure 10 demonstrates that the private and public sector investment rates exhibit unmistakably opposite responses to greater uncertainty. The underlying economics can be understood as follows. Rising uncertainty lowers the risk-free rate through the standard precautionary savings channel, as shown in panel (b). Ceteris paribus, a lower risk-free rate should contribute to lower costs of capital, thereby benefiting investment in both sectors. However, rising uncertainty also pushes up the risk premium on private sector capital to an extent that more than offsets the decline in the risk-free rate. This leads to a higher, rather than lower, cost of capital in the private sector, which is reflected by the depressed capital value relative to dividends as revealed in panel (b). The depressed capital value imposes a negative influence on private sector investment via Tobin’s \(q\) (Leahy and Whited, 1996). As a result, the private sector investment rate falls due to enormous increases in the risk premium, whereas the public sector investment rate is largely buttressed by a lower interest rate.

The real-flight-to-safety effect echoes the emphasis of Cochrane (2017) on time-varying risk premiums as a driver of real investment. It also captures the notion that a rising demand for safe assets may constrain risky physical investment required for economic growth (Caballero, Farhi, and Gourinchas, 2017). In my model, with fluctuations in uncertainty affecting the investment rates, the distribution of capital between the private and public sectors constantly deviates from the output-maximizing level, taking a toll on economic growth. Finally, it also relates to the uncertainty-driven business cycle the-

\[37\] The pattern presented in panel (b) of Figure 10 is commonly referred to as a “flight to safety” in the literature (e.g., Barsky, 1989; Baele et al., 2018).
Figure 9: **Risk premium on private sector capital.** This figure plots against the public sector capital share (χ) a group of variables related to the risk premium on private sector capital while holding the level of uncertainty (ς) at 0.01 (dotted line), ς (solid line), and 0.1 (dashed line). Panel (a) to (c) display the betas associated with the return to private sector capital. Panel (d) to (f) display the three components of the risk premium. Panel (g) and (h) display the risk premium on and the expected return to private sector capital, respectively. The expressions for these variables are given in equation (22). The solid vertical lines indicate the unconditional mean of χ.

Public sector investment rate and the risk premium. Figure [11] plots against the state...
variables the public sector investment rate and the risk premium on private sector capital. Clearly, both the public sector investment rate and the risk premium are decreasing in the public sector capital share, but increasing in uncertainty. As already explained, the behavior of the public sector investment rate is driven by the balancing effect and the real-flight-to-safety effect, while the behavior of the risk premium is explained by changes in the risk exposure and the risk prices. Together they point to a positive relationship between the public sector investment rate and the risk premium. This implication is consistent with the empirical evidence documented in Belo and Yu (2013). They find that a higher public sector investment rate is associated with higher risk premiums both at the aggregate and the firm-level. Belo and Yu (2013) rationalize this finding in a partial equilibrium framework with a prespecified process for public sector investment. In contrast, I demonstrate that this relation can endogenously arise in general equilibrium.

**Public sector investment share and the interest rate.** Figure 12 displays together the public sector investment share \( I^s_{I\ell + I^p} \) and the risk-free interest rate. Driven by the real-flight-to-safety effect, greater uncertainty is associated with a higher public sector investment share, consistent with the pattern uncovered in Figure 1. Controlling for uncertainty, the investment share and the capital share of the public sector move in lockstep because capital adjustment costs penalize their divergences. The behavior of the public sector in-
Figure 11: **Public sector investment rate and the risk premium.** Panel (a) plots against the public sector capital share ($\chi$) the public sector investment rate ($\iota^g$) and the risk premium on private sector capital while holding the level of uncertainty ($\varsigma$) at its steady state value. Panel (b) plots these two variables against the level of uncertainty while holding the public sector capital share at its steady state value. The public sector investment rate is represented by solid lines with its magnitude indicated on the left vertical axes. The risk premium is represented by dashed lines with its magnitude indicated on the right vertical axes. The solid vertical lines indicate the unconditional means of $\chi$ and $\varsigma$.

Investment share, when viewed together with that of the risk-free rate, generates a testable prediction: greater uncertainty drives up the public sector investment share but lowers the risk-free rate; controlling for uncertainty, however, a higher public sector investment share is associated with a higher risk-free rate. In Appendix B I take this prediction to data and find supporting evidence.
Figure 12: **Public sector investment share and the risk-free interest rate.** Panel (a) plots the public sector investment share \((\frac{I^g}{I^p + I^g})\) and the risk-free interest rate \((r)\) against the public sector capital share \((\chi)\) while holding the level of uncertainty \((\varsigma)\) at its steady state value. Panel (b) plots these two variables against the level of uncertainty while holding the public sector capital share at its steady state value. The public sector investment share is represented by solid lines with its magnitude indicated on the left vertical axes. The risk-free interest rate is represented by dashed lines with its magnitude indicated on the right vertical axes. The solid vertical lines indicate the unconditional means of \(\chi\) and \(\varsigma\).

3 Empirical Investigation: Regression-Based Approach

In this section, I empirically investigate whether and how PUB shocks are priced. The GE theory developed above has demonstrated how the share of public sector capital may enter the pricing kernel and thus become a risk factor priced in assets’ expected returns. Guided by this theory, I propose a three-factor asset pricing model with PUB shocks, uncertainty shocks, and the market excess return as risk factors; they represent innovations to those three state variables that govern the GE pricing kernel \((11)\). In what follows I confront this factor model with a variety of test assets.

### 3.1 Primary variables and risk factors

I start by defining main variables and explaining the construction of risk factors. Other variables are introduced later when they enter my analysis.

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Footnote: "Strictly speaking, the pricing kernel shown in \((11)\) is entered by the capital share, not the investment share, of the public sector. But the stocks of productive capital are unobservable and hard to measure, so I use the investment share instead. This choice is backed by the GE model, in which the investment share and the capital share of the public sector move in lockstep due to capital adjustment costs (see Figure 12)."
**Investment.** The measurements of private and public sector investments come from the National Income and Product Accounts (NIPA) data provided by the U.S. Bureau of Economic Analysis (BEA). I follow Belo and Yu (2013) in defining *private sector investment* as the seasonally adjusted private fixed nonresidential investment (NIPA: Table 1.1.5, line 9), and *public sector investment* as the seasonally adjusted government nondefense investment (NIPA: Table 3.9.5, line 3 minus line 19). I define *national investment* as the sum of private and public sector investments per Aschauer (1989a), and the *public sector investment share* as the ratio of public sector investment to national investment. All variables are in real terms (deflated by corresponding price indexes) with quarterly observations that span the period 1947Q1 to 2018Q4.

**Economic uncertainty.** The measure of economic uncertainty is from Jurado, Ludvigson, and Ng (2015). They construct comprehensive and model-free macroeconomic uncertainty indexes that capture the common variation in uncertainty among a variety of economic indicators. This measure is well-suited for the study of aggregate uncertainty and its comovement with other variables. I pick their 1-month-ahead macro uncertainty index and aggregate it to a quarterly frequency (by simple average). The resulting measure spans the period 1960Q3 to 2018Q4.

It is worth mentioning that economic uncertainty is difficult to quantify. Researchers have taken various approaches to measure it, resulting in a variety of uncertainty indicators yet little consensus on which one is the best (Caldara et al., 2016). The only agreement on this matter is probably that uncertainty is countercyclical (Bloom, 2014). That said, I choose the JLN measure for good reason: it has relatively long sample period and also possesses more predictive content than other measures.

**Constructing risk factors.** Figure 1 plots the public sector investment share as well as the JLN uncertainty index. From these two variables, I construct two risk factors, denoted by PubFac and UncFac, as shocks to the public sector investment share (PUB shocks) and economic uncertainty, respectively. Shocks to each variable are defined as innovations in the AR(1) representation of that variable’s dynamics. For convenience, I standardize PubFac and UncFac to unit variance. Together with the market excess return, these factors

---

39 Specifically, Jurado, Ludvigson, and Ng (2015) define individual uncertainty as the conditional volatility of the forecast error for each indicator. They estimate the forecast error by fitting a diffusion index model to the time series of these indicators. Then, with the estimated forecast error, they infer its conditional volatility using a stochastic volatility model. The final products, the macroeconomic uncertainty indexes, are constructed by aggregating together these individual uncertainty measures.

40 Caldara, Fuentes-Albero, Gilchrist, and Zakrjeski (2016) conduct a “horse race” exercise, demonstrating that the JLN measure is more informative about future economic activity.
constitute the three-factor model that underpins my subsequent analysis.

Figure 13 displays the time series of PubFac and UncFac. Both factors seem countercyclical because they often witness sizeable positive spikes during recessions. Most notably, in the Great Recession, UncFac reached its nadir at the height of the crisis. PubFac also showed a big increase, especially at the passage of the American Recovery and Reinvestment Act (ARRA), a fiscal stimulus bill that includes large public sector investment.

Figure 13: Risk factors. This figure plots two risk factors denoted by PubFac and UncFac, which are defined as innovations in the AR(1) representations of the public sector investment share and economic uncertainty, respectively. PubFac and UncFac are standardized to unit variance. Shaded areas indicate U.S. recessions defined by NBER.

Table documents the correlations of PubFac and UncFac with a selection of economic indicators. Both PubFac and UncFac are negatively related to GDP growth and positively related to changes in the unemployment rate, confirming the countercyclicality of the public sector investment share and economic uncertainty. PubFac positively correlates with government consumption and the fiscal deficit (relative to GDP), suggesting that a higher public sector investment share tends to coincide with increased government consumption and a larger deficit.

3.2 Empirical approach

To examine the asset pricing role of PUB shocks, I follow a standard two-pass regression approach. The first pass estimates the betas (that is, exposure to risks) for each test asset \( i \)
Table 1: Risk factors’ correlations with common economic indicators. This table presents the correlations of two risk factors, PubFac and UncFac—which are defined as innovations in the AR(1) representations of the public sector investment share and economic uncertainty, respectively—with a selection of economic indicators including: the market excess return (from Ken French’s website); the growth (log change) of GDP (NIPA: Table 1.1.5, line 1) and government nondefense consumption (NIPA: Table 3.9.5, line 2 minus line 18), both of which are in real terms; and the changes in civilian unemployment rate (from FRED) and the deficit-to-GDP ratio (NIPA: Table 3.1, (-) line 43 to GDP).

<table>
<thead>
<tr>
<th></th>
<th>PubFac</th>
<th>UncFac</th>
</tr>
</thead>
<tbody>
<tr>
<td>PubFac</td>
<td>1.00</td>
<td>0.00</td>
</tr>
<tr>
<td>UncFac</td>
<td>0.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Market excess return</td>
<td>0.16</td>
<td>-0.23</td>
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<tr>
<td>GDP (log change)</td>
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<td>-0.37</td>
</tr>
<tr>
<td>Unemployment rate (change)</td>
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<td>0.20</td>
</tr>
<tr>
<td>Govt. consumption (log change)</td>
<td>0.13</td>
<td>0.06</td>
</tr>
<tr>
<td>Deficit/GDP (change)</td>
<td>0.26</td>
<td>0.14</td>
</tr>
</tbody>
</table>

via a time-series regression of the asset’s excess returns, $r_{i,t}^e$, on the risk factors:

$$r_{i,t}^e = a_i + f_i' \beta_i + \xi_{i,t}, \ t = 1, ..., T$$

where $f$ is a vector of risk factors, and $\beta_i$ is a vector of betas (to be estimated) for asset $i$. The second pass estimates the risk prices via a cross-sectional regression of assets’ (time-series) average excess returns on their estimated betas:

$$\bar{r}_i^e = \alpha + \beta_i' \lambda + \epsilon_i, \ i = 1, ..., N$$

where $\bar{r}_i^e$ is the unconditional mean excess return for asset $i$, $\beta_i$ denotes the estimated betas from the first pass, and $\lambda$ is a vector of risk prices to be estimated. My primary factor model consists of PubFac, UncFac, and the market excess return ($f = [\text{PubFac}, \text{UncFac}, \text{MktRf}]$), while I also consider the Fama and French (1993) model ($f = [\text{SMB}, \text{HML}, \text{MktRf}]$) as a comparison.

This regression approach is standard and widely commended for its transparency, but like any other approach, it has limitations. A well-known one is that betas are estimated via time-series regressions and hence are inaccurate by definition. This is particularly relevant when nontraded factors are used (as is the case here), because, if a nontraded factor contains substantial noise, the estimated betas will be understated while the corresponding risk prices overstated. To assess the extent to which this limitation bites, I use Shanken (1992)’s correction to adjust standard errors, checking if it makes a big difference.
Another limitation is that implicit in this approach is a presumption of constant betas for each asset, whereby the estimated $\lambda$ gives the time-series averages of risk prices. One can reasonably argue that this presumption is untenable, but relaxing it requires more sophisticated estimators or granular data; both seem beyond reach at this point. So I leave for future research the exploration of alternative approaches.

**Test assets.** For test assets I consider a wide range of standard equity portfolios formed on size, BM, momentum, investment, and profitability. These portfolios are known to exhibit sizeable differences in average returns (Fama and French, 2015). Besides, I also consider portfolios formed on past exposure to PubFac. Specifically, at each quarter end, I sort stocks in the Center for Research in Security Prices (CRSP) database by their past exposure to PubFac (or $\beta_{Pub}$) and then stratify them into decile portfolios. I obtain the pre-formation $\beta_{Pub}$ for each stock via a rolling regression of its excess returns on PubFac, UncFac, and the market excess return with a 40-quarter trailing window (I require at least 32 quarters of data); the pre-formation $\beta_{Pub}$ is measured by the coefficient on PubFac. These portfolios are rebalanced every quarter, and their returns are computed as the value-weighted averages of their constituent stocks' returns.

### 3.3 Results

I start by pricing 25 size and value sorted portfolios with my primary factor model, comparing it with the Fama and French (1993) model; Table 2 presents the results. Panel (a) reports the mean excess returns and the estimated betas for all portfolios. Consistent with the literature, average return generally falls from small stocks to big stocks while rises from growth stocks to value stocks. As for betas, an interesting observation is that exposure to PUB shocks seems to negatively correlate with size: small stocks tend to be more sensitive to variations in the public sector investment share. Similar patterns can be found in almost every investment, profitability, and momentum quintile, as shown in Table B.4. This implies that augmenting public sector capital is likely to benefit small firms more than big firms.

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41I do not consider other asset classes like corporate bonds and derivatives because, according to Adrian, Etula, and Muir (2014) and He, Kelly, and Manela (2017), financial intermediaries tend to be the marginal investors in these more sophisticated asset markets rather than households.

42I only include stocks with share codes 10 or 11 and listed on the NYSE, AMEX, or NASDAQ.

43Anecdotal evidence also supports the idea that small firms may benefit more from greater public sector investment. A good example is the construction industry, an undoubted beneficiary that has “the largest small business concentration of any industry” (Mills, 2014). According to The Economist (2017), the construction industry has highly fragmented structure: “less than 5% of builders work for construction firms that employ over 10,000 workers.”
Panel (b) reports the estimated risk prices and several test diagnostics. The price of risk for PUB shocks ($\lambda_{pub}$), which is my main focus, is positive and statistically significant. The $t$-statistics, whether based on Fama and MacBeth (1973) standard errors adjusted for autocorrelation or ordinary least squares (OLS) standard errors adjusted for beta estimation errors per Shanken (1992), are both above 2. The economic magnitude of $\lambda_{pub}$ is also sizeable at 1.06% per quarter. With PUB betas ranging from -0.43 to 0.95 for this group of assets, this amounts to a roughly $6(\approx 1.38 \times 1.06 \times 4)$ percent differential in expected annual returns. (As a reference point, the range of the mean excess returns across these assets is about 9 percent per year.) This result points to PUB shocks as good news from investors’ perspective, as they demand higher returns from assets that load more positively on PUB shocks. Thus an expansion in public sector investment (relative to private) is likely to accompany a favorable shift in investors’ welfare.

The pricing performance of my factor model is modestly strong. The mean absolute pricing error (MAPE) is low at 0.28% per quarter, while the adjusted $R^2$ is moderate at 51%. The $\chi^2$ statistic is at a particularly low level of 20.50, indicating that the hypothesis of zero joint pricing errors across assets is not rejected. These statistics are close to that for the Fama and French (1993) model reported in panel (c), which is pretty impressive given the fact that the Fama and French (1993) model is statistically tailored to price this cross section while my factor model is theoretically motivated. However, I do not want to stretch too far because the estimation also reveals a large intercept ($\alpha$) that indicates a certain degree of misspecification. (The same problem attends the Fama-French model.) So I conduct more tests to check the robustness of these findings.

**Robustness: other assets.** Next, I confront my primary model with more test assets and see how it fares. The results, reported in Table 3, echo and strengthen the previous findings. The risk price for PubFac remains positive and statistically significant across different sets of test assets. This is true even when all portfolios are included in the tests. Interestingly, in an unreported result, I find in this larger cross section that my primary model provides a better fit (in terms of higher $R^2$) relative to the Fama and French (1993) model. This finding is also mirrored in Figure 14, which plots the realized mean excess returns on all portfolios against their model-implied counterparts. When priced by my primary model, these assets line up closer to the 45-degree line.

In summary, a theoretically founded factor model that includes PubFac, UncFac, and the market excess return performs fairly well in pricing a wide range of standard equity portfolios. The estimated risk price for PUB shocks is consistently positive and significant. This finding suggests that increases in the share of public sector investment tend to concur
Figure 14: **Realized versus model-implied mean excess returns.** This figure compares the realized versus the model-implied mean excess returns for all test assets including 25 size and value sorted portfolios, 10 $\beta_{Pub}$ sorted portfolios, 10 momentum portfolios, 10 investment portfolios, and 10 profitability portfolios. Two factor models are considered: the primary model displayed in panel (a) consists of $PubFac$, $UncFac$, and $MktRf$; the Fama and French (1993) model displayed in panel (b) consists of $SMB$, $HML$, and $MktRf$. The sample is quarterly and spans the period 1969Q1 to 2018Q4.

with better welfare for investors.
Table 2: Two-pass asset pricing analysis: 25 Size-BM equity portfolios. This table presents the results of a two-pass asset pricing analysis. Panel (a) reports the test assets’ mean quarterly excess returns ($\bar{r}_i$) and estimated betas. The latter are obtained by running a time-series regression specified as $r_{i,t} = \alpha_i + f_i \beta_i + \xi_i$ for each asset $i$, where $r_{i,t}$ is the asset’s excess return, $f_i$ represents a vector of risk factors, and $\beta_i$ denotes a vector of beta estimates. Panel (b) reports the risk prices estimated from a cross-sectional regression of test assets’ mean excess returns on estimated betas, that is, $\bar{r}_i = \alpha + \beta_i \lambda + \epsilon_i$. The t-statistics are based on either Fama and MacBeth (1973) standard errors with Newey and West (1987) correction (one lag) or ordinary least squares (OLS) standard errors with Shanken (1992) correction. Also reported are test diagnostics including mean absolute pricing error (MAPE), adjusted $R^2$, and a $\chi^2$ statistic along with the $p$-value that tests whether the pricing errors are jointly zero. The primary factor model comprises PubFac, UncFac and the market excess return. The test assets include 25 size and value sorted equity portfolios. The sample is quarterly and spans the period 1960Q4 to 2018Q4. As a comparison, panel (c) reports the analogous statistics for the Fama and French (1993) model.

(a) Mean excess returns and betas by asset

<table>
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<tr>
<th></th>
<th>$\bar{r}_i$</th>
<th>$\beta_{Pub}$</th>
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<tr>
<td></td>
<td>Small</td>
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<tr>
<td>Growth</td>
<td>0.89</td>
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<td></td>
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<td>2.26</td>
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<tr>
<td>BM</td>
<td>2.26</td>
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<tr>
<td></td>
<td>2.93</td>
<td>2.71</td>
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<th>$\beta_{Unc}$</th>
<th>$\beta_{Mkt}$</th>
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</thead>
<tbody>
<tr>
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</tr>
<tr>
<td>Growth</td>
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<tr>
<td></td>
<td>0.06</td>
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<td>BM</td>
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<td>-0.28</td>
</tr>
<tr>
<td></td>
<td>-0.07</td>
<td>-0.22</td>
</tr>
<tr>
<td>Value</td>
<td>-0.84</td>
<td>-0.27</td>
</tr>
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</table>

(b) Risk prices and test diagnostics

<table>
<thead>
<tr>
<th></th>
<th>$\lambda_{Pub}$</th>
<th>$\lambda_{Unc}$</th>
<th>$\lambda_{Mkt}$</th>
<th>$\alpha$</th>
<th>Test diagnostics</th>
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<tr>
<td>Coefficient</td>
<td>1.06</td>
<td>-0.35</td>
<td>-1.61</td>
<td>3.52</td>
<td>MAPE 0.28 $\chi^2$ 20.50</td>
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<tr>
<td>[t-FMNW]</td>
<td>[3.80]</td>
<td>[-1.55]</td>
<td>[-1.61]</td>
<td>[3.98]</td>
<td>Adj. $R^2$ 0.51 $p$-value 0.55</td>
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<tr>
<td>[t-Shanken]</td>
<td>[2.16]</td>
<td>[-0.92]</td>
<td>[-1.03]</td>
<td>[2.33]</td>
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(c) Comparison with the Fama and French (1993) model

<table>
<thead>
<tr>
<th></th>
<th>$\lambda_{SMB}$</th>
<th>$\lambda_{HML}$</th>
<th>$\lambda_{Mkt}$</th>
<th>$\alpha$</th>
<th>Test diagnostics</th>
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<tr>
<td>Coefficient</td>
<td>0.39</td>
<td>1.06</td>
<td>-1.74</td>
<td>3.41</td>
<td>MAPE 0.22 $\chi^2$ 58.12</td>
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<td>[t-Shanken]</td>
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<td>[2.83]</td>
<td>[-1.61]</td>
<td>[3.65]</td>
<td></td>
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</table>
Table 3: **Two-pass asset pricing analysis: other portfolios.** This table presents the results of a two-pass asset pricing analysis. The procedure and relevant statistics are described in more detail in Table 2. Panel (a) summarizes the test assets' mean (quarterly) excess returns and estimated betas. $\mu[\cdot]$ and $\sigma[\cdot]$ denote the cross-sectional mean and standard deviation, respectively. Panel (b) reports the estimated risk prices. The factor model comprises $PubFac$, $UncFac$ and the market excess return. The test assets are 25 size and value sorted equity portfolios (Column 1) plus 10 $\beta_{Pub}$ sorted portfolios (Column 2), or 10 momentum portfolios (Column 3), or 10 investment portfolios (Column 4), or 10 profitability portfolios (Column 5), or all 65 portfolios together (Column 6). The sample is quarterly and spans the period 1969Q1 to 2018Q4; the start is dictated by the $\beta_{Pub}$ portfolios.

### (a) Mean excess returns and betas by asset

<table>
<thead>
<tr>
<th></th>
<th>SZBM25</th>
<th>PUB10</th>
<th>MOM10</th>
<th>INV10</th>
<th>OP10</th>
<th>All</th>
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<tbody>
<tr>
<td>$\mu[r_e]$</td>
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<td>2.24</td>
<td>1.41</td>
<td>1.69</td>
<td>1.45</td>
<td>1.74</td>
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<tr>
<td>$\sigma[r_e]$</td>
<td>0.58</td>
<td>0.24</td>
<td>0.97</td>
<td>0.41</td>
<td>0.39</td>
<td>0.63</td>
</tr>
<tr>
<td>$\mu[\beta_{Pub}]$</td>
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<td>0.16</td>
<td>0.06</td>
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<td>$\sigma[\beta_{Pub}]$</td>
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<td>0.39</td>
<td>0.40</td>
<td>0.24</td>
<td>0.46</td>
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<td>$\mu[\beta_{Unc}]$</td>
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<td>$\sigma[\beta_{Unc}]$</td>
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<td>0.30</td>
<td>0.51</td>
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<td>0.37</td>
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<td>$\mu[\beta_{Mkt}]$</td>
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### (b) Risk prices and test diagnostics

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<th></th>
<th>SZBM25</th>
<th>SZBM25 + PUB10</th>
<th>SZBM25 + MOM10</th>
<th>SZBM25 + INV10</th>
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<td>$\lambda_{Pub}$</td>
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<td>$t$-Shanken</td>
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<td>[1.79]</td>
<td>[1.96]</td>
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<td>$t$-Shanken</td>
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<td>[0.21]</td>
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<td>-2.60</td>
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<tr>
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<td>[-2.27]</td>
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<td>[-2.28]</td>
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</tr>
<tr>
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<tr>
<td>$t$-FMNW</td>
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<td>[5.01]</td>
<td>[5.59]</td>
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<tr>
<td>$t$-Shanken</td>
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<td>[3.75]</td>
<td>[3.59]</td>
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<td>[4.11]</td>
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<td>MAPE</td>
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<td>Adj. $R^2$</td>
<td>0.56</td>
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</tr>
<tr>
<td>$\chi^2$</td>
<td>28.43</td>
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<td>44.53</td>
<td>38.59</td>
<td>38.81</td>
<td>160.71</td>
</tr>
<tr>
<td>$p$-value</td>
<td>0.16</td>
<td>0.00</td>
<td>0.07</td>
<td>0.20</td>
<td>0.19</td>
<td>0.00</td>
</tr>
</tbody>
</table>
4 Empirical Investigation: Portfolio-Based Approach

The regression-based approach has its limitations (as already mentioned) that might raise concerns about the validity of its results. So in this section, I provide additional evidence via a portfolio-based approach using a sample of U.S. government contractors. The idea is as follows. I postulate that firms with heavier reliance on sales to the U.S. government load more positively on PUB shocks. Thereby if the price of risk for PUB shocks is positive, high-dependency firms should carry higher risk premiums compared to low-dependency firms. This is exactly what I find.

4.1 Sample construction and portfolio formation

From the CRSP/Compustat Merged (CCM) database, I collect a sample of U.S. government contractors. Using their stocks I form portfolios based on the extent of their dependency on government customers for revenue.

Identifying government contractors. The Financial Accounting Standards Board (1997) requires firms to report their sales to major customers including the U.S. government (federal, state, and local). This information, which is in the Compustat Customer Segment file, together with other accounting information from the Compustat Fundamental Annual file allows me to compute for each firm-year the fraction of sales accounted for by government customers (denoted by StG). Every year I define government contractors as firms that reported positive sales to government at least once over the past three years. I exclude firms in the healthcare and pharmaceutical industries, the consumer goods and services industries as well as the defense industry, because transactions between these firms and government, if any, are more likely to stem from other types of government spending than public sector investment. For example, healthcare and pharmaceutical companies have business connections with government mainly because of their involvements in social security programs such as Medicaid and Medicare (Goldman, 2019). Government purchases from consumer goods and services firms are more likely to be categorized as government consumption rather than investment. As for firms in the defense industry, their transactions with government apparently come from defense spending.

44The Financial Accounting Standards Board (1997) dictates that “an enterprise shall provide information about the extent of its reliance on its major customers. If revenues from transactions with a single external customer amount to 10 percent or more of an enterprise’s revenues, the enterprise shall disclose that fact, the total amount of revenues from each such customer ... For purposes of this Statement, ... the federal government, a state government, a local government (for example, a county or municipality), or a foreign government each shall be considered as a single customer.” (para. 39)
After this exclusion (and other standard filters), I find 1,242 government contractors with 9,944 firm-year observations spanning 1980 to 2017; these firms are mainly from the construction and manufacturing industries (with SIC between 1500 and 3999).  

Panel (a) in Table 4 provides summary statistics for this sample of government contractors. As shown, there is substantial variation in StG. The median government contractor has 18.5% of its sales generated by government customers. About a quarter of government contractors derive more (less) than 45% (5%) of their sales revenue from government. For a tenth of government contractors, sales to government account for more than 75% of their total sales. Regarding other firm characteristics, the average government contractor has a book-to-market ratio of 0.72 and market leverage of 0.21; its book value of assets (total sales) grows 14.7% (14.1%) year-on-year; its profitability ratio and return on assets are 0.16 and 0.3%, respectively. These numbers are similar to those in Goldman (2019), who reported, for a sample of government contractors in 2005 and 2006, an average sales growth of 19%, return on assets of -3.1%, and leverage of 0.23.

**Forming portfolios on government dependency.** Using these government contractors’ stocks (which are ordinary common shares listed on the NYSE, AMEX, or NASDAQ), I form portfolios based on the extent to which they depend on government customers for revenue. Every year I measure a firm’s government dependency by $\bar{StG}_{t-3:t-1}$, a three-year trailing average of StG. Following the convention in the literature, I form stock portfolios at the end of June in each year t based on the quintiles of government dependency computed for the previous year (that is, $\bar{StG}_{t-3:t-1}$). I also consider a zero-investment portfolio that is long stocks in the highest-dependency quintile and short stocks in the lowest-dependency quintile. These portfolios are held from July of year t to June of year t + 1, by which time the next formation happens. The first set of portfolios were formed in 1981, and the last in 2018.

Panel (b) in Table 4 compares firms in different government dependency portfolios. Unsurprisingly, high-dependency firms tend to have high StG in the year before formation. In other aspects, however, firms are similar across portfolios. Although firms with the highest dependency are somewhat smaller and have slightly lower leverage and higher asset growth and operating profitability compared to firms with the lowest depen-

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45 Appendix B provides more details on the sample construction.

46 I choose this moving-average measure for good reason. First, a firm only needs to report its sales to government customers when that accounts for more than 10% of its total sales in a fiscal year. For years with no reported sales to government, StG is zero though the real value can be larger than that. Also, there are some data errors as noted by Goldman (2019). For example, occasionally foreign governments are mistaken for the U.S. government, and the U.S. government agencies are mistaken for private companies. Using a moving average can help smooth out, at least in part, some of these data omissions and errors.
dency, the differences are minor. This is confirmed by Figure [B.1], which uses box plots to compare the distributional properties of firm characteristics across portfolios; it shows that other firm characteristics are not systematically related to government dependency.

4.2 Portfolio analysis

Given these government dependency portfolios, I first establish the link between government dependency and exposure to public sector investment. Then I infer investors’ opinion on public sector investment by comparing the average returns on different dependency portfolios.

Is government dependency a relevant proxy? I hypothesize that the extent of a firm’s dependency on government is a relevant proxy for its exposure to changes in public sector investment. Now I provide support for this hypothesis. First, I show that government dependency is persistent. Specifically, I examine whether past dependency predicts future dependency via a predictive regression specified as

\[
StG_{i,t+h} = \alpha_h + \beta_h \overline{StG}_{i,t-2\rightarrow t} + \epsilon_{i,t+h}
\]  

(23)

where \( h \) is the forecast horizon, and \( \overline{StG}_{i,t-2\rightarrow t} \) is the average fraction of sales to government over the past three years ending in year \( t \). If government dependency is persistent, then \( \beta_h \) would be positive and close to one. This is exactly the case. As shown in Table [5] at the one-year horizon, a one percentage point increase in \( \overline{StG}_{-2,0} \) is associated with a 0.93 percentage point increase in \( StG \); this figure remains high at 0.86 even for the three-year horizon. It suggests that a firm with high government dependency in the past also tends to have a large fraction of sales contributed by government in the near future.

Second, I show that high-dependency firms are more sensitive to changes in public sector investment. I examine the relation between firms’ performance and public sector investment, and more importantly, whether the magnitude of this relation is greater for high-dependency firms. I consider the following regression

\[
\nabla \left[ \frac{\text{sales/earnings}}{} \right]_{i,t+1} = \alpha + \beta_1 \overline{StG}_{i,t-2\rightarrow t} + \beta_2 \nabla r_{i+1} + \beta_3 \nabla i_{i+1} \times \overline{StG}_{i,t-2\rightarrow t} + \epsilon_{t+1}
\]

(24)

where \( \nabla \left[ \frac{\text{sales/earnings}}{} \right]_{i,t+1} \) is the sales or earnings (EBITDA) growth for firm \( i \) in year \( t + 1 \), and \( \nabla i_{i+1} \) is the contemporaneous public sector investment growth. The last two columns of Table [5] report the results. To understand, consider two average firms: one from the lowest dependency quintile and another from the highest. The estimated coef-
ficients indicate that, for the former ($\bar{StG}_{-2,0} = 0.03$), a one percentage point increase in the growth rate of public sector investment accompanies a 0.29 (0.10) percentage point increase in its sales (earnings) growth; whereas for the latter ($\bar{StG}_{-2,0} = 0.74$), the same increase in public sector investment growth is associated with a 1.01 (0.87) percentage point increase in its sales (earnings) growth.\footnote{It clearly suggests that firms with higher government dependency are more sensitive to variations in public sector investment.}

Comparing returns on government dependency portfolios Having established the link between government dependency and exposure to public sector investment for this sample of government contractors, I then turn to examining the average returns on dependency portfolios. I obtain stock-level data from the Center for Research in Security Prices (CRSP) Monthly Stock file.\footnote{I consider both value- and equal-weighted portfolios.}

Panel (a) of Table 6 reports the mean excess returns along with the Sharpe ratios for value-weighted portfolios over the full sample period (1981-2018). One can see that stocks in high-dependency portfolios tend to provide higher average returns. The long-short portfolio (long the highest-dependency portfolio and short the lowest-dependency portfolio) delivers an average return of 0.62% per month (that is, 7.43% per year) and has a Sharpe ratio (annualized) of 36.14%. A similar pattern emerges from panel (a) of Table 7, where I consider equal-weighted portfolios; it also reveals a positive relation between government dependency and average return. The long-short portfolio provides an average return of 0.35% per month (that is, 4.21% per year) and has a Sharpe ratio of 31.70%. These dependency patterns in average returns are graphically shown in Figure 15.

Given these sizable spreads in average returns, a natural question is whether they are driven by government contractors’ differential loadings on classic risk factors regardless of their exposure to public sector investment. I address this question by estimating the portfolio alphas with respect to a set of standard risk factors in the literature, including the market factor ($MKT$), the size factor ($SMB$), and the value factor ($HML$) from\footnote{The results for earnings growth are not statistically significant at conventional levels, which may be caused by the fact that earnings growth is much more noisy than sales growth: there are a lot more instances of missing or negative values for EBITDA than for sales.} Fama and French (1993) as well as the momentum factor ($MOM$) from Carhart (1997) and the liquidity factor ($LIQ$) from Pastor and Stambaugh (2003). The results for value-weighted portfolios are shown in panel (b) of Table 6; it confirms that the spread in average returns between high- and low-dependency firms is not accounted for by loadings on these risk factors. The long-short portfolio’s alpha is 0.82% monthly with a $t$-statistic of 2.42. For

\footnote{Monthly stock returns are corrected for delisting (Shumway, 1997) and winsorized at 1st and 99th percentiles. But these adjustments make little difference.}
equal-weighted portfolios the conclusion is the same: the dependency premium cannot be explained by exposure to classic risk factors. The long-short portfolio’s alpha, shown in panel (b) of Table 7, is 0.56% monthly with a t-statistic of 2.40. Figure 15 provides a clear picture of this pattern in portfolio alphas.

**Implications for public investment policy.** If the spread in average returns between high- and low-dependency firms is driven by a PUB risk premium, then, as the GE theory suggests, its sign and magnitude reflect investors’ opinion on whether public sector capital is underinvested. If yes, high-dependency firms should provide higher expected returns compared to low-dependency firms. Following this logic, the results above seem to suggest that investors perceive an overall shortfall in public sector investment during the 1981-2018 period. But a natural question is whether this shortfall is getting better or worse over time. This question is particularly relevant because policymakers are recently considering potential increases in public investment. If, for example, the PUB risk premium was high in earlier years but had diminished in more recent years, then the case for greater public sector investment would be weakened by such observation. Nevertheless, what I find is the opposite.

I split the sample into two subperiods of equal length: 1981 to 1999 and 2000 to 2018, and repeat the analysis above for these two subperiods separately. The results are reported in Table 8 and graphically displayed in Figure 16 and 17. I find that the differences in average returns across government dependency portfolios are small for the 1981-1999 period, but they become large in the 2000-2018 period. In particular, when using equal weight, the long-short portfolio actually has a slightly negative average return of -0.03% per month for the 1981-1999 period. In comparison, for the 2000-2018 period, the long-short portfolio provides a notably higher average return: 0.87% per month (that is, 10.44% per year) when using value weight and 0.73% per month (that is, 8.76% per year) when using equal weight. And again I confirm that these return spreads cannot be explained by exposure to classic risk factors.

So the results of this exercise suggest that the inadequacy in public investment, if any, is minor in the 1980s and 1990s, but it has become more severe in recent years.

This finding accords with the notion that the cost of government spending was high in the 1980s and 1990s, so the net benefits of public investment were probably low. As noted by Furman and Summers (2019), the fiscal consolidation efforts at that time might have been beneficial and contributed to higher economic growth. Also, this finding is

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49 Table B.2 and B.3 along with Figure B.2 show that the results remain unchanged when I include the profitability and investment factors from Fama and French (2015).
consistent with the declining trend in the public sector investment share. Intuitively, if the optimal capital allocation between the private and public sectors remains constant, then the declining share of public sector investment implies that a shortfall in public investment is more likely to exist in more recent periods. Indeed, Figure 18 demonstrates an evident negative correlation between the public sector investment share and the average future return (over the subsequent seven years) on the equal-weighted long-short dependency portfolio; the correlation coefficient is 0.56 and highly significant. It reveals that a lower public sector investment share tends to precede a larger return spread between high- and and low-dependency firms.
Table 4: **Summary statistics.** Panel (a) summarizes a selection of firm characteristics for a sample of U.S. government contractors. Every year government contractors are defined as firms with positive sales to government over the past three years. The reported characteristics include $StG$ ratio (sales to government divided by total sales), market capitalization (in billions of 2012 dollars, deflated by GDP price index), book-to-market ratio, market leverage, asset growth, sales growth, operating profitability, and return on assets. Panel (b) compares the means of these characteristics across portfolios formed on government dependency (that is, the extent to which a firm depends on government customers for revenue). Government dependency is measured by $StG_{-2,0}$, a three-year trailing average of $StG$. This government contractor sample consists of 9,944 firm-year observations spanning 1980 to 2017. The first portfolio formation was at the end of June in 1981, and it was based on government dependency computed for 1980; the same procedure are repeated every year thereafter until 2018. Detailed sample construction and variable calculations are in Appendix B.

### (a) Government contractors

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Mean</th>
<th>S.D.</th>
<th>10th</th>
<th>25th</th>
<th>50th</th>
<th>75th</th>
<th>90th</th>
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<tr>
<td>$StG$</td>
<td>0.284</td>
<td>0.283</td>
<td>0.000</td>
<td>0.059</td>
<td>0.185</td>
<td>0.442</td>
<td>0.758</td>
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<tr>
<td>Market capitalization</td>
<td>1.435</td>
<td>4.604</td>
<td>0.010</td>
<td>0.029</td>
<td>0.110</td>
<td>0.588</td>
<td>2.790</td>
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<tr>
<td>Book-to-market</td>
<td>0.715</td>
<td>0.539</td>
<td>0.187</td>
<td>0.345</td>
<td>0.590</td>
<td>0.935</td>
<td>1.391</td>
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<tr>
<td>Market leverage</td>
<td>0.212</td>
<td>0.212</td>
<td>0.000</td>
<td>0.032</td>
<td>0.151</td>
<td>0.331</td>
<td>0.530</td>
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<tr>
<td>Asset growth</td>
<td>0.147</td>
<td>0.386</td>
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<td>-0.034</td>
<td>0.066</td>
<td>0.208</td>
<td>0.512</td>
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<tr>
<td>Sales growth</td>
<td>0.141</td>
<td>0.358</td>
<td>-0.182</td>
<td>-0.035</td>
<td>0.084</td>
<td>0.235</td>
<td>0.478</td>
</tr>
<tr>
<td>Operating profitability</td>
<td>0.163</td>
<td>0.488</td>
<td>-0.222</td>
<td>0.049</td>
<td>0.196</td>
<td>0.346</td>
<td>0.543</td>
</tr>
<tr>
<td>Return on assets</td>
<td>0.003</td>
<td>0.180</td>
<td>-0.177</td>
<td>-0.015</td>
<td>0.044</td>
<td>0.086</td>
<td>0.137</td>
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</tbody>
</table>

### (b) Government dependency portfolios

<table>
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<tr>
<th>Characteristics</th>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>5 (high)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$StG$</td>
<td>0.030</td>
<td>0.099</td>
<td>0.197</td>
<td>0.371</td>
<td>0.726</td>
</tr>
<tr>
<td>Market capitalization</td>
<td>1.786</td>
<td>1.401</td>
<td>1.593</td>
<td>1.028</td>
<td>1.368</td>
</tr>
<tr>
<td>Book-to-market</td>
<td>0.707</td>
<td>0.708</td>
<td>0.721</td>
<td>0.731</td>
<td>0.707</td>
</tr>
<tr>
<td>Market leverage</td>
<td>0.220</td>
<td>0.216</td>
<td>0.227</td>
<td>0.197</td>
<td>0.201</td>
</tr>
<tr>
<td>Asset growth</td>
<td>0.140</td>
<td>0.136</td>
<td>0.133</td>
<td>0.145</td>
<td>0.181</td>
</tr>
<tr>
<td>Sales growth</td>
<td>0.147</td>
<td>0.152</td>
<td>0.128</td>
<td>0.130</td>
<td>0.149</td>
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<tr>
<td>Operating profitability</td>
<td>0.156</td>
<td>0.141</td>
<td>0.172</td>
<td>0.146</td>
<td>0.199</td>
</tr>
<tr>
<td>Return on assets</td>
<td>0.009</td>
<td>-0.007</td>
<td>0.003</td>
<td>-0.003</td>
<td>0.016</td>
</tr>
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</table>
Table 5: **Government dependency is a persistent proxy for exposure to public sector investment.** This table reports the estimation results of a predictive regression: $StG_{i,t+h} = \alpha_h + \beta_h \overline{StG}_{i,t-2\rightarrow t} + \epsilon_{i,t+h}$, where $StG_{i,t+h}$ is the fraction of sales to government in year $t+h$ for firm $i$, $\overline{StG}_{i,t-2\rightarrow t}$ is the average fraction of sales to government from year $t-2$ to $t$, and $h$ is the forecast horizon. It also reports the results of the following regression: $\nabla[\frac{sales}{earnings}]_{i,t+1} = \alpha + \beta_1 \overline{StG}_{i,t-2\rightarrow t} + \beta_2 \nabla iG_{i,t+1} + \beta_3 \nabla iG_{i,t+1} \times \overline{StG}_{i,t-2\rightarrow t} + \epsilon_{t+1}$ where $\nabla[\frac{sales}{earnings}]_{i,t+1}$ is the sales or earnings (EBITDA) growth for firm $i$ in year $t+1$, and $\nabla iG_{i,t+1}$ is the contemporaneous public sector investment growth. The sample consists of 9,944 firm-year observations spanning 1980 to 2017. Industries are classified by two-digit SIC code. In parentheses are robust standard errors clustered at the firm level. Attached stars (*, **, ***) indicate (1, 5, 10%) statistical significance.

<table>
<thead>
<tr>
<th>$\overline{StG}_{i,t-2\rightarrow t}$</th>
<th>$\overline{StG}_{i,t-2\rightarrow t}$</th>
<th>$\nabla sales_{i,t+1}$</th>
<th>$\nabla earnings_{i,t+1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h = 1$</td>
<td>$h = 2$</td>
<td>$h = 3$</td>
<td></td>
</tr>
<tr>
<td>0.93***</td>
<td>0.89***</td>
<td>0.86***</td>
<td>-0.05***</td>
</tr>
<tr>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.02)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>$\nabla iG_{i,t+1}$</td>
<td></td>
<td>0.26*</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.15)</td>
<td>(0.28)</td>
</tr>
<tr>
<td>$\nabla iG_{i,t+1} \times \overline{StG}_{i,t-2\rightarrow t}$</td>
<td>1.01***</td>
<td>1.08</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.38)</td>
<td>(0.70)</td>
<td></td>
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</table>

Fixed effects: Year Year Year Industry Industry
Figure 15: **Government dependency portfolios: average returns and alphas.** Panel (a) and (c) display the mean excess returns on government dependency portfolios as well as the mean return on a zero-investment portfolio that is long stocks in the highest-dependency quintile and short stocks in the lowest-dependency quintile. Panel (b) and (d) display the alphas estimated from fitting a five-factor model to these portfolio returns; the five risk factors are the market, size, and value factors from Fama and French (1993); the momentum factor from Carhart (1997); and the liquidity factor from Pastor and Stambaugh (2003). Also displayed are 90% and 95% confidence intervals (indicated by the grey bar and the whiskers, respectively) computed with heteroskedasticity- and autocorrelation-consistent (HAC) standard errors following the routine of Newey and West (1987, 1994). Returns are monthly. Portfolios are value-weighted in panel (a) and (b), and equal-weighted in panel (c) and (d). The first portfolio formation was at the end of June in 1981, and it was based on government dependency ($\text{StG}_{-2,0}$) computed for 1980; the same procedure are repeated every year thereafter until 2018.
Table 6: **Government dependency portfolios: value-weighted portfolios.** Panel (a) reports the mean excess returns on government dependency portfolios as well as the mean return on a zero-investment portfolio that is long stocks in the highest-dependency quintile and short stocks in the lowest-dependency quintile. Also reported are Sharpe ratios calculated from monthly returns but expressed in annualized percentages to facilitate comparison. Panel (b) reports the estimation results of regressing these portfolio returns on five classic risk factors including the market, size, and value factors \( (MKT, \, SMB, \, HML) \) from Fama and French (1993); the momentum factor \( (MOM) \) from Carhart (1997); and the liquidity factor \( (LIQ) \) from Pastor and Stambaugh (2003). In square brackets are \( t \)-statistics computed with heteroskedasticity- and autocorrelation-consistent (HAC) standard errors following the routine of Newey and West (1987, 1994). Returns are monthly. Portfolios are value-weighted. Risk factor data are obtained from Kenneth French’s and Lubos Pastor’s websites. The first portfolio formation was at the end of June in 1981, and it was based on government dependency \( (StG_{-2,0}) \) computed for 1980; the same procedure are repeated every year thereafter until 2018.

<table>
<thead>
<tr>
<th></th>
<th>1 (low)</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5 (high)</th>
<th>High minus Low</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean excess return</strong> ((\text{monthly})%)</td>
<td>0.43</td>
<td>0.69</td>
<td>0.67</td>
<td>0.90</td>
<td>1.06</td>
<td>0.62</td>
</tr>
<tr>
<td><strong>Sharpe ratio</strong> ((\text{annualized})%)</td>
<td>22.01</td>
<td>38.11</td>
<td>35.57</td>
<td>45.55</td>
<td>65.50</td>
<td>36.14</td>
</tr>
<tr>
<td><strong>( \alpha )</strong></td>
<td>-0.40</td>
<td>0.07</td>
<td>-0.11</td>
<td>0.37</td>
<td>0.43</td>
<td>0.82</td>
</tr>
<tr>
<td>([-2.06]) ([0.69]) ([-0.63]) ([1.89]) ([1.89]) ([2.42])</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>( \beta_{MKT} )</strong></td>
<td>1.19</td>
<td>1.11</td>
<td>1.18</td>
<td>0.95</td>
<td>0.89</td>
<td>-0.30</td>
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<tr>
<td>([23.59]) ([22.09]) ([19.88]) ([17.48]) ([15.96]) ([-3.93])</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td><strong>( \beta_{SMB} )</strong></td>
<td>0.29</td>
<td>0.18</td>
<td>0.15</td>
<td>0.45</td>
<td>0.18</td>
<td>-0.10</td>
</tr>
<tr>
<td>([3.65]) ([2.01]) ([1.72]) ([2.54]) ([2.74]) ([-0.92])</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>( \beta_{HML} )</strong></td>
<td>-0.05</td>
<td>0.01</td>
<td>0.18</td>
<td>-0.22</td>
<td>0.23</td>
<td>0.28</td>
</tr>
<tr>
<td>([-0.62]) ([0.08]) ([1.49]) ([-1.33]) ([1.49]) ([1.60])</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>( \beta_{MOM} )</strong></td>
<td>-0.03</td>
<td>-0.18</td>
<td>-0.13</td>
<td>-0.07</td>
<td>0.07</td>
<td>0.10</td>
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<tr>
<td>([-0.51]) ([-2.84]) ([-2.23]) ([-1.63]) ([0.70]) ([0.90])</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>( \beta_{LIQ} )</strong></td>
<td>0.20</td>
<td>0.01</td>
<td>0.12</td>
<td>0.02</td>
<td>-0.12</td>
<td>-0.32</td>
</tr>
<tr>
<td>([2.35]) ([0.19]) ([1.49]) ([0.26]) ([-1.36]) ([-2.73])</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Adj. ( R^2 )</strong></td>
<td>0.67</td>
<td>0.68</td>
<td>0.66</td>
<td>0.51</td>
<td>0.47</td>
<td>0.13</td>
</tr>
</tbody>
</table>

48
Table 7: Government dependency portfolios: equal-weighted portfolios. Panel (a) reports the mean excess returns on government dependency portfolios as well as the mean return on a zero-investment portfolio that is long stocks in the highest-dependency quintile and short stocks in the lowest-dependency quintile. Also reported are Sharpe ratios calculated from monthly returns but expressed in annualized percentages. Panel (b) reports the estimation results of regressing these portfolio returns on five classic risk factors. Portfolios are equal-weighted. Other specifics are the same as in Table 6.

<table>
<thead>
<tr>
<th>Govt. dependency portfolios</th>
<th>1 (low)</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5 (high)</th>
<th>High minus Low</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>(a) Return moments</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean excess return</td>
<td>0.41</td>
<td>0.53</td>
<td>0.66</td>
<td>0.57</td>
<td>0.76</td>
<td>0.35</td>
</tr>
<tr>
<td>(monthly %)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>22.49</td>
<td>29.44</td>
<td>37.72</td>
<td>32.02</td>
<td>45.45</td>
<td>31.70</td>
</tr>
<tr>
<td>(annualized %)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| **(b) Controlling for classic risk factors** |
| $\alpha$ | -0.26 | 0.06 | 0.12 | 0.05 | 0.30 | 0.56 |
|          | [-2.04] | [0.52] | [0.67] | [0.29] | [1.40] | [2.40] |
| $\beta_{MKT}$ | 1.03 | 0.93 | 0.95 | 0.86 | 0.81 | -0.22 |
|          | [32.63] | [24.28] | [30.79] | [19.31] | [15.98] | [-3.90] |
| $\beta_{SMB}$ | 0.81 | 0.88 | 0.83 | 1.02 | 0.76 | -0.06 |
|          | [12.85] | [11.20] | [10.52] | [20.75] | [8.11] | [-0.89] |
| $\beta_{HML}$ | 0.03 | -0.04 | -0.02 | -0.05 | -0.04 | -0.07 |
|          | [0.39] | [-0.46] | [-0.28] | [-0.62] | [-0.42] | [-1.10] |
| $\beta_{MOM}$ | -0.14 | -0.26 | -0.17 | -0.16 | -0.09 | 0.05 |
|          | [-3.09] | [-6.87] | [-2.98] | [-5.82] | [-1.41] | [1.16] |
| $\beta_{LIQ}$ | 0.09 | -0.03 | -0.02 | 0.05 | -0.09 | -0.18 |
|          | [1.30] | [-0.64] | [-0.44] | [0.88] | [-1.81] | [-3.40] |
| Adj. $R^2$ | 0.80 | 0.79 | 0.80 | 0.78 | 0.67 | 0.10 |
Figure 16: **Government dependency portfolios: value-weighted portfolios; subperiods: 1981 to 1999 vs. 2000 to 2018.** Panel (a) and (b) display the mean excess returns on government dependency portfolios as well as the mean return on a zero-investment portfolio that is long stocks in the highest-dependency quintile and short stocks in the lowest-dependency quintile. Panel (c) and (d) display the alphas estimated from fitting a five-factor model to these portfolio returns. Also displayed are 90% and 95% confidence intervals indicated by the grey bar and the whiskers, respectively. The sample period is 1981 to 1999 in panel (a) and (c), and 2000 to 2018 in panel (b) and (d). Portfolios are value-weighted. Other specifics are the same as in Figure 15.
Figure 17: **Government dependency portfolios: equal-weighted portfolios; subperiods: 1981 to 1999 vs. 2000 to 2018.** Panel (a) and (b) display the mean excess returns on government dependency portfolios as well as the mean return on a zero-investment portfolio that is long stocks in the highest-dependency quintile and short stocks in the lowest-dependency quintile. Panel (c) and (d) display the alphas estimated from fitting a five-factor model to these portfolio returns. Also displayed are 90% and 95% confidence intervals indicated by the grey bar and the whiskers, respectively. The sample period is 1981 to 1999 in panel (a) and (c), and 2000 to 2018 in panel (b) and (d). Portfolios are equal-weighted. Other specifics are the same as in Figure [15](#).
Table 8: **Government dependency portfolios: 1981-1999 vs. 2000-2018.** Panel (a) reports for two subperiods, 1981-1999 and 2000-2018, the mean excess returns on government dependency portfolios as well as the mean return on a zero-investment portfolio that is long stocks in the highest-dependency quintile and short stocks in the lowest-dependency quintile. Panel (b) reports the corresponding alphas estimated by regressing these portfolio returns on five classic risk factors. Portfolios are either value-weighted or equal-weighted. Other specifics are the same as in Table 6.

<table>
<thead>
<tr>
<th></th>
<th>Govt. dependency portfolios</th>
<th>High minus Low</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 (low)</td>
<td>2</td>
</tr>
<tr>
<td><strong>Value weight</strong></td>
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<td></td>
</tr>
<tr>
<td>1981-1999</td>
<td>0.71</td>
<td>0.60</td>
</tr>
<tr>
<td>2000-2018</td>
<td>0.15</td>
<td>0.79</td>
</tr>
<tr>
<td><strong>Equal weight</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1981-1999</td>
<td>0.61</td>
<td>0.50</td>
</tr>
<tr>
<td>2000-2018</td>
<td>0.21</td>
<td>0.56</td>
</tr>
</tbody>
</table>

**Panel (b) Alphas w.r.t. five classic risk factors**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-0.05</td>
<td>-0.56</td>
<td>0.11</td>
<td>-0.49</td>
</tr>
<tr>
<td></td>
<td>[0.06]</td>
<td>[0.23]</td>
<td>[0.11]</td>
<td>[-0.08]</td>
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<tr>
<td></td>
<td>[-0.12]</td>
<td>[0.08]</td>
<td>[0.01]</td>
<td>[0.09]</td>
</tr>
<tr>
<td></td>
<td>[-0.13]</td>
<td>[0.58]</td>
<td>[-0.08]</td>
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</tr>
<tr>
<td></td>
<td>[0.49]</td>
<td>[0.55]</td>
<td>[0.19]</td>
<td>[1.11]</td>
</tr>
<tr>
<td></td>
<td>[0.55]</td>
<td>[1.90]</td>
<td>[0.73]</td>
<td>[3.30]</td>
</tr>
</tbody>
</table>

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Figure 18: **Expected return on long-short government dependency portfolio and the public sector investment share.** The solid line represents the average future return (over the subsequent seven years) on a zero-investment portfolio that is long stocks in the highest-dependency quintile and short stocks in the lowest-dependency quintile. The dashed line represents the public sector investment share, that is, the ratio of public sector investment to the sum of public and private sector investments. The magnitude of the former (in monthly percent) is indicated on the left axis while the latter (in percent) on the right axis.
5 Conclusion

In this paper, I employ asset pricing theory and empirics to infer from asset prices whether investors perceive a shortfall in public sector investment. I develop a parsimonious two-sector GE model to provide a theoretical argument for why and how public sector investment may drive agents’ marginal utility and thus affect asset pricing. From this GE theory I derive a factor pricing model and confront it with a wide range of test assets. The results indicate that shocks to the share of public sector investment are priced in the cross-section of stock returns with a consistently positive price of risk. It points to an increase in the public sector investment share as good news for investors. To strengthen and expand this finding, I conduct a portfolio analysis (which is model-free) using a sample of U.S. government contractors. I find that firms with heavier reliance on the U.S. government for revenue are more sensitive to changes in public sector investment and provide higher stock returns on average. I also find that the spread in average returns between high- and low-government-dependency stocks has widened in recent years, implying a bigger shortfall in public investment.

That said, one should not use my findings to guide the investment decision on a particular public sector project, which ought to be based on specific cost-benefit analyses. My results should instead be interpreted as an indicator of an overall shortfall in public sector investment, and that allocating more resources to augment public sector capital may improve welfare.

An unanswered question in this study is why the public sector is underinvested. In theory, an inadequate stock of public sector capital should attract more investment for its high marginal product (as well as other benefits). But the public sector investment share shown in Figure 1 has been declining since the 1960s with no sign of a pickup whatsoever. What is missing here? I can think of two possible drivers that are absent from my model. One is political factors. Public investment decision-making is often influenced by political considerations that dominate economic ones in many cases, if not all. For one thing, when it comes to winning votes, tax cuts are arguably more appealing than infrastructure spending. Another reason is that inefficiencies and perversities attending the existing public sector projects may stymie any attempt to increase spending. One can reasonably argue that resolving these problems should take priority over passing big spending bills. In any case, the evidence provided in this paper suggests that augmenting the relative stock of public sector capital, by either investing more or spending more efficiently, has a nontrivial, positive impact on investors’ welfare.
References


The Economist (2019, May). Many governments could bear more debt. That does not mean they should. The risks posed by higher public debt are distant but real. [https://www.economist.com/leaders/2019/05/18/many-governments-could-bear-more-debt-that-does-not-mean-they-should](https://www.economist.com/leaders/2019/05/18/many-governments-could-bear-more-debt-that-does-not-mean-they-should).


A Appendix

In this appendix, I present expressions omitted in the main text. I also provide details on the calibration of the two-sector general equilibrium model.

A.1 Omitted expressions

The first set of omitted expressions are the drift and diffusion coefficients of $\xi_t$, $q_t$, and $c_t$.

$$
\mu_{\xi,t} = \frac{\partial \xi_t}{\xi_t} \mu_{\chi,t} + \frac{\partial \xi_t}{\xi_t} \kappa (\xi_t - c_t) + \frac{1}{2} \frac{\partial^2 \xi_t}{\xi_t} \xi_t^2 + \frac{1}{2} \frac{\partial^2 \xi_t}{\xi_t} \nu^2 \xi_t \\
\sigma_{\xi,t} = -\frac{\partial \xi_t}{\xi_t} \nu \xi_t \sqrt{\xi_t} \quad \zeta_{\xi,t} = -\frac{\partial \xi_t}{\xi_t} \xi_t \chi_t
$$

$$
\mu_{q,t} = \frac{\partial q_t}{q_t} \mu_{\chi,t} + \frac{\partial q_t}{q_t} \kappa (\xi_t - c_t) + \frac{1}{2} \frac{\partial^2 q_t}{q_t} \xi_t^2 + \frac{1}{2} \frac{\partial^2 q_t}{q_t} \nu^2 \xi_t \\
\sigma_{q,t} = -\frac{\partial q_t}{q_t} \nu \xi_t \sqrt{\xi_t} \quad \zeta_{q,t} = -\frac{\partial q_t}{q_t} \xi_t \chi_t
$$

$$
\mu_{c,t} = \frac{\partial c_t}{c_t} \mu_{\chi,t} + \frac{\partial c_t}{c_t} \kappa (\xi_t - c_t) + \frac{1}{2} \frac{\partial^2 c_t}{c_t} \xi_t^2 + \frac{1}{2} \frac{\partial^2 c_t}{c_t} \nu^2 \xi_t \\
\sigma_{c,t} = -\frac{\partial c_t}{c_t} \nu \xi_t \sqrt{\xi_t} \quad \zeta_{c,t} = -\frac{\partial c_t}{c_t} \xi_t \chi_t
$$

Next is the pricing kernel $\Lambda_t$, which is defined as per Duffie and Epstein (1992)

$$\Lambda_t = \exp \left[ \int_0^t u_V(C_t, V_t) d\tau \right] u_C(C_t, V_t)$$

with

$$u_C(C, V) \equiv \frac{\partial u(C, V)}{\partial C} = \frac{\beta C^{-1/\psi}}{[(1-\gamma)V]^{\gamma-1/\psi}}$$

$$u_V(C, V) \equiv \frac{\partial u(C, V)}{\partial V} = \frac{\beta}{1-1/\psi} \left[ (1/\psi - \gamma) \frac{C^{1-1/\psi}}{[(1-\gamma)V]^{\gamma-1/\psi}} - (1-\gamma) \right].$$

The law of motion of the pricing kernel can be derived using Ito’s lemma

$$\frac{d\Lambda_t}{\Lambda_t} = u_V(C_t, V_t) dt + \frac{du_C(C_t, V_t)}{u_C(C_t, V_t)} = -r_t dt - \eta_t dZ_t - \theta_t dW_t.$$
The expressions for the risk-free rate and the risk prices are shown in equation (20) and (21). One can easily verify that when $1/\psi = \gamma$, these three expressions collapse to those derived from a standard continuous-time Lucas (1978) economy with power utility.

A.2 Model calibration

TBA
B Online Appendix

In this online appendix, I provide a heuristic derivation of the HJB equation associated with the utility maximization problem of an agent with recursive preferences. I also empirically examine the relationships between the public sector investment share, the real risk-free rate, and economic uncertainty, which turn out to be consistent with the model predictions. In addition, I provide details on the numerical solution of the two-sector GE model. Lastly, I elaborate on the construction of the government contractor sample and the calculations of related variables. Additional empirical results, tables and figures, are also presented here.

B.1 Derivation of the HJB Equation with Recursive Preferences

I start from a discrete-time setting and derive the continuous-time limit, following a similar route as Obstfeld (1994); technical details are addressed by Duffie and Epstein (1992).

Consider the utility maximization problem of an agent with recursive preferences:

\[ V_t = \max \left[ \left(1 - e^{-\beta \Delta} \right) C_t^{1 - 1/\psi} + e^{-\beta \Delta} (E_t V_{t+\Delta}^{1-\gamma}) \right]^{1/1-1/\psi} \]  \hspace{1cm} (B.1)

where the time length per period is \( \Delta \), and other parameters are defined as usual. Define a new value function \( V_t \equiv \frac{V_t^{1-\gamma}}{1-\gamma} \), and rewrite (B.1) as

\[ [(1 - \gamma) V_t]^{1/\psi} = \max \left\{ \left(1 - e^{-\beta \Delta} \right) C_t^{1 - 1/\psi} + e^{-\beta \Delta} \left[ (1 - \gamma) V_t^{1-\gamma} \right] \right\}^{1/1-1/\psi}. \] \hspace{1cm} (B.2)

Define another function \( G(X) \equiv \left[(1 - \gamma) X\right]^{1/1-1/\psi} \), and rewrite again:

\[ G(V_t) = \max \left[ \left(1 - e^{-\beta \Delta} \right) C_t^{1 - 1/\psi} + e^{-\beta \Delta} G(V_t^{1-\gamma}) \right]^{1/1-1/\psi}. \] \hspace{1cm} (B.2)

Because \( \frac{X^{1-1/\psi}}{1-1/\psi} \) is a monotonic transformation of \( X \), maximizing \( G(V_t) \) and \( G(V_t)^{1/1-1/\psi} \) are equivalent; so (B.2) is equivalent to

\[ \frac{G(V_t)}{1-1/\psi} = \max \left[ \left(1 - e^{-\beta \Delta} \right) C_t^{1 - 1/\psi} + e^{-\beta \Delta} \frac{G(V_t^{1-\gamma})}{1-1/\psi} \right]. \] \hspace{1cm} (B.3)
Subtract $e^{-\beta \Delta} \frac{G(V_t)}{1 - 1/\psi}$ from both sides:

$$(1 - e^{-\beta \Delta}) \frac{G(V_t)}{1 - 1/\psi} = \max \left\{ (1 - e^{-\beta \Delta}) \frac{C_t^{1-1/\psi}}{1 - 1/\psi} + e^{-\beta \Delta} \left[ \frac{G(\mathbb{E}_t V_{t+\Delta})}{1 - 1/\psi} - \frac{G(V_t)}{1 - 1/\psi} \right] \right\}.$$  

Divide both sides by $\Delta$ and take $\Delta \to 0$:

$$\lim_{\Delta \to 0} \frac{1 - e^{-\beta \Delta}}{\Delta} \frac{G(V_t)}{1 - 1/\psi} = \max \left\{ \lim_{\Delta \to 0} \frac{1 - e^{-\beta \Delta}}{\Delta} \frac{C_t^{1-1/\psi}}{1 - 1/\psi} + \lim_{\Delta \to 0} e^{-\beta \Delta} \frac{G(\mathbb{E}_t V_{t+\Delta})}{1 - 1/\psi} - \frac{G(V_t)}{1 - 1/\psi} \right\}.$$  

Use Taylor’s theorem:

$$\frac{\beta}{1 - 1/\psi} G(V_t) = \max \left\{ \frac{\beta}{1 - 1/\psi} C_t^{1-1/\psi} + \frac{1}{1 - 1/\psi} G'(V_t) \mathbb{E}_t dV_t \right\}.$$  

Substitute in function $G(\cdot)$ and rearrange terms:

$$0 = \max \left\{ \frac{\beta (1 - \gamma) V_t}{1 - 1/\psi} \left\{ \frac{C_t^{1-1/\psi}}{[(1 - \gamma) V_t]^{1-1/\psi}} - 1 \right\} + \frac{\mathbb{E}_t dV_t}{d t} \right\}. \quad (B.4)$$  

From (B.4), I obtain equation (9) in the main text.

**B.2 Testing model predictions**

The GE model presented in the paper predicts that, when facing greater uncertainty, the public sector investment share rises while the risk-free rate declines; but controlling for uncertainty, it predicts a positive association between these two variables (see Figure 12). Here I take this prediction to the U.S. data.

**B.2.1 Specifications**

I start by examining the role of uncertainty as a predictor of the public sector investment share and the real risk-free rate. I use a standard predictive regression specified as

$$A^h(Y_t) = \alpha + \beta \times UNC_t + \epsilon_t + h,$$  

where $A^h(Y_t) \equiv \frac{1}{n+h} \sum_{t=0}^{h} Y_{t+h}$ is the average value of a predicted variable $Y$ over a forecast horizon of $h$ periods (e.g., $A^1(Y_t) = (Y_t + Y_{t+1})/2$), $UNC_t$ is an uncertainty index
from Jurado, Ludvigson, and Ng (2015), and $\epsilon_{t+h}$ is the forecast error. The predicted variables include $PubIS_{t}^{cyc}$, the cyclical component of the public sector investment share, and $r_t$, the real risk-free rate. All variables are already defined in Section 3 and Appendix A.

I then test the relation between the public sector investment share and the real risk-free rate controlling for uncertainty. Specifically, I estimate the following regression:

$$A^h(r_t) = \alpha + \beta_1 \times A^h(PubIS_{t}^{cyc}) + \beta_2 \times UNC_t + \epsilon_t. \quad (B.6)$$

My main interest is the slope coefficient $\beta_1$, which is predicted to be positive according to my model. I run this regression under different horizons because, in practice, both the public sector investment share and the risk-free rate may not respond instantaneously to changes in economic conditions. Allowing some flexibility in the time frame may help identify the correlation implied by the model.

B.2.2 Results

Table B.1 presents the estimation results based on a sample from 1960Q3 to 2018Q4; the first observation is dictated by the start of the uncertainty measure. I trimmed the 1979Q4 to 1982Q4 episode to avoid a spell of drastic movements in interest rates caused by a well-documented monetary policy shock. My model does not incorporate monetary policy risk, so it cannot speak to changes in that period.

Conforming to the model prediction, panel (a) in Table B.1 shows that the public sector investment share and the real risk-free rate react differently to higher uncertainty: the former goes up, whereas the latter goes down. The estimated slope coefficients are statistically significant for all horizons, and their magnitudes increase in horizon. As for the economic significance, at the two-year horizon, a one-standard-deviation ($\approx 0.075$) increase in the JLN uncertainty index is associated with a 66 basis point (bps) decrease in the (annualized) real risk-free rate and a 0.67 percentage point increase in the public sector investment share. The adjusted $R^2$ also increases in horizon, ranging from 0.08 to 0.14 for the real risk-free rate and 0.08 to 0.17 for the public sector investment share.

Panel (b) examines the relation between the real risk-free rate and the public sector investment share. As shown, without any control, the real risk-free rate is barely related to the contemporaneous public sector investment share for all horizons. But controlling

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50Clarida, Gali, and Gertler (2000) point out that this episode was characterized by a sharp, one-shot “Volcker shock” that brought inflation down by more than 5 percent in a relatively short period of time. Also, the operating procedures of the Federal Reserve briefly changed to targeting non-borrowed reserves in lieu of the usual instrument, Federal Funds rate. These monetary factors caused exceptional disturbances to the real interest rates. Also see Christiano, Eichenbaum, and Evans (1999) and Romer (2016).
Table B.1: Interest rate, public sector investment share, and economic uncertainty. Panel (a) reports the estimation results of a predictive regression (B.5). The dependent variable is $A^h(Y_t)$, the average value of a predicted variable $Y$ over a forecast horizon of $h$ periods; $Y$ is either the (annualized) real risk-free rate or the cyclical component of the public sector investment share, and $h$ equals 2, 4, or 8 quarters. The regressor ($UNC$) is an economic uncertainty index from Jurado, Ludvigson, and Ng (2015). Panel (b) reports the estimation results of another regression (B.6). The $t$-statistics are based on heteroskedasticity- and autocorrelation-consistent (HAC) standard errors (Newey and West, 1987, 1994). The sample is from 1960Q3 to 2018Q4 with the period from 1979Q4 to 1982Q4 trimmed due to a significant monetary policy shock.

(a) Economic uncertainty as a predictor

<table>
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<tr>
<th>Forecast horizon ($h$)</th>
<th>Real risk-free rate (annualized, %)</th>
<th>2-quarter</th>
<th>4-quarter</th>
<th>8-quarter</th>
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<tbody>
<tr>
<td>UNC</td>
<td>-7.46</td>
<td>-7.97</td>
<td>-8.76</td>
<td></td>
</tr>
<tr>
<td>$[t]$</td>
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<td>[-2.32]</td>
<td>[-3.00]</td>
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<tr>
<td>Adj. $R^2$</td>
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<td>0.10</td>
<td>0.14</td>
<td></td>
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</table>

<table>
<thead>
<tr>
<th>Forecast horizon ($h$)</th>
<th>Public sector inv. share (cyc., %)</th>
<th>2-quarter</th>
<th>4-quarter</th>
<th>8-quarter</th>
</tr>
</thead>
<tbody>
<tr>
<td>UNC</td>
<td>7.89</td>
<td>9.89</td>
<td>9.03</td>
<td></td>
</tr>
<tr>
<td>$[t]$</td>
<td>[3.20]</td>
<td>[4.96]</td>
<td>[4.98]</td>
<td></td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>0.08</td>
<td>0.15</td>
<td>0.18</td>
<td></td>
</tr>
</tbody>
</table>

(b) The relation between the risk-free rate and the public sector investment share

<table>
<thead>
<tr>
<th>Forecast horizon ($h$)</th>
<th>Real risk-free rate (annualized, %)</th>
<th>2-quarter</th>
<th>4-quarter</th>
<th>8-quarter</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A^h(PubIS^{cyc})$</td>
<td>0.09</td>
<td>0.19</td>
<td>0.09</td>
<td>0.26</td>
</tr>
<tr>
<td>$[t]$</td>
<td>[0.69]</td>
<td>[1.41]</td>
<td>[0.64]</td>
<td>[1.81]</td>
</tr>
<tr>
<td>UNC</td>
<td>-8.93</td>
<td>-10.48</td>
<td>-12.36</td>
<td></td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>0.01</td>
<td>0.11</td>
<td>0.01</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Interpretation: Greater uncertainty precedes a lower risk-free rate but a higher public sector investment share. Controlling for uncertainty, a higher public sector investment share coincides with a higher risk-free rate.

for uncertainty, the real risk-free rate displays a positive association with the public sector investment share. In particular, at the two-year horizon, a one-standard-deviation ($\approx$...
2.2%) increase in the public sector investment share is associated with a 88 bps higher real risk-free rate. This is again consistent with the model prediction.

### B.3 Numerical Methods

The two-sector general equilibrium model presented in the paper is numerically solved using an iterative method. The procedure is as follows. I start by putting together a system of partial differential equations (PDEs) that characterizes a Markov equilibrium. It consists of the HJB equation associated with the central planning problem and the corresponding first-order conditions (FOCs):

\[
\frac{\beta}{1 - 1/\psi} = \max_{\pi^p, \xi^t} \frac{\beta}{1 - 1/\psi} \left( \frac{\xi_t}{\xi_t} \right)^{1-1/\psi} + \mu_{K,t} + \mu_{\xi,t} - \frac{\gamma}{2} \left[ \sigma^2 + (1 - \chi_t)^2 \xi_t^2 + \sigma_{\xi,t}^2 + \xi_{\xi,t}^2 \right] 
\]

\[
+ (1 - \gamma) \left( \sigma \sigma_{\xi,t} + (1 - \chi_t) \xi_t \xi_{\xi,t} \right),
\]

\[
\left( \frac{c_t}{\xi_t} \right)^{1/\psi} = \frac{\beta}{\phi'(t^p_i)} \frac{1}{\xi_t - \chi_t} \frac{\partial}{\partial \xi_t}
\]

\[
\left( \frac{c_t}{\xi_t} \right)^{1/\psi} = \frac{\beta}{\phi'(t^p_i)} \frac{1}{\xi_t + (1 - \chi_t)} \frac{\partial}{\partial \xi_t}
\]

(B.7) (B.8)

where \(c_t \equiv [M(\chi_t) - \xi_t^p \chi_t - \xi_t^q (1 - \chi_t)]\) is the consumption-capital ratio, and \(\xi_t \equiv \xi(\chi_t, \xi_t)\) is the unknown function to be obtained. Ideally, with the state of this system determined by \(\chi_t\) and \(\xi_t\), one should seek the true solution—that is, a well-behaved analytical function \(\xi^*(\chi_t, \xi_t)\) that satisfies (B.7) and (B.8). But in this case such a solution is difficult to find, if not impossible. Thus my goal instead is to find a numerical solution that approximates the true solution as close as possible.

**Discretization.** The first step is to choose a set of grid points in the state space. Specifically, I choose \(I \times J\) grid points from the state space; each point, denoted by \((i, j)\), represents a unique state of the system characterized by \(\chi(i)\) and \(\xi(j)\), where

\[
\chi(i) = \frac{3}{I^2} - 2 \frac{\beta}{I^3}, \quad i = 1, ..., I, \quad \xi(j) = \frac{j^2}{J^2}, \quad j = 1, ..., J.
\]

This scheme constructs a nonuniform grid that is denser near boundaries where function \(\xi\) is expected to have more curvature. \(^{51}\) Alternatively, one can also use uniform grids that are simpler to construct but lend less accuracy.

\(^{51}\)See Brunnermeier and Sannikov (2016b) for another example of using this scheme. There is a whole area of research concentrated on the optimal grid generation (see, e.g., Thompson, Warsi, and Mastin 1985). The presented method may not be optimal but works well enough in this context.
Iterative method. The next step is to find the approximate values of function $\xi$ at these grid points. I adapt an iterative method from Brunnermeier and Sannikov (2016a) and Achdou et al. (2017); the key idea is to add a pseudo time dimension to the system and iterate it until convergence. Specifically, I assume that $\xi$ is directly dependent on time, that is, $\xi_t$ equals $\xi(\chi_t, \varsigma_t, t)$ instead of $\xi(\chi_t, \varsigma_t)$. Then I modify equation (B.7) accordingly and write it as a linear combination of the first- and second-order partial derivatives of $\xi$:

$$H_{0,t} = \frac{\partial \xi_t}{\partial t} + H_{1,t} \frac{\partial \xi_t}{\partial \chi_t} + H_{2,t} \frac{\partial \xi_t}{\partial \varsigma_t} + H_{3,t} \frac{\partial^2 \xi_t}{\partial \chi_t^2} + H_{4,t} \frac{\partial^2 \xi_t}{\partial \varsigma_t^2}$$  \hspace{1cm} (B.9)

where

$$H_{0,t} = \xi_t \left\{ \frac{\beta}{1 - 1/\psi} - \frac{\beta}{1 - 1/\psi} \left( \frac{\xi_t}{\xi_t} \right)^{1-1/\psi} - [(1 - \chi_t)\phi(t_p^p) + \chi_t\phi(t_s^p) - \delta] 
+ \frac{\gamma}{2} \left[ \sigma^2 + (1 - \chi_t)^2 \varsigma_t^2 \left( \frac{\partial \xi_t}{\xi_t} \right)^2 v^2 \chi_t + \left( \frac{\partial \chi_t}{\xi_t} \right)^2 \varsigma_t^2 \right] \right\} \hspace{1cm} (B.10)$$

$$H_{1,t} = \mu_{\chi,t} - (1 - \gamma)(1 - \chi_t)\varsigma_t\chi_t \hspace{1cm} H_{2,t} = \kappa(\zeta - \varsigma_t) - (1 - \gamma)\sigma v \sqrt{\xi_t}$$

$$H_{3,t} = \frac{\varsigma_t}{2} \hspace{1cm} H_{4,t} = \frac{v^2 \varsigma_t}{2}.$$  

The core step is to design an algorithm that takes in some guessed values of $\xi$ and generates updated ones, for which there are two options: the explicit and implicit methods.

The explicit method is relatively easy to implement. Specifically, I evaluate the revised HJB equation (B.9) at every grid point, transforming it into a set of difference equations. For a given grid point $(i, j)$, I substitute $\chi(i), \zeta(j)$, and the guessed value of $\xi(i, j)$ into (B.8), (B.9), and (B.10) to attain a difference equation:

$$H_0(i, j) = \frac{\partial \xi}{\partial t} \bigg|_{(i, j)} + H_1(i, j) \frac{\partial \xi}{\partial \chi} \bigg|_{(i, j)} + H_2(i, j) \frac{\partial \xi}{\partial \zeta} \bigg|_{(i, j)} + H_3(i, j) \frac{\partial^2 \xi}{\partial \chi^2} \bigg|_{(i, j)} + H_4(i, j) \frac{\partial^2 \xi}{\partial \zeta^2} \bigg|_{(i, j)},$$  \hspace{1cm} (B.11)
where the derivatives are approximated using the finite difference method.  

\[
\frac{\partial \xi}{\partial \chi} \big|_{(i,j)} \approx \begin{cases} 
\frac{\xi(i+1,j)-\xi(i,j)}{\chi(i+1)-\chi(i)}, & i = 1 \\
\frac{\xi(i+1,j)-\xi(i,j)}{\chi(i+1)-\chi(i)}, & 1 < i < I \\
\frac{\xi(i,j)-\xi(i-1,j)}{\chi(i)-\chi(i-1)}, & i = I \end{cases}
\]

\[
\frac{\partial^2 \xi}{\partial \chi^2} \big|_{(i,j)} \approx \begin{cases} 
\frac{[\chi(i+1)-\chi(i)]\xi(i+2,j)-[\chi(i+2)-\chi(i)]\xi(i+1,j)+[\chi(i+2)-\chi(i+1)]\xi(i,j)}{[\chi(i+2)-\chi(i)][\chi(i+2)-\chi(i+1)][\chi(i+1)-\chi(i)]}, & i = 1 \\
\frac{[\chi(i)-\chi(i-1)]\xi(i+1,j)-[\chi(i+1)-\chi(i-1)]\xi(i,j)+[\chi(i+1)-\chi(i)]\xi(i-1,j)}{[\chi(i+1)-\chi(i-1)][\chi(i+1)-\chi(i)][\chi(i)-\chi(i-1)]}, & 1 < i < I \\
\frac{[\chi(i)-\chi(i-2)]\xi(i,j)-[\chi(i)-\chi(i-2)]\xi(i-1,j)+[\chi(i)-\chi(i-1)]\xi(i-2,j)}{[\chi(i)-\chi(i-2)][\chi(i)-\chi(i-1)][\chi(i-1)-\chi(i-2)]}, & i = I \end{cases}
\]

\[
\frac{\partial \xi}{\partial t} \big|_{(i,j)} \approx \frac{\xi(i,j) - \xi^u(i,j)}{\Delta}
\]

I first use (B.8) to attain the values of \( \rho^p(i,j) \) and \( \rho^s(i,j) \), which then are used to compute (B.10). Plugging (B.10) into (B.9) gives (B.11), in which the updated value—denoted by \( \xi^u(i,j) \)—is the only unknown and hence can be “explicitly” computed. Repeating this calculation for all grid points gives a full set of updated values, \( \{ \xi^u(i,j); i = 1, \ldots, I \text{ and } j = 1, \ldots, J \} \).

Another approach to attain updates is the implicit method. Compared with the explicit method, the only difference here is that four of the partial derivatives in (B.11) are now approximated using the updated values in lieu of the guessed ones, that is,  

\[
H_0(i,j) = \frac{\partial \xi}{\partial t} \big|_{(i,j)} + H_1(i,j) \frac{\partial \xi^{u}}{\partial \chi} \big|_{(i,j)} + H_2(i,j) \frac{\partial^2 \xi^{u}}{\partial \chi^2} \big|_{(i,j)} + H_3(i,j) \frac{\partial \xi^{u}}{\partial \xi} \big|_{(i,j)} + H_4(i,j) \frac{\partial^2 \xi^{u}}{\partial \xi^2} \big|_{(i,j)}
\]

I mainly used central differences in this paper. But I also tried the “upwind scheme”, a method that is widely considered as the most reliable one (in terms of stability) when it comes to this type of problems (Achdou et al., 2017). Since in the context of my model the central differences perform reasonably well, I skip the explanation of the “upwind scheme” for brevity.

It can be shown that the explicit method converges only if \( \Delta \) is sufficiently small, while the implicit method is not subject to this constraint.

Note that \( \xi \) is replaced by \( \xi^u \) at four places. Strictly speaking, the presented method is only “semi-implicit”. A fully implicit method requires the partial derivatives in (B.10) to be calculated using the updated values as well. But that would produce a nonlinear optimization problem instead of the linear one presented here.
Such changes result in interdependence among the corresponding difference equations, which makes it impossible to calculate $\xi^{u}(i,j)$ point by point. Instead I stack all difference equations together and treat them as a system that can be written in matrix form

$$A\xi^{u} = B,$$  \hfill (B.13)

where $A$ is an $(I \times J) \times (I \times J)$ sparse matrix, and $B$ is an $(I \times J) \times 1$ vector. (B.13) can be solved efficiently by taking advantage of the sparse matrix operations in Matlab. The solution $\xi^{u} \equiv [\xi^{u}(1,1), \ldots, \xi^{u}(I,J)]$ is a vector of updated values.

**Summary.** Put together, an algorithm to find the numerical solution to (B.7) and (B.8) is summarized below.

Start with an initial guess of $\xi$, follow these steps:

1. For all $i = 1, \ldots, I$ and $j = 1, \ldots, J$, compute $\iota^{p}$ and $\iota^{g}$ using (B.8), and $H_{0}$ to $H_{4}$ using (B.10). Replace partial derivatives with finite differences.

2. Find $\xi^{u}(i,j)$ for every grid point using either the explicit method (B.11) or the implicit method (B.12).

3. If $\xi^{u}$ is close enough to the guessed $\xi$, then stop. Otherwise, use $\xi^{u}$ as the new guess and go back to step 1.

Several implementation notes are in order. First, although this algorithm is not rigorously validated (e.g., convergence, stability, etc.), it demonstrates smooth and stable convergence when confronted with a wide range of parameter configurations. This is especially true for the implicit method. (In comparison, the explicit method fails to converge for some parameter values.) Hence, based on my experience, the implicit method is preferred over the explicit method for its better stability as well as higher efficiency (since a larger step size can be used). But these advantages come with some cost: the implicit method is much less penetrable and harder to code and debug. So probably a better strategy is to carry out the explicit method first to help one think through the whole process. And with that as a foundation, it becomes more straightforward to modify the code and apply the implicit method.

Second, the accuracy of the numerical approximation of partial derivatives is essential to the success of this algorithm. In particular, both the implicit and explicit methods need to calculate (B.8) and (B.10) using the guessed $\xi$, in which the evaluations of its partial derivatives are involved. I experiment two schemes to reduce the approximation errors. The first scheme is to fit a polynomial to the guessed $\xi$, and then use that polynomial as a
proxy to compute derivatives at any given point. The advantage of this scheme is that the
derivatives are perfectly calculated with no approximation whatsoever. But it only works
as well as the fitting, the performance of which drops drastically outside of the region
where \( \xi \) has mild curvature. The second scheme is to apply a sophisticated interpolation
method (like spline) to the guessed \( \xi \), and then calculate derivatives numerically with
ultra-fine grids. This scheme works reasonably well even when \( \xi \) has extreme curvature.
Given the properties of these two schemes, my strategy is to start with the former (that is, when the guessed \( \xi \) is far from the exact solution) and use a small grid that only covers the
region where \( \xi \) has mild curvature. Then I switch to the latter scheme, using the result
from the former one as a start point and a broader grid that includes more points from
uncovered region. This strategy leverages the strengths of both schemes and fares very
well in my application.

B.4 Additional Details on Government Contractor Sample

This section complements my portfolio-based analysis in the main text, which uses a sam-
ple of U.S. government contractors. I provide more details on the sample construction and
variable calculations.

**Constructing the government contractor sample.** To identify firms with sales to the U.S.
government, I source accounting data from the Compustat database. I begin by selecting
firms that meet standard criteria in the literature: that is, firms incorporated in the U.S.
and with common stocks listed on the NYSE, AMEX, or NASDAQ; firms involved in sig-
nificant mergers/acquisitions or seriously affected by the 1988 accounting change are ex-
cluded; firms in the finance or utilities industry, with \( \text{SIC} \in [6000, 7000) \cup [4900, 4950) \), are also dropped. For selected firms I obtain their annual accounting records from
the fundamental annual file (\text{funda}) as well as the segment customer file (\text{seg\_customer});
the latter provides information on firms’ sales to the U.S. government (federal, state, and
local). These accounting data allow me to compute for each firm-year the fraction of
sales accounted for by government (denoted by \( StG \)). Every year I define government
contractors as firms that reported positive \( StG \) at least once over the past three years; ac-

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55 If a firm experienced a **significant** merger or acquisition in a fiscal year, it would be assigned a footnote
code of AB, FD, FE, or FF. According to Covas and Den Haan (2011), firms that were most affected by the
1988 accounting change (i.e., FAS94) include GM, GE, Ford, and Chrysler (also see Bernanke et al., 1990).
56 I obtain the Standard Industrial Classification (SIC) code from the fundamental annual file (\text{funda}), or
the name file (\text{names}) if the former is not available.
57 I only consider records showing positive total assets (item at) *and* net sales (item \text{sale}).
58 Data on government customers start from 1978.
59 If no transaction with government is reported, then \( StG \) is set to zero.
cording to this definition I find about 2,400 firms. However, transactions between these firms and government may stem from various types of government expenditures that are hardly related to public sector investment. So to be more specific, I exclude firms in the healthcare and pharmaceutical industries, personal and business services industries, and the defense industry (as defined by the Fama-French 48-industry classification). I also exclude firms in the consumer goods industry (as defined by the Fama-French 5-industry classification). Government contractors in these industries are least relevant with respect to public sector investment. The resulting sample consists of 1,242 government contractors with 9,944 firm-year observations spanning 1980 to 2017.

Calculating related variables. Using the Compustat data, I calculate a selection of firm characteristics for these government contractors; the following explains the calculations in detail. \( StG \) ratio, as already mentioned, is sales to government divided by total sales (item `sale`). \( \overline{StG}_{-2,0} \) is a 3-year trailing average of \( StG \) and serves as my measure of government dependency. The book-to-market ratio is the book value of equity divided by the market value of equity. The book value of equity is stockholders’ equity (item `seq`) plus deferred taxes and investment tax credit (item `txditc`) minus preferred stock redemption/liquidation/par value (item `pstkrv/pstk1/pstk`). The market value of equity is market price per share times number of shares outstanding; I obtain these two items from the Compustat fundamental annual file (`funda`), or the security monthly file (`secm`), or the CRSP monthly stock file (`msf`), based on availability in that order. The market value of equity is also referred to as market capitalization, a measure of firm size. Market leverage is the book value of debt divided by the sum of the book value of debt and the market value of equity; the book value of debt is the sum of short-term and long-term debt (item `dlc` plus item `dltt`). Asset growth is the annual relative change in total assets (item `at`). Sales growth is the annual relative change in net sales (item `sale`). Operating profitability is measured by the ratio of total revenue (item `revt`) or sales (item `sale`) minus cost of goods sold (item `cogs`) minus selling, general and administrative expense (item `xsga`) minus interest and related expense (item `xint`) to the book value of equity. Return on assets is the ratio of income before extraordinary items (item `ib`) to lagged total assets.

B.5 Additional tables and figures

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\(^{60}\)To minimize the instances of missing value, I impute missing items using other related items based on accounting identities whenever possible. For example, if item `seq` is missing, I use item `ceq` plus item `pstk`, or item `at` minus item `lt` minus item `mib` instead.
Figure B.1: Firm characteristics across government dependency portfolios. This figure compares via box plots the distributional properties of a selection of firm characteristics across portfolios formed on government dependency. In each panel, diamonds mark the medians of the corresponding characteristic, boxes span from the first to third quartiles, whiskers extend to the upper and lower adjacent values as defined by Tukey (1977). Detailed sample construction and variable calculations are in Appendix B.
Figure B.1: (Continued)
Figure B.2: Government dependency portfolios: controlling for more risk factors. This figure displays the alphas estimated by regressing the excess returns on government dependency portfolios as well as the return on a zero-investment portfolio that is long stocks in the highest-dependency quintile and short stocks in the lowest-dependency quintile on seven classic risk factors including the market, size, and value factors from Fama and French (1993); the momentum factor from Carhart (1997); the liquidity factor from Pastor and Stambaugh (2003); and the profitability and investment factors from Fama and French (2015). Also displayed are 90% and 95% confidence intervals (indicated by the grey bar and the whiskers, respectively) computed with heteroskedasticity- and autocorrelation-consistent (HAC) standard errors following the routine of Newey and West (1987, 1994). Returns are monthly. Portfolios are value-weighted in panel (a) and (b), and equal-weighted in panel (c) and (d). The sample period is 1981 to 1999 in panel (a) and (c), and 2000 to 2018 in panel (b) and (d). The first portfolio formation was at the end of June in 1981, and it was based on government dependency (StG_{-2,0}) computed for 1980; the same procedure are repeated every year thereafter until 2018.
Table B.2: **Government dependency portfolios: controlling for more risk factors; value-weighted portfolios.** This table presents the estimation results of regressing the excess returns on government dependency portfolios as well as the return on a zero-investment portfolio that is long stocks in the highest-dependency quintile and short stocks in the lowest-dependency quintile on seven classic risk factors including the market, size, and value factors \((MKT, SMB, HML)\) from Fama and French (1993); the momentum factor \((MOM)\) from Carhart (1997); the liquidity factor \((LIQ)\) from Pastor and Stambaugh (2003); and the profitability and investment factors \((RMW, CMA)\) from Fama and French (2015). Panel (a) reports the alphas estimated separately for two subperiods, 1981-1999 and 2000-2018. Panel (b) reports the betas estimated for the full sample period, 1981-2018. Portfolios are value-weighted. Other specifics are the same as in Table 6.

<table>
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<tr>
<th>Govt. dependency portfolios</th>
<th>1 (low)</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5 (high)</th>
<th>High minus Low</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>(a) Alphas</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>(\alpha) (1981-1999)</td>
<td>0.23</td>
<td>0.06</td>
<td>-0.15</td>
<td>0.38</td>
<td>0.33</td>
<td>0.10</td>
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<tr>
<td></td>
<td>[1.05]</td>
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<td>[-0.60]</td>
<td>[1.31]</td>
<td>[1.08]</td>
<td>[0.25]</td>
</tr>
<tr>
<td>(\alpha) (2000-2018)</td>
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<td>-0.21</td>
<td>0.81</td>
<td>0.34</td>
<td>0.81</td>
</tr>
<tr>
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<td>[-0.64]</td>
<td>[2.99]</td>
<td>[1.21]</td>
<td>[2.06]</td>
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<tr>
<td><strong>(b) Betas</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\beta_{MKT})</td>
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<td>1.25</td>
<td>0.88</td>
<td>0.94</td>
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<td>[19.16]</td>
<td>[-3.53]</td>
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<td>0.33</td>
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<td>[3.13]</td>
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<td>[3.13]</td>
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<td>-0.05</td>
<td>-0.11</td>
<td>0.14</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
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<td>[-0.53]</td>
<td>[-0.66]</td>
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<td>-0.03</td>
<td>0.04</td>
<td>0.04</td>
</tr>
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<td>[0.46]</td>
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<tr>
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<td>[1.46]</td>
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<td>[-1.22]</td>
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<tr>
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<td>0.70</td>
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<td>0.09</td>
<td>0.33</td>
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<td>Adj. (R^2)</td>
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<td>0.68</td>
<td>0.67</td>
<td>0.53</td>
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Table B.3: **Government dependency portfolios: controlling for more risk factors; equal-weighted portfolios.** This table presents the estimation results of regressing the excess returns on government dependency portfolios as well as the return on a zero-investment portfolio that is long stocks in the highest-dependency quintile and short stocks in the lowest-dependency quintile on seven classic risk factors including the market, size, and value factors \((MKT, SMB, HML)\) from Fama and French (1993); the momentum factor \((MOM)\) from Carhart (1997); the liquidity factor \((LIQ)\) from Pastor and Stambaugh (2003); and the profitability and investment factors \((RMW, CMA)\) from Fama and French (2015). Panel (a) reports the alphas estimated separately for two subperiods, 1981-1999 and 2000-2018. Panel (b) reports the betas estimated for the full sample period, 1981-2018. Portfolios are equal-weighted. Other specifics are the same as in Table 6.

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<tr>
<td>(\alpha) (1981-1999)</td>
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<td>-0.07</td>
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<tr>
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<td>0.86</td>
<td>0.83</td>
<td>-0.18</td>
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<tr>
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<tr>
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<td>0.80</td>
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Table B.4: **Mean excess returns and $\beta_{Pub}$ for 25 Size-(Inv/OP/Mom) equity portfolios.** This table reports the test assets’ mean excess returns ($r_i^e$) and estimated $\beta_{Pub}$. The latter are obtained by running a time-series regression specified as $r_{i,t}^e = a_i + f_i^T \beta_i + \xi_{i,t}$ for each asset $i$, where $r_{i,t}^e$ is the asset’s excess return, $f_i$ represents a vector of risk factors including PubFac, UncFac and the market excess return, and $\beta_i$ denotes a vector of beta estimates. The test assets include 25 size and investment (Inv) or profitability (OP) or momentum (Mom) sorted equity portfolios. The sample is quarterly and spans the period 1960Q4 to 2018Q4.

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