Learning and Efficiency with Market Feedback

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Abstract

We analyze the joint information acquisition problem of a firm manager and traders in financial markets, when the firm manager conditions her investment decisions on information revealed through stock prices and the firm is exposed to multiple sources of uncertainty. We highlight a fundamental mismatch: while traders want to collect the same information as the manager to maximize trading profits, the manager optimally diversifies her information sources and tries to acquire orthogonal information. We analyze the impact of this tension on price efficiency and real efficiency. Most importantly, we emphasize a delicate connection between these two efficiency measures. For instance, we show that market efficiency is maximized if the firm is only exposed to a single source of uncertainty, while real efficiency is minimized in this case and maximized if all shocks matter equally.

Keywords: feedback effect; information acquisition; information diversity; real efficiency; price informativeness.

JEL Classification: D83, G14, G31.
1 Introduction

It has long been recognized that the informational content of stock prices ("market efficiency") relies crucially on the willingness of traders to acquire costly information [see e.g., Grossman and Stiglitz, 1980; Verrecchia, 1982]. Similarly, it has been recognized that the profitability of firm managers’ capital investment decisions ("real efficiency") depends on the quality of their private information and their ability to select optimal projects [see e.g., Lambert, 1986; Harris and Raviv, 1996]. A more recent literature argues that there might be a connection between these two efficiency concepts because some of the information in prices might be useful for managers and can help them to invest more efficiently. This "feedback effect" [Bond et al., 2012] has received significant support from the recent empirical literature [see e.g., Luo, 2005; Chen et al., 2007; Foucault and Fresard, 2012].

While existing theoretical work has focussed on the information acquisition decision of either traders or firm managers in separation, our model allows both types to acquire information simultaneously. In particular, we show that the information acquisition decision of one type generates spill-overs for the other type. On the one hand, traders’ acquisition of private information renders the equilibrium stock price more informative and allows the firm manager to extract valuable information about the investment opportunities. On the other hand, the manager’s acquisition of private information is reflected in a more efficient investment decision, which affects the firm’s future value and thus the traders’ payoffs.

We study this joint information acquisition problem in a setting with multiple sources of uncertainty. More specifically, a firm manager can invest in two independent investment projects. The return on both projects is determined by different fundamentals (or "shocks") and all agents, traders and the firm manager, are ex ante uninformed about these shocks and not sure whether it is worthwhile to invest in the projects. Both types can acquire private signals about these shocks but it is too costly for them to collect information along both dimensions. Thus, all traders and the firm manager have to decide how to spend their limited resources most efficiently and what type

\footnote{See also Edmans et al. [2012], Foucault and Fresard [2014], Edmans et al. [2017], and Dessaint et al. [2019] for empirical evidence of market feedback.}
of uncertainty they would like to reduce privately.

Our main insight is that this two-way interaction entails a fundamental tension of incentives. On the one hand, the firm manager can most efficiently learn from the stock price if all traders acquire private information about the project she is not acquiring private information about herself. This way the manager can rely on price information regarding the investment decision for one project and on private information for the other project. Hence, the market’s information choice is a strategic substitute from the manager’s perspective. On the other hand, each individual trader has an incentive to mimic the manager’s information choice and acquire information about the same shock. This way traders maximize their trading profits because they can forecast a more significant portion of the future payoff with their private signal. Hence, the manager’s information choice is a strategic complement for individual traders.

The fundamental mismatch in the incentives to acquire information leads to a very nuanced equilibrium information choice. More specifically, we show that the traders’ and the manager’s information choice depends on two model parameters, the traders’ information precision capacity (relative to that of the manager) and the relative payoff of the two projects which we interpret as a measure of diversity. If one of the two projects offers a particularly high payoff and the precision capacity is sufficiently small, all traders acquire information about this project. In this case, the firm manager also learns about this shock because this project offers a sufficiently high return to offset the strategic substitutability effect mentioned earlier. If, however, the traders’ information capacity is sufficiently high and the projects offer a more similar return, there is no pure-strategy equilibrium and all agents randomize between both shocks. In particular, the firm manager diversifies her information choice almost equally between both projects and acquires slightly more information about the less-profitable project. In response, traders acquire more information about the more-profitable project.

We abstract from any agency conflicts between the firm and the manager such that she invests in either project if the expected net present value is positive. In our model the manager’s expectation comprises two types of signal. First, a private signal about one of the two projects and second, a
public signal based on the firm’s stock price. In particular, we assume that the manager cannot acquire a perfect signal about both projects’ returns such that she has to rely on price information in some circumstances. As mentioned before, traders acquire more information about the more-profitable project which renders the price more informative along this dimension. Moreover, we show that a particularly high stock price represents good news about both project, while a slightly above-average price indicates that the return on the more-profitable project is high. In both cases, the manager can infer the return on at least one of the projects and invest.

More generally, the firm’s equilibrium stock price is a step function of total order flow, i.e. the sum of informed traders’ demand and that of liquidity traders. A competitive market maker observes total order flow and sets the stock price equal to the expected firm value. Importantly, the market maker can more easily interpret movements in total order flow if all traders acquire information about the same project. In this case, particularly high values reveal a "high" project fundamental and vice versa for low values. If, however, traders pursue a mixed-strategy, total order flow is affected by both fundamentals and it is not clear whether a slightly above-average order flow is the result of a "high" fundamental for project one and a "low" fundamental for project two, or vice versa. Thus, the traders’ information mix has a direct effect on the informational content of total order flow and the equilibrium stock price.

Our main focus is on the implications of information acquisition on two efficiency measures. First, we consider market efficiency which is defined as the reduction in payoff variance that can be achieved by conditioning on the stock price. This measure represents the degree to which asset prices predict future payoffs and is widely used in the empirical literature to gauge the informational content of stock prices. Second, we consider real efficiency which is defined as the firm’s expected long-run value. This measure reflects the efficiency of the firm manager’s investment decision in the two risky projects. The conventional view is that market efficiency is a good proxy for real efficiency because a higher price-payoff correlation is expected to reveal more information to real-decision makers. One of our main contributions is to show that this conventional wisdom is generally incorrect.
For example, we show that market efficiency is maximized if the firm is, by and large, affected by a single shock and the manager is restricted to invest in a single project. In this case, the manager and all traders acquire information about the same shock and the price-payoff correlation is maximized. However, we also show that this outcome minimizes real efficiency and hence firm value. Intuitively, there is no feedback role for the stock price in this setting because the price reveals information that is already part of the manager’s (private) information set. As a result, the stock price merely predicts the future payoff but does not causally affect it. As a consequence, real efficiency is highest when both shocks are equally important for the firm. In this case, the price reveals novel information to the firm manager which increases investment efficiency at the expense of market efficiency.

The fundamental insight that the type of price information matters for market feedback is not new. Following Bond et al. [2012], who differentiate between forecasting price efficiency (FPE) and revelatory price efficiency (RPE), several empirical papers have tried to separate the two concepts [see e.g., Bai et al., 2016; Edmans et al., 2017]. We provide a formal framework that emphasizes a fundamental misalignment of incentives behind the information acquisition decisions. While a (benevolent) firm manager acquires private information to maximize RPE, traders acquire information to maximize FPE. Most importantly, we show that these two concepts lead to the exact opposite outcome in terms of preferred information choices. FPE is maximized when the manager and traders acquire the same information while RPE is maximized if they acquire information about different risks. Of course, in equilibrium the manager and traders maximize their objectives such that both types generally acquire information along both dimensions and neither efficiency concept is maximized.

The model makes three important assumptions. First, we consider a firm affected by multiple shocks that govern the return on the different investment opportunities. Potential examples of these different dimensions of uncertainty are multinational firms that are exposed to different country-level shocks or conglomerates that are exposed to different industry-level shocks. More generally, a firm might have the opportunity to launch several new products and faces uncertainty
about the respective profitability. Second, we allow the firm manager and, more importantly, traders to acquire private information about the same set of fundamental shocks. Therefore, we do not preclude the trader from certain types of shocks and consider all of them learnable. Thereby we are able to ascertain the equilibrium information acquisition decision of the different types with as little restrictions as possible. Our third assumption is that neither the manager nor informed traders have sufficient resources to collect perfect information about all shocks. This assumption is important for two reasons. First, it implies that the manager can learn additional information from the stock price. Second, it also creates a trade-off for the two types and renders the information acquisition decision non-trivial. The existing literature has highlighted several frictions (like limited attention or resources) that might lead to such a constraint [see e.g., Aghion and Stein, 2008; Mondria, 2010; Kacperczyk et al., 2016, among others].

In an extension of the main model, we consider a firm manager who misjudges the informational content of the stock price. Perhaps surprisingly, we show that a manager with less confidence in market feedback leads to higher real efficiency. As a result, the firm’s shareholders are better off hiring a biased manager. This result is driven by the fact that this bias encourages the manager to shift more attention towards the more profitable shock because she anticipates a less informative price signal. Traders respond by shifting more attention towards the same shock such that in equilibrium the stock price becomes more informative about the more profitable shock.

Our papers contributes to two strands of the literature. First, it is related to the literature studying the real effects of financial markets, where trading and prices affect the firms’ investment decisions, which in turn affect the firms’ cash flows. This is known as the "feedback effect" and Bond et al. [2012] provide a review of this literature. The first theoretical contributions take the agents’ information endowment as given and study market feedback with respect to a single fundamental. Building on these models, Dow et al. [2017] and Gao and Liang [2013] endogenize the traders’ signal precision in a single-shock setting, while Goldstein and Yang [2019] consider a setting with fixed information endowments but two sources of uncertainty. In contrast to these

\[\text{Goldstein and Yang [2015]}\text{discuss the importance of studying different dimensions of uncertainty in a model without market feedback.}\]

\[\text{See e.g., Dow and Gorton [1997], Subrahmanyam and Titman [2001], Goldstein and Guembel [2008], Goldstein et al. [2013], and Edmans et al. [2015].}\]
papers, we endogenize the information endowment of the manager and traders. As a result, we get several novel predictions relative to the existing work. In particular, our framework leads to a novel distinction between market efficiency and real efficiency and emphasizes the fact that it matters what kind of information financial market participants collect. In related work Benhabib et al. [2018] also study a feedback model with mutual learning by managers and traders. However, in contrast to our work, they assume that both types acquire information about different shocks. As a result, they do not focus on the strategic coordination between them and the efficiency implications of this interaction which at the core of our paper.

Second, our paper is also related to the economics and finance literature studying strategic information acquisition with multiple sources of uncertainty. Goldstein and Yang [2015] study a trading model in the spirit of Grossman and Stiglitz [1980] and show that the presence of multiple sources of uncertainty gives rise to strategic complementarities. van Nieuwerburgh and Veldkamp [2010], van Nieuwerburgh and Veldkamp [2010], and Kacperczyk et al. [2016] study trading models with multiple pieces of uncertainty and traders with limited attention capacity. Goldman [2004] and Goldman [2005] study a setting featuring a multi-division firm and endogenous information acquisition by traders. Our contribution relative to this literature is twofold. First, we allow for a real effect of the information collected by traders because of the feedback effect. Second, we also study the simultaneous information acquisition on the real side and its repercussions to the financial market. As a result, none of these papers addresses the relationship between market efficiency and real efficiency.

The remainder of the paper is organized as follows. In Section 2 we provide a description of the model. Section 3 characterizes the equilibrium outcome. In Section 4, we analyze an extension of the main model in which we consider a firm manager who misjudges the strength of market feedback. Section 5 concludes and all proofs are contained in Appendix A.
2 Model Setup

The model considers three dates, \( t \in \{0, 1, 2\} \), and a single firm. The firm’s stock is traded in a secondary financial market and a manager (“she”) can increase the firm’s value through investment in two uncertain growth opportunities. The financial market is populated by informed traders, uninformed noise traders, and a competitive market maker (“he”). At \( t = 0 \), the firm’s manager and informed traders make an information acquisition decision and receive information about one of the two growth opportunities. At \( t = 1 \), trading in the financial market occurs and subsequently the firm manager decides on investment in the two growth opportunities. The manager’s decision may be influenced by the realization of the stock price which creates a feedback effect [Bond et al., 2012]. At \( t = 2 \), the firm’s terminal value \( V \) is realized and paid out as a liquidating dividend. Figure 1 provides a timeline for the key events of the model and Appendix A.1 summarizes the notation.

![Timeline for the main model.](image)

2.1 The Firm’s Decision

In our model, the firm is operated by a benevolent manager who maximizes the firm’s expected long-term value \( V \). The firm has access to two growth opportunities, \( A \) and \( B \), and can choose to invest one unit in both of them, one of them, or not at all. We index the individual growth opportunity by \( j \in \{A, B\} \) and denote the respective investment decision by \( K_j \in \{0, 1\} \). The return on investment \( (x^{\theta_j}) \) depends on the realization of the binary random variable \( \theta_j \in \{L, H\} \) which takes on the two values "high" \( (H) \) and "low" \( (L) \) with equal probability, \( \mathbb{P}(\theta_j = H) = \mathbb{P}(\theta_j = L) = \frac{1}{2} \). For simplicity, we assume that \( \theta_A \) and \( \theta_B \) are independent of each other and all other random variables in the model. Moreover, we make the following three assumptions regarding the return
on investment.

**Assumption 1 (Return on Growth Opportunities)** We assume that the returns on the growth opportunities satisfy the following three conditions:

(i) \( x_j^L < 0 < x_j^H \) for \( j \in \{A, B\} \): ex post, it is efficient to invest in project \( j \) if \( \theta_j = H \) and inefficient if \( \theta_j = L \);

(ii) \( x_j^L + x_j^H < 0 \) for \( j \in \{A, B\} \): ex ante, it is inefficient to invest in project \( j \);

(iii) \( x_A^H \geq x_B^H \): in the “high” state investment opportunity \( A \) offers a (weakly) higher return than investment opportunity \( B \).

The first assumption is necessary to make the firm’s investment problem interesting. If \( x_j \) was always positive (negative), the firm would always (never) invest and there would be no role for learning about \( \theta_j \) which is the focus of this paper. The second assumption is made for tractability. It implies that the firm manager does not invest based on prior information, i.e. it requires a positive signal about \( \theta_j \) to induce the manager to invest. Our results are robust to the alternative assumption that the ex ante NPV is positive in which case it requires a negative signal to induce the manager not to invest. The third assumption allows for an asymmetric return on the two growth opportunities. Without loss of generality, we define \( A \) to be the project with the (weakly) higher return if \( \theta = H \).\(^4\) In the following, we will sometimes refer to the relative return of both projects, as the degree of *diversity* which is formally defined as:\(^5\)

\[
\kappa_x \equiv \frac{x_B^H}{x_A^H} \in (0, 1].
\]  

(1)

Intuitively, a higher value of \( \kappa_x \) indicates that the two growth opportunities offer a more similar return on investment. In the limit \( \kappa_x \to 0 \), there is no diversity because it is never efficient for the firm to invest in project \( B \) and there is effectively only one (potentially profitable) project.

\(^4\)It will become clear below that the firm never invests if \( \theta = L \) and thus the actual levels of \( x_j^L \) are irrelevant; it only matters that they are (sufficiently) negative.

\(^5\)It is worth noting that our diversity measure is based on payoff primitives, while Goldstein and Yang [2015] define information diversity in terms of the equilibrium mass of traders along both dimensions. We will show below that the two diversity measures are closely connected.
The firm’s terminal value comprises three parts, the return on the assets-in-place and the return on both growth opportunities:

\[ V = V_0 + \sum_{j \in \{A, B\}} K_j x_j^{\Theta_j}. \]  

(2)

The constant \( V_0 \) represents the return on the firm’s assets-in-place. To highlight the feedback effect from stock prices to real investment and to keep the equilibrium expressions more compact, we focus purely on the growth opportunities and set \( V_0 = 0 \) in the following.

Since the manager chooses \( K_j \) to maximize the expected firm value, she invests \((K_j = 1)\) if her conditional expectation of \( x_j^{\Theta_j} \) is positive and does not invest \((K_j = 0)\) otherwise. It will become clear below that the information structure implies the manager either learns \( \Theta_j \) perfectly or not at all. Thus, the manager only invests in project \( j \) when she learns that \( \Theta_j = H \) from the financial market or her private signal.

Next, we describe the two signals that influence the manager’s investment decision in more detail. The first signal type is an endogenous feedback signal from the financial market based on the stock price \( P \). The second type is a private signal \( \sigma_m \in \{I_m \Theta_A + (1 - I_m) \Theta_B, 0\} \) with \( I_m \in \{0, 1\} \). Thus, the manager’s choice of \( I_m \) determines whether she receives an informative signal about growth opportunity \( A \) or \( B \). This assumption captures the fact that in reality firm managers face several constraints limiting their ability to acquire precise private information about all dimensions of uncertainty.\(^6\) This assumption allows us to focus on the coordination problem between the firm manager and informed traders with regards to their information choice. Moreover, we set the precision of the manager’s private signal equal to \( \Gamma_m \) such that \( \mathbb{P}(\sigma_m = 0) = 1 - \Gamma_m \). The manager’s information choice maximizes the firm’s expected value at \( t = 0 \):

\[
\max_{I_m \in \{0,1\}} \mathbb{E}[V] \tag{3}
\]

where \( V \) is defined in equation (2). It is important to note that the firm’s terminal value \( V \) depends on the optimal real investment decisions \( K_j \). As we will show below, receiving informative signals allows the manager to invest more efficiently which, in turn, raises the firm’s expected value. In

\(^6\)One possible friction could be capacity constraint as in the literature on limited attention [see e.g. van Nieuwerburgh and Veldkamp, 2010; Kacperczyk et al., 2016, among others].
the following, we normalize $\Gamma_m = 1$ and model the firm manager as the best-informed agent in the economy who always receives a perfect signal about one of the two projects. At the same time, she might be able to learn additional information about the other project from the stock price.⁷

### 2.2 The Financial Market

Trading at $t = 1$ is modeled in the spirit of Kyle [1985]. The financial market consists of the following three types of traders who trade claims to the firm’s liquidating dividend $V$ at a price $P$. First, a unit continuum of risk-neutral informed traders, indexed by $i \in [0, 1]$. Each trader can either buy up to one unit, sell up to one unit, or not trade at all, $s_i \in [-1, 1]$.⁸ Because traders do not have price impact and are risk-neutral, they will always trade up to the limits if they decide to trade.⁹

In addition to informed traders, noise traders collectively demand $z \sim U[-1, 1]$ which generates non-fundamental variation in total order flow. Lastly, a risk-neutral, competitive market maker sets the stock price based on aggregate order flow $X = \int_0^1 s_i di + z$ to break even in expectation:

$$P = \mathbb{E}[V | X].$$

(4)

Informed traders face the same learning technology as the firm manager. Each trader receives a private signal $\sigma_i \in \{I_i \theta_A + (1 - I_i) \theta_B, \emptyset\}$ with $I_i \in \{0, 1\}$. Just like the firm manager, each trader has to decide whether to acquire information about project $A$ or $B$. This private signal reveals information about the fundamental with probability $\Gamma \in (0, 1]$ such that $\mathbb{P}(\sigma_i = \emptyset) = 1 - \Gamma$. Traders choose $I_i$ to maximize their expected trading profit $\Pi_i = s_i (V - P)$:

$$\max_{I_i \in \{0, 1\}} \mathbb{E}[\Pi_i].$$

(5)

It is worth noting that the expected trading profits also depend on the manager’s real investment decisions, which impact $V$. Thus, the traders’ objective is not only affected by their own information choice but also by the manager’s. We will elaborate more on this strategic interaction below. It follows from our previous assumption $\Gamma_m = 1$ that the traders’ information capacity $\Gamma$ can be interpreted as their degree of sophistication relative to the manager’s.

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⁷This normalization also allows us to turn off market feedback along the dimension the manager specializes in.

⁸A potential justification for this position limit are borrowing or short-sell constraints. See also Goldstein et al. [2013] and Goldstein and Yang [2019] for feedback models with a common assumption.

⁹Following the existing literature, we assume that traders do not trade if they are indifferent.
2.3 Equilibrium

Our equilibrium concept is Perfect Bayesian Equilibrium ("PBE"). We conjecture, and verify in Proposition 3, that there might be circumstances without a pure-strategy information equilibrium. In these cases, we let the firm manager and informed traders choose mixing probabilities \( \{ \omega_A, \omega_B \} \) and \( \{ q_{i,A}, q_{i,B} \} \), respectively. Moreover, we restrict attention to symmetric information acquisition decisions for traders and assume that their private signals are uncorrelated among each other. Thus for a given mixing probability \( q_{i,A} = q_A \) and \( q_{i,B} = q_B \) the mass of traders with a perfect signal about \( \theta_A \) and \( \theta_B \) is equal to \( q_A \Gamma \) and \( q_B \Gamma \), respectively by the law of large numbers. Furthermore, we also conjecture, and verify in Proposition 3, that \( q_A \geq q_B \) which is intuitive because we assume project \( A \) to yield the weakly higher ex post return. Of course, a pure-strategy information equilibrium for traders corresponds to \( q_A = 1 \) (or \( q_A = 0 \)). The firm manager can also randomize between the two signals by choosing the mixing probability \( \omega_A \) associated with \( I_m = 1 \). It follows from our assumption that \( \Gamma_m = 1 \) that the manager observes \( \theta_A \) with probability \( \omega_A \) and \( \theta_B \) with probability \( \omega_B = 1 - \omega_A \).

**Definition 1 (Perfect Bayesian Equilibrium)** A PBE consists of the following two sub-equilibria.

1. Trading and investment equilibrium at \( t = 1 \):
   - informed traders choose their asset demands to maximize expected trading profits;
   - the market maker sets the price to break even in expectation;
   - the firm manager chooses capital investments to maximize the expected firm value.

2. Information acquisition equilibrium at \( t = 0 \):
   - traders acquire information to maximize expected profits anticipating the equilibrium at \( t = 1 \);
   - the manager acquires information to maximize the expected firm value anticipating the equilibrium at \( t = 1 \).

We assume that all agents have rational expectations in that each player’s belief about the other players’ strategies is correct in equilibrium and out-of-equilibrium beliefs satisfy the conditions stated in the Appendix.
3 Model Solution and Implications

In this section we characterize the equilibrium in the main model and discuss its implications. More specifically, we solve the model backwards and start with the financial market and investment equilibrium at $t = 1$ in Section 3.1. In Section 3.2, we derive the information acquisition equilibrium at $t = 0$. Section 3.3 discusses the model’s efficiency implications.

3.1 Trading and Investment Equilibrium

As a first step, we take all agents’ information choices at $t = 0$ as given. Therefore, each trader acquires information about the two fundamental shocks with probabilities $q_A$ and $q_B$, respectively.\(^\text{10}\) Similarly, the firm manager’s information choice is described by the mixing probabilities $\omega_A$ and $\omega_B$. Based on these choices, each trader has to decide whether to trade and, if yes, whether to buy or sell the asset. Furthermore, the market maker sets the equilibrium stock price based on the observed order flow and the firm manager decides whether to invest in the growth opportunities.

Proposition 1 (Trading and Investment Equilibrium) Given information choices by traders ($q_A$) and the manager ($\omega_A$), there is a trading and investment equilibrium in which:

1. Each trader buys the firm’s stock if $\sigma_i = H$, sells if $\sigma_i = L$, and does not trade if $\sigma_i = \emptyset$:

$$s_i = \begin{cases} +1 & \text{if } \sigma_i = H \\ -1 & \text{if } \sigma_i = L \\ 0 & \text{if } \sigma_i = \emptyset \end{cases}$$

which leads to the following total order flow:

$$X = \begin{cases} \Gamma + z & \text{if } \theta_A = H, \theta_B = H \\ (2q_A - 1) \Gamma + z & \text{if } \theta_A = H, \theta_B = L \\ (1 - 2q_A) \Gamma + z & \text{if } \theta_A = L, \theta_B = H \\ -\Gamma + z & \text{if } \theta_A = L, \theta_B = L \end{cases}$$

\(^{10}\)As mentioned earlier, we conjecture, and verify in Proposition 3, that $q_A \geq \frac{1}{2}$ in equilibrium.
2. The firm’s stock price satisfies:

\[
P = \begin{cases} 
p^{HH} = x^H_A + x^H_B & \text{if } X \geq 1 + (2q_A - 1) \Gamma \\
p^H = x^H_A + \frac{1-w_A}{2} x^H_B & \text{if } 1 - (2q_A - 1) \Gamma < X < 1 + (2q_A - 1) \Gamma \\
p^M = \frac{w_A}{2} x^H_A + \frac{1-w_A}{2} x^H_B & \text{if } -1 + (2q_A - 1) \Gamma \leq X \leq 1 - (2q_A - 1) \Gamma \\
p^L = \frac{1-w_A}{2} x^H_B & \text{if } -1 - (2q_A - 1) \Gamma < X < -1 + (2q_A - 1) \Gamma \\
p^{LL} = 0 & \text{if } X \leq -1 - (2q_A - 1) \Gamma \end{cases}
\]

3. The firm’s investment decision satisfies:

\[
K_A = \begin{cases} 
1 & \text{if } P \in \{p^{HH}, p^H\} \text{ or } \sigma_m = \theta_A = H \\
0 & \text{otherwise}
\end{cases} \quad \text{and} \quad K_B = \begin{cases} 
1 & \text{if } P = p^{HH} \text{ or } \sigma_m = \theta_B = H \\
0 & \text{otherwise}
\end{cases}
\]

**Proof:** See Appendix A.2.1.

Proposition 1 shows that each trader optimally buys (sells) on positive (negative) private information about the firm’s fundamental in anticipation of a higher (lower) payoff \( V \). It follows that total order flow \( X \) depends on the informed traders’ private signals and therefore the fundamentals \( \theta_j \). In a mixed-strategy information equilibrium, the resulting aggregate order flow will be a function of \( \theta_A \) and \( \theta_B \) because a fraction \( q_A \Gamma \) receives a perfect signal about \( \theta_A \) and a fraction \( q_B \Gamma \) receives a perfect signal about \( \theta_B \). In this case, a particularly high order flow indicates that both shocks must be equal to \( \theta_j = H \) because it must be a result of buy orders from both types of informed investors. The same intuition applies to a particularly negative order flow such that both shocks must be equal to \( \theta_j = L \). It should be noted that there is an inherent asymmetry between the two shocks \( \theta_A \) and \( \theta_B \). Due to the assumption that project \( A \) offers a weakly higher ex post return, we will show below in Proposition 3 that there are always weakly more \( A \)-informed traders in a mixed-strategy equilibrium (\( q_A \geq q_B \)) such that the stock price reveals weakly more information about \( \theta_A \). More precisely, slightly above or below-average values of \( X \) already reveal
\( \theta_A = H \) or \( \theta_A = L \) to the market maker. However, it requires more extreme values for \( X \) to reveal the value of \( \theta_B \) together with that of \( \theta_A \).

There are two interesting corner solutions with regards to the traders’ information choice. First, if \( q_A = q_B = \frac{1}{2} \), traders randomize evenly between both shocks. In this case, the market maker cannot disentangle \((\theta_A = H, \theta_B = L)\) from \((\theta_A = L, \theta_B = H)\) because both combinations lead to the same total order flow \( X = z \sim U[-1,1] \). As a result, he learns that both shocks are "high" if \( X > 1 \) and that both shocks are "low" if \( X < -1 \). It follows that the equilibrium stock price only takes on three distinct values. If the market maker learns that both shocks are "high," he sets the price equal to \( p^{HH} \), if he learns that both shocks are "low," he sets the price equal to \( p^{LL} \), and if he cannot infer any information from \( X \), the price is set equal to \( p^M \). The second interesting case is the pure-strategy information equilibrium, \( q_A = 1 \). In this case, all traders specialize in \( \theta_A \) and total order flow cannot reveal any information about \( \theta_B \). As a result, the stock price takes on the value \( p^H \) if the market maker learns that \( \theta_A = H \), \( p^L \) if he learns that \( \theta_A = L \), and \( p^M \) otherwise.

More generally, we can see from Proposition 1 that the equilibrium stock price \( P \) is an increasing step function of total order flow \( X \). The two most extreme values \( p^{HH} \) and \( p^{LL} \) reveal that the market maker has learnt \( \theta_A = \theta_B = H \) and \( \theta_A = \theta_B = L \), while the two less extreme values \( p^H \) and \( p^L \) reveal that he has learnt \( \theta_A = H \) and \( \theta_A = L \). The intermediate value \( p^M \) is uninformative. Hence, the firm manager either learns the true value of \( \theta_j \) from her private signal or the stock price. Given that she invests benevolently, the manager invests in project \( j \) if she learns \( \theta_j = H \) and does not invest otherwise. Therefore, she invests in growth opportunity \( A \) if the price is equal to \( p^H \) or \( p^{HH} \) while she invests in \( B \) if the price is equal to \( p^{HH} \). Of course, the decision to invest in either growth opportunity can also be triggered by a positive realization of the manager’s private signal \( \sigma_m \). In a pure-strategy equilibrium this signal either reveals \( \theta_A \) or \( \theta_B \) perfectly. In a mixed-strategy equilibrium, it reveals \( \theta_A \) with probability \( \omega_A \) and \( \theta_B \) otherwise.

### 3.2 Information Choice Equilibrium

Next, we analyze the information acquisition decisions at the initial date \( t = 0 \) and compute the firm manager’s and traders’ expected utilities. Starting with the former, we use the manager’s
optimal investment policy in Proposition 1 and the definition of $V$ in equation (2) to show that the expected firm value is given by:

$$
\mathbb{E}[V] = \frac{((2 + \Gamma)x_A + 3q_A \Gamma(1 - x_A) - \Gamma)x_A^H + (2 - (2 + q_A \Gamma) - \Gamma)x_A}{4}.
$$

(6)

Next, we differentiate this expression with respect to $x_A$ to obtain the manager’s marginal benefit of acquiring information about $\theta_A$:

$$
\frac{\partial \mathbb{E}[V]}{\partial x_A} = \frac{(2 + \Gamma - 3q_A \Gamma - \Gamma)x_A^H - (2 + q_A \Gamma - \Gamma)x_A^H}{4}.
$$

We can use this expression to solve for the manager’s best response to the traders’ collective information choice $q_A$:

$$
\omega_A^* = \begin{cases} 
1 & \text{if } q_A < \frac{(2+\Gamma) - (2-\Gamma)\kappa_x}{\Gamma(3+\kappa_x)} \\
\in [0, 1] & \text{if } q_A = \frac{(2+\Gamma) - (2-\Gamma)\kappa_x}{\Gamma(3+\kappa_x)} \\
0 & \text{if } q_A > \frac{(2+\Gamma) - (2-\Gamma)\kappa_x}{\Gamma(3+\kappa_x)}.
\end{cases}
$$

(7)

Because the manager’s objective function is linear in her mixing probability $\omega_A$, we either get a corner solution or the manager is indifferent between all values for the mixing probability. In particular, equation (7) shows that if $q_A$ is below (above) the threshold value $\frac{(2+\Gamma) - (2-\Gamma)\kappa_x}{\Gamma(3+\kappa_x)}$, the manager chooses to acquire information about $\theta_A$ ($\theta_B$) all the time. If $q_A$ is equal to this value, the manager is indifferent between any value for $\omega_A$ and willing to pursue a mixed strategy.

Next, we compute the expected trading profits for trader $i$ to find the best response of each individual trader to the manager’s information choice. Under the most general assumption that trader $i$ chooses a mixing probability $q_{i,A}$, the trader receives a perfect signal about $\theta_A$ with probability $q_{i,A} \Gamma$ and a perfect signal about $\theta_B$ with probability $(1 - q_{i,A}) \Gamma$. Hence, we can write the trader’s objective function as

$$
\mathbb{E}[\Pi_i] = q_{i,A} \Gamma \mathbb{E}[\Pi_i | \sigma_i = \theta_A] + (1 - q_{i,A}) \Gamma \mathbb{E}[\Pi_i | \sigma_i = \theta_B]
$$

(8)

because, according to Proposition 1, traders do not trade ($s_i = 0$) if they receive an uninformative signal ($\sigma_i = \emptyset$). We can differentiate this expression with respect to $q_{i,A}$ to get

$$
\frac{\partial \mathbb{E}[\Pi_i]}{\partial q_{i,A}} = \Gamma (\mathbb{E}[\Pi_i | \sigma_i = \theta_A] - \mathbb{E}[\Pi_i | \sigma_i = \theta_B]).
$$
We can see that at an interior optimum each trader is indifferent between specializing in either project, i.e. \( \mathbb{E}[\Pi_i|\sigma_i = \theta_A] = \mathbb{E}[\Pi_i|\sigma_i = \theta_B] \). Of course, if \( \mathbb{E}[\Pi_i|\sigma_i = \theta_A] > \mathbb{E}[\Pi_i|\sigma_i = \theta_B] \) it is optimal to specialize in \( \theta_A \), i.e. \( q_{i,A} = 1 \), and vice versa if \( \mathbb{E}[\Pi_i|\sigma_i = \theta_A] < \mathbb{E}[\Pi_i|\sigma_i = \theta_B] \).

Next, we can plug in trader \( i \)'s realized profit \( \Pi_i = s_i(V - P) \) together with the results in Proposition 1 to see that the individual trader’s expected profit conditional on \( \sigma_i \) only depends on the collective information choice of all traders (\( q_A \)), that of the manager (\( \omega_A \)), and the model parameters \( \kappa_x \) and \( \Gamma \). Lastly, we set \( q_{i,A}^* = q_A^* \) and derive the traders’ best-response function to the manager’s information choice:

\[
q_A^* = \begin{cases} 
1 & \text{if } \omega_A > \frac{\kappa_x}{1+\kappa_x-\Gamma} \Leftrightarrow \mathbb{E}[\Pi_i|\sigma_i = \theta_A] > \mathbb{E}[\Pi_i|\sigma_i = \theta_B] \\
\frac{1}{2\Gamma} \left( 1 + \kappa - \frac{\omega_A}{\omega_A^*} \right) & \text{if } \frac{\kappa_x}{1+\kappa_x-\Gamma} \leq \omega_A \leq \frac{\kappa_x}{1+\kappa_x} \Leftrightarrow \mathbb{E}[\Pi_i|\sigma_i = \theta_A] = \mathbb{E}[\Pi_i|\sigma_i = \theta_B] \\
0 & \text{if } \omega_A < \frac{\kappa_x}{1+\kappa_x} \Leftrightarrow \mathbb{E}[\Pi_i|\sigma_i = \theta_A] < \mathbb{E}[\Pi_i|\sigma_i = \theta_B].
\end{cases}
\]

(9)

Thus, the traders’ optimal mixing probability depends on the manager’s mixing probability. For intermediate values of \( \omega_A \), the mixing probability is equal to \( \frac{1}{2\Gamma} \left( 1 + \kappa - \frac{\omega_A}{\omega_A^*} \right) \). Since this expression is increasing in \( \omega_A \), we find that informed traders choose a pure strategy and specialize in \( \theta_A \) if \( \omega_A \) is sufficiently high. If \( \omega_A \) is sufficiently small, informed traders move to the other extreme and choose \( q_A = 0 \). We will show below that this scenario never occurs in equilibrium. Overall, we can see from the expressions for \( q_A^* \) and \( \omega_A^* \) that the manager’s and the market’s information choice are interdependent. The following proposition formalizes this interdependence.

**Proposition 2 (Substitutability vs. Complementarity)** From the firm’s perspective, market information and the manager’s private information are strategic substitutes:

\[
\frac{\partial \omega_A^*}{\partial q_A} \leq 0.
\]

From the traders’ perspective, market information and the manager’s private information are strategic complements:

\[
\frac{\partial q_A^*}{\partial \omega_A} \geq 0.
\]

**Proof:** See Appendix A.2.2.
Proposition 2 shows a key driver behind our main results. There is a fundamental tension between the firm manager and the market with regards to their incentives to acquire information. On the one hand, the firm manager has an incentive to diversify her information sources about the two investment opportunities. Thus, if the stock price is particularly informative about $\theta_A$ because $q_A$ is high, she would rather pay more attention to $\theta_B$ and vice versa. This result is intuitive because the manager wants to avoid "overlap" in the information conveyed by the price signal and her private signal. For instance, if the price signal already reveals $\theta_A = H$ (e.g. because $P = p^H$) receiving the same information from the private signal does not add any value. On the other hand, traders have an incentive to focus on the same shock as the manager because it allows them to predict the payoff, that depends on the manager’s investment choice, more precisely. In the extreme case in which the manager solely focuses on $\theta_A$, traders don’t have an incentive to acquire information about $\theta_B$ at all. If they did, the manager would only invest in project $B$ in response to the price signal $P = p^{HH}$. However, in that case this decision is fully priced-in because the market maker is also able to infer $\theta_B$ from total order flow whenever the manager invests. As a result, learning about $\theta_B$ does not carry any profits for informed traders in this extreme case. Next, we can combine the manager’s and traders’ demand for information in (7) and (9) to solve for the equilibrium values of $q_A$ and $\omega_A$.

**Proposition 3 (Information Choice Equilibrium)** There are two mutually exclusive information choice equilibria depending on the traders’ signal precision $\Gamma$ and the degree of diversity $\kappa_x = \frac{x^H}{x^A} \in (0, 1]$.

1. If $\Gamma < 1 - \kappa_x$, there is a pure-strategy information acquisition equilibrium. The manager and each trader acquire information solely about $\theta_A$:

$$\omega^*_A = q^*_A = 1.$$  

2. If $\Gamma \geq 1 - \kappa_x$, there is a mixed-strategy information acquisition equilibrium with:

$$\omega^*_A = \frac{(3 + \kappa_x) \kappa_x}{(8 - \Gamma) \kappa_x - (1 - \Gamma) + \kappa_x^2},$$

$$q^*_A = \frac{(2 + \Gamma) - (2 - \Gamma) \kappa_x}{\Gamma (3 + \kappa_x)}.$$
with $\frac{\partial \omega^*_A}{\partial \kappa_x} > 0$ if $\Gamma \in \left(\frac{5 \kappa_x^2 - 2 \kappa_x - 3}{\kappa_x^2 - 2 \kappa_x}, 1\right)$ and $\leq 0$ otherwise, and $\frac{\partial q^*_A}{\partial \Gamma} \leq 0$; $\frac{\partial q^*_A}{\partial \kappa_x} \leq 0$ and $\frac{\partial q^*_A}{\partial \Gamma} \leq 0$. Moreover $\omega^*_A \in \left[\frac{3}{7}, \frac{1}{2}\right]$ and $q^*_A \in \left[\frac{1}{2}, 1\right]$.

**Proof:** See Appendix A.2.3.

Proposition 3 gives the equilibrium values of the mixing probabilities $\omega^*_A$ and $q^*_A$. We can see that there are two different regions. First, if the traders’ precision capacity $\Gamma$ is sufficiently low, all agents focus their attention on shock $A$, i.e. $\omega^*_A = q^*_A = 1$. This result is intuitive: in anticipation of the market’s low precision capacity, the manager’s incentive to shift her attention towards shock $B$ is reduced because she does not expect the equilibrium price to be very informative about shock $A$. Therefore, it is optimal for the manager to specialize in shock $A$ which, in turn, incentivizes all traders to mimic the manager’s choice and specialize in shock $A$, too. Second, if the traders’ information capacity ($\Gamma$) is above a certain threshold, the manager and each trader randomize between shock $A$ and $B$. In this case, the mixing probabilities depend on the degree of diversity in the two projects $\kappa_x$ and the traders’ signal precision $\Gamma$.

The comparative statics of the two mixing probabilities can be most easily seen in Figure 2. The traders’ mixing probability for project $A$ is monotonically decreasing in both $\kappa_x$ and $\Gamma$. As project $B$ becomes relatively more profitable, traders start shifting more attention towards this project and $q^*_A$ converges towards $\frac{1}{2}$ from above. Similarly a decrease in signal precision $\Gamma$ makes it less likely that an individual traders’ signal is revealed to the market maker. As a result, it is less costly to pay attention to the relatively more profitable shock $A$. One can also see that our initial conjecture holds and $q^*_A \geq \frac{1}{2}$ for all permissible values of $\Gamma$ and $\kappa_x$. The manager’s mixing probability looks quite different and is rather flat in both dimensions. We can show analytically that $\omega^*_A$ is always between $\frac{3}{7} \approx 0.43$ and $0.5$ in the feedback region $\Gamma \geq 1 - \kappa_x$ (blue area) and equal to unity in the non-feedback region $\Gamma < 1 - \kappa_x$ (red area).

As mentioned above, Goldstein and Yang [2015] use a slightly different concept of *information diversity* ($\Delta$) that relies on the equilibrium mass of $A$–informed and $B$–informed traders. Using
Figure 2: Equilibrium values for $\omega^*_A$ (left plot) and $q^*_A$ (right plot) as a function of $\kappa_x = \frac{\mu}{\sigma_A}$ and $\Gamma$.

Using the results in Proposition 3, it is straightforward to show that this measure is increasing in $\kappa_x$, i.e. $\frac{d\Delta}{d\kappa_x} \geq 0$. As a result, both measures of diversity are closely connected.

### 3.3 Efficiency Implications

Next, we analyze the efficiency implications of the manager’s and the traders’ strategic information acquisition decisions. In particular, we focus on two widely used efficiency measures, *market efficiency* and *real efficiency*. Both measures are formally defined next.

**Definition 2 (Efficiency)** We define the following two measures of efficiency.

1. **Market efficiency** is defined as the negative ratio of the (expected) conditional and unconditional payoff variance:

   $$ ME \equiv -\mathbb{E} \left[ \frac{\text{Var}(V|P)}{\text{Var}(P)} \right], $$

2. **Real efficiency** is defined as the ex ante expected firm value:

   $$ RE \equiv \mathbb{E}[V]. $$
First, our definition of market efficiency (ME) is the informational content of the price, i.e. the negative payoff variance conditioned on the stock price $P$, scaled by the unconditional variance to obtain a relative measure. Note that in our setting the conditional variance is a random variable that depends on the specific price realization. For example, $\text{Var}(V|P = p^{HH}) = 0$ because $p^{HH}$ perfectly reveals $\theta_A$ and $\theta_B$ (and thus $V$) while $\text{Var}(V|P = p^M) = \text{Var}(V)$ because $p^M$ does not reveal any information. This measure of market efficiency has been often used in the existing literature as a proxy for the informational content of the stock price (see e.g. Peress [2010], Ozsoylev and Walden [2011], and Edmans et al. [2016]).\textsuperscript{11} In our setting, the expected variance ratio is also proportional to the price-payoff correlation. It is straightforward to show that $ME = \text{Corr}(V,P)^2 - 1$.\textsuperscript{12} Our measure of market efficiency is maximized at $ME = 0$, if the price always reveals the future payoff perfectly, and minimized at $ME = -1$, if observing the price does not add any information and $P = p^M$ with probability one. Second, we define real efficiency ($RE$) as the ex ante expectation of the firm’s realized long-term value $V$. It measures the efficiency of the firm’s allocation of capital and is maximized at $\frac{1}{2} (x_A^H + x_B^H)$ if the firm manager always invests in project $j$ when $\theta_j = H$ (which happens with probability $\frac{1}{2}$).\textsuperscript{13}

**Proposition 4 (Efficiency Measures)** In the main model the two efficiency measures are given by:

1. **Market efficiency:**

   $$ME^* = \begin{cases} 
   \frac{(4-\Gamma)\omega^2(\kappa,3)\Gamma(1+\kappa,(-2-4)\Gamma+\kappa,(-10-4)\Gamma+\kappa,(-12-4)\Gamma+\kappa,41)+27)}{(\Gamma+\kappa,(-1+\kappa,8)-1)((\Gamma-10\Gamma+\kappa,2)+6\Gamma)-9)} & \text{if } \Gamma \geq 1 - \kappa, \\
   \Gamma - 1 & \text{if } \Gamma < 1 - \kappa
   \end{cases}$$

   with $\frac{dME^*}{d\kappa} > 0$ if $\kappa$ and $\Gamma$ are sufficiently large and $\leq 0$ otherwise, and $\frac{dME^*}{d\Gamma} > 0$. Moreover, $ME^* \in [-1,0]$.

---

\textsuperscript{11}In recent empirical work, Davila and Parlato [2019a] and Davila and Parlato [2019b] use price volatility and regression $R^2$-squareds to identify price efficiency.

\textsuperscript{12}Note that $-\frac{\text{Var}[\text{Var}(V|P)]}{\text{Var}(P)} = \frac{\text{Var}(\text{Var}(P))}{\text{Var}(P)} - 1 = \frac{\text{Var}(P)}{\text{Var}(P)} - 1$. Moreover, it holds that $\text{Cov}(V,P) = \text{Var}(P)$ because $E[V|P] = E[V] = E[E[P|V]]$ such that $\text{Corr}(V,P) = \sqrt{\frac{\text{Var}(P)}{\text{Var}(V)}}$.

\textsuperscript{13}Note that the expected firm value can be written as $E[V] = \frac{1}{2} \sum_{j \in \{A, B\}} P(\theta_j = H) x_j^H$ because the manager never invests if $\theta_j = L$. Hence, real efficiency captures the extent to which the firm invests in the growth opportunities when their ex post return is in fact positive.
2. Real efficiency:

\[ RE^* = \begin{cases} 
\frac{3 + \Gamma \kappa_x + \kappa_x^2}{6 + 2\kappa_x} x_A^H & \text{if } \Gamma \geq 1 - \kappa_x \\
\frac{1}{2} x_A^H & \text{if } \Gamma < 1 - \kappa_x 
\end{cases} \]

with \( \frac{\partial RE^*}{\partial x_A^H} > 0 \), \( \frac{\partial RE^*}{\partial x_B^H} \geq 0 \), and \( \frac{\partial RE^*}{\partial \Gamma} \geq 0 \). Moreover, \( RE^* \in \left[ \frac{1}{2} x_A^H, \frac{5}{8} x_A^H \right] \).

Again, \( \kappa_x = \frac{x_B^H}{x_A^H} \in (0, 1] \) denotes the degree of diversity in the two projects.

**Proof:** See Appendix A.2.4.

Proposition 4 provides analytic expressions for our two efficiency measures. As before, these expressions depend on the traders’ signal precision \( \Gamma \) and the degree of project diversity \( \kappa_x \). If \( \Gamma < 1 - \kappa_x \), the manager and traders specialize in \( \theta_A \) and there is no market feedback. In this case, real efficiency is solely a function of the return on project \( A \), \( x_A^H \), and market efficiency is solely a function of the traders’ information capacity \( \Gamma \). If \( \Gamma \geq 1 - \kappa_x \), the manager randomizes between an informative signal about \( \theta_A \) and an informative signal about \( \theta_B \). As a result, she is able to learn additional information about the shock that is not perfectly revealed by her private signal from the stock price. In this scenario, real efficiency depends positively on the precision of the informed traders’ signal because it affects the informational content of the price. Furthermore, as expected, real efficiency also increases in the high-state payoff of both projects. With price feedback, market efficiency depends on the degree of diversity \( \kappa_x \) and the following corollary describes this dependence in more detail.

**Corollary 1 (Impact of Information Diversity)** In the main model, information diversity \( \kappa_x = \frac{x_B^H}{x_A^H} \) affects the two efficiency measures in the following way:

1. For a given \( \Gamma \), market efficiency is maximized without information diversity \( (\kappa_x \to 0) \).

2. For a given \( \Gamma \), real efficiency is maximized with full information diversity \( (\kappa_x = 1) \) and minimized without information diversity \( (\kappa_x \to 0) \).

**Proof:** See Appendix A.2.5.

Quite interestingly, the two efficiency measures respond differently to changes in the degree of information diversity \( \kappa_x \). More specifically, real efficiency is strictly increasing in \( \kappa_x \) such that the
firm’s expected value is maximized if the relative payoff of the two projects is equal to unity and minimized if it is equal to zero. Market efficiency, on the other hand, is non-monotone in $\kappa_x$. Most strikingly, market efficiency is \textit{maximized} when real efficiency is \textit{minimized}, i.e. in the absence of information diversity. In this limiting case, the manager and traders acquire information about the same shock and the price reveals relatively more information about the payoff. At the same time, real efficiency is minimized in this case because the manager cannot infer any useful information from the stock price.

Figure 3 plots our two efficiency measures against the traders’ signal precision $\Gamma$ and the degree of information diversity $\kappa_x$. First, the pure-strategy, no-feedback case corresponds to the triangle in the front left of both plots ($\Gamma < 1 - \kappa_x$). In this region, market efficiency increases linearly in $\Gamma$ and does not depend on $\kappa_x$. Real efficiency only depends on $x^H_A$, which is set to unity in this plot, because the firm manager invests in project $A$ only based on her private signal. Second, the mixed-strategy, feedback case corresponds to the triangle in the back right of both plots ($\Gamma \geq 1 - \kappa_x$). We can see that the two efficiency measures differ quite dramatically in this region. On the one hand, market efficiency is maximized at $\Gamma \rightarrow 1$ and $\kappa_x \rightarrow 0$. In this case, the firm’s payoff only depends on shock $A$ such that traders only acquire information about $\theta_A$. If, in addition, $\Gamma = 1$ the stock price is fully information about this shock and the firm manager can extract the informed traders’ information perfectly. As a result, the price efficiency is maximized and $ME^* = 0$. On the other hand, real efficiency is maximized at $\Gamma \rightarrow 1$ and $\kappa_x \rightarrow 1$. In this case, traders randomize between both shocks and it is more difficult for the manager to infer information from the price which is reflected in the reduced level of market efficiency. At the same time, however, the manager is able to learn about projects $A$ and $B$ from the stock price (in addition to her private information) which increases the overall amount of \textit{useful} information for the manager and hence increases the ex ante firm value.
4 Extension: Misjudgment of Information Capacity

In this section, we introduce a friction relative to the main model. In particular, we assume that the firm manager misjudges the market’s capacity to process information and acts under the belief that $P(\sigma_i \neq \emptyset) = \tilde{\Gamma} \in [0, 1]$. In general, the manager’s assessment is therefore different from the true value $\Gamma$ and this extension collapses to the main model if $\tilde{\Gamma} = \Gamma$. We assume that every trader is aware of the manager’s potentially biased assessment of $\Gamma$ and, vice versa, the manager understands that traders consider $\Gamma$ to be the true precision. Put differently, the two sides "agree to disagree" about the true value of $\Gamma$.

It should be noted that the manager’s belief $\tilde{\Gamma}$ only affects the equilibrium information acquisition choices $\omega_A$ and $q_A$ but not the trading and investment equilibrium in Proposition 1. We assume that the manager reacts rationally to movements in the equilibrium stock price and continues to invest in project $A$ if $P \in \{p^H, p^{HH}\}$ and in project $B$ if $P = p^{HH}$ such that the manager only deviates from the rational benchmark ex ante, i.e. at $t = 0$.

**Proposition 5 (Information Choice with Misjudgment) If the manager believes that the traders’ information capacity is given by $\tilde{\Gamma}$ instead of $\Gamma$, there are two mutually exclusive information choice equilibria.**

1. If $\tilde{\Gamma} < 1 - \kappa_x$, there is a pure-strategy information acquisition equilibrium. The manager and each
trader acquire information about \( \theta_A \):

\[
\tilde{\omega}_A = \tilde{q}_A = 1.
\]

2. If \( \tilde{\Gamma} \geq 1 - \kappa_x \), there is a mixed-strategy information acquisition equilibrium with:

\[
\begin{align*}
\tilde{\omega}_A &= \frac{(3 + \kappa_x)\kappa_x \tilde{\Gamma}}{(1 + \kappa_x)(3 + \kappa_x)\tilde{\Gamma} - (1 - \kappa_x)(4 - \tilde{\Gamma})\tilde{\Gamma}}, \\
\tilde{q}_A &= \frac{(2 + \tilde{\Gamma}) - (2 - \tilde{\Gamma})\kappa_x}{\tilde{\Gamma}(3 + \kappa_x)}.
\end{align*}
\]

with \( \frac{\partial \tilde{\omega}_A}{\partial \tilde{\Gamma}} < 0, \frac{\partial \tilde{\omega}_A}{\partial \Gamma} > 0, \frac{\partial \tilde{q}_A}{\partial \Gamma} < 0 \), and \( \frac{\partial \tilde{q}_A}{\partial \kappa_x} < 0 \).

As before, \( \kappa_x \equiv \frac{x_H}{x_A} \in (0, 1] \) denotes the degree of diversity in the two projects.

**Proof:** See Appendix A.2.6.

Proposition 5 provides closed-form solutions for the optimal information choices when the manager misjudges the market’s information capacity. Similar to the results in the benchmark equilibrium, the manager does not rely on market feedback if she deems the information capacity too low (\( \tilde{\Gamma} \leq 1 - \frac{x_H}{x_A} \)). In this case, the manager and each trader shift all capacity towards \( \theta_A \). If the manager deems the market’s capacity sufficiently high, the equilibrium information choices depend on the manager’s assumed precision capacity, the actual precision capacity, and the degree of diversity. Next, we analyze the implications for our two efficiency measures.

**Proposition 6 (Efficiency with Misjudgment)** If the manager believes that the traders’ information capacity is given by \( \tilde{\Gamma} \) instead of \( \Gamma \), the two efficiency measures are given by:

1. **Market efficiency:**

\[
\overline{ME} = \begin{cases} 
\mathcal{M}\left(\tilde{\Gamma}, \Gamma, \kappa_x\right) & \text{if } \tilde{\Gamma} \geq 1 - \kappa_x \\
\Gamma - 1 & \text{if } \tilde{\Gamma} < 1 - \kappa_x 
\end{cases}
\]

with \( \mathcal{M} \in [-1, 0] \).

2. **Real efficiency:**

\[
\overline{RE} = \begin{cases} 
\mathcal{R}\left(\tilde{\Gamma}, \Gamma, \kappa_x, x_H^H\right) & \text{if } \tilde{\Gamma} \geq 1 - \kappa_x \\
\frac{1}{2}x_A^H & \text{if } \tilde{\Gamma} < 1 - \kappa_x 
\end{cases}
\]
with \( R \in \left(0, \frac{1}{2}(1 + \kappa_x)x^H_A\right]\).

As before, \( \kappa_x \equiv \frac{x^H_B}{x^H_A} \in (0, 1] \) denotes the degree of diversity in the two projects. Analytic expressions for functions \( M \) and \( R \) are given in the Appendix.

**Proof:** See Appendix A.2.7

Proposition 6 analyzes the efficiency implications of the manager’s misjudgment of \( \Gamma \). We can see that in the extended model with misjudgment both efficiency measures depend on the true and the misjudged information capacity. If the manager’s assessment of \( \Gamma \) is sufficiently low, both efficiency measures are identical to their rational counterparts. Figure 4 shows a graphical representation of \( \tilde{ME} \) and \( \tilde{RE} \) as a function of \( \tilde{\Gamma} \). The left plot represents market efficiency and each line corresponds to a different value of \( \Gamma \). First, we can see that market efficiency increases in the actual information capacity. Second, we can see that the dependence on the assessed capacity depends on \( \Gamma \). In particular, for rather high values of \( \Gamma \) (dashed and dotted line), market efficiency is highest if the manager has a too pessimistic view of \( \Gamma \), i.e. \( \tilde{\Gamma} < \Gamma \). However, if \( \Gamma \) is smaller (solid line), market efficiency is higher if the manager is too optimistic about \( \Gamma \). The right plot corresponds to real efficiency. Again, we can see that a higher \( \Gamma \) increases the firm’s expected value, as in the main model. More interestingly, we can see that real efficiency can be increased if the manager systematically underestimates the true information capacity. This observation is formalized next.

![Figure 4](image-url)

**Corollary 2 (Efficient Misjudgment)** In the extended model with managerial misjudgment, real efficiency is maximized at \( \tilde{\Gamma}^* \leq \Gamma \). The analytic expression for \( \tilde{\Gamma}^* \) is given in the Appendix.
**Proof**: See Appendix A.2.8.

Corollary 2 shows that the firm is weakly better off hiring a manager who is too pessimistic about the market’s informational endowment and underestimates it. The reason for this quite surprising finding is the agents’ strategic information choice (Proposition 5). A manager who is skeptical about \( \Gamma \) will focus more on the more profitable project \( A \) and increase \( \tilde{\omega}_A \), i.e. \( \frac{\partial \tilde{\omega}_A}{\partial \tilde{\Gamma}} \leq 0 \). Traders respond by increasing their attention to \( \theta_A \) because \( \frac{\partial \tilde{q}_A}{\partial \tilde{\Gamma}} \leq 0 \). Overall, the manager ends up with more information overall and is able to invest more efficiently.

5 Conclusion

We consider a model in which a real-decision maker (the firm manager) and traders in a financial market are allowed to collect information simultaneously. The resulting information acquisition equilibrium highlights an inherent coordination problem: while the manager wants traders to acquire information that differs from her own choice, traders want to collect the same information as the manager. We show that this tension leads to a mismatch between market efficiency and real efficiency. Moreover, the firm is always better off hiring a manager who is overly skeptical about market feedback in order to overcome this mismatch.
References


A Appendix

A.1 Notation Summary

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A.2 Proofs

A.2.1 Proof of Proposition 1

We first conjecture that trader $i$’s optimal trading policy is to buy if $\sigma_i = H$, sell if $\sigma_i = L$, and not to trade if $\sigma_i = \emptyset$. Moreover, we take the traders’ and the manager’s information choice as given. It
follows that a mass $q_A \Gamma$ receives an informative signal about $\theta_A$ and a mass $(1 - q_A)\Gamma$ receives an informative signal about $\theta_B$. Noise traders always trade a random amount $z \sim U[-1, 1]$ such that we get the following four possibilities for total order flow $X$: (i) if $\theta_A = \theta_B = H: X = \Gamma + z$, (ii) if $\theta_A = H$ and $\theta_B = L: X = (2q_A - 1)\Gamma + z$, (iii) if $\theta_A = L$ and $\theta_B = H: X = (1 - 2q_A)\Gamma + z$, and (iv) if $\theta_A = \theta_B = L: X = -\Gamma + z$.

The market maker observes total order flow and sets the price equal to the expected future firm value $E[V|X]$. It follows from the distributional assumption on $z$ and $q_A \in \left[\frac{1}{2}, 1\right]$, that the market maker can make the following inferences: (i) if $X > (2q_A - 1)\Gamma + 1$: $\theta_A = \theta_B = H$, (ii) if $X < (1 - 2q_A)\Gamma - 1$: $\theta_A = \theta_B = L$, (iii) if $(2q_A - 1)\Gamma + 1 > X > (1 - 2q_A)\Gamma + 1$: $\theta_A = H$, and (iv) if $(2q_A - 1)\Gamma - 1 > X > (1 - 2q_A)\Gamma - 1$: $\theta_A = L$. Thus, there are five distinct information sets for the manager depending on total order flow which leads to the five different prices $p_{HH}^H, p_H^H, p_M^M, p_L^L$, and $p_{LL}^L$.

Next, we conjecture that $p_{HH}^H$ reveals $\theta_A = \theta_B = H$ and $p_H^H$ reveals $\theta_A = H$ to the firm manager. Thus, the firm manager chooses $K_A = 1$ in response to $p_{HH}^H$ or $p_H^H$ and $K_B = 1$ in response to $p_{HH}^H = 1$. The firm manager does not invest if the price reveals $\theta_j = L$ because the ex post NPV is negative in this case.

As a result, the expected firm values from the market maker’s perspective are given by:

1. If $X > (2q_A - 1)\Gamma + 1$:

$$p_{HH}^H = E[V|\theta_A = \theta_B = H] = x_A^H + x_B^H$$

2. If $(2q_A - 1)\Gamma + 1 > X > (1 - 2q_A)\Gamma + 1$:

$$p_H^H = E[V|\theta_A = H] = x_A^H + E[K_Bx^{q_B}] = x_A^H + P(\theta_B = H)E[K_B|\theta_B = H]x_B^H = x_A^H + \frac{\omega_B}{2}x_B^H$$

3. If $(2q_A - 1)\Gamma - 1 < X < 1 - (2q_A - 1)\Gamma$:

$$p_M^M = E[V] = P(\theta_A = H)E[K_A|\theta_B = H]x_A^H + P(\theta_B = H)E[K_B|\theta_B = H]x_B^H = \frac{\omega_A x_A^H + \omega_B x_B^H}{2}$$

4. If $(1 - 2q_A)\Gamma - 1 < X < (2q_A - 1)\Gamma - 1$:

$$p_L^L = E[V|\theta_A = L] = P(\theta_B = H)E[K_B|\theta_B = H]x_B^H = \frac{\omega_B}{2}x_B^H$$
5. If $X < (1 - 2q_A)\Gamma - 1$:

$$p^{LL} = \mathbb{E}[V|\theta_A = \theta_B = L] = 0.$$ 

Now we have to verify the conjectured trading policy. To this end, we compute trader $i$’s expected trading profits $\Pi_i = s_i(V - P)$ conditional on their private signal $\sigma_i$.

1. $\sigma_i = \theta_A = H$: $s_i = 1$

$$\mathbb{E}[\Pi_i|\sigma_i = \theta_A = H] = \mathbb{E}[V - P|\theta_A = H] = \frac{(2 + \Gamma - 3q_A\Gamma) \omega_A x_H^A - (1 - q_A) \Gamma \omega_B x_H^B}{4}$$

2. $\sigma_i = \theta_A = L$: $s_i = -1$

$$\mathbb{E}[\Pi_i|\sigma_i = \theta_A = L] = \mathbb{E}[P - V|\theta_A = L] = \frac{(2 + \Gamma - 3q_A\Gamma) \omega_A x_H^A - (1 - q_A) \Gamma \omega_B x_H^B}{4}$$

3. $\sigma_i = \theta_B = H$: $s_i = 1$

$$\mathbb{E}[\Pi_i|\sigma_i = \theta_B = H] = \mathbb{E}[V - P|\theta_B = H] = \frac{(2 - \Gamma + q_A\Gamma) \omega_B x_H^B - (1 - q_A) \Gamma \omega_A x_H^A}{4}$$

4. $\sigma_i = \theta_B = L$: $s_i = -1$

$$\mathbb{E}[\Pi_i|\sigma_i = \theta_B = L] = \mathbb{E}[P - V|\theta_B = L] = \frac{(2 - \Gamma + q_A\Gamma) \omega_B x_H^B - (1 - q_A) \Gamma \omega_A x_H^A}{4}$$

Note that the trader’s expected profits are equal to zero if he does not trade and simply multiplied by $-1$ if he follows the opposite strategy. We show in Appendix A.2.3 that all four expected profits are positive in equilibrium.

### A.2.2 Proof of Proposition 2

These two expressions follow directly from equation (7) and equation (9) given in the text.
A.2.3 Proof of Proposition 3

First, we compute the trader’s ex ante expected profit based on receiving $\sigma_i = \theta_A$ and $\sigma_i = \theta_B$ based on the expressions derived in Appendix A.2.1.

$$
E[\Pi_i | \sigma_i = \theta_A] = \frac{(2 + \Gamma - 3q_A\Gamma) \omega_A x_A^H - (1 - q_A) \Gamma \omega_B x_B^H}{4}
$$

and

$$
E[\Pi_i | \sigma_i = \theta_B] = \frac{(2 - \Gamma + q_A \Gamma) \omega_B x_B^H - (1 - q_A) \Gamma \omega_A x_A^H}{4}.
$$

Setting these two expressions equal to each other leads to:

$$
q_A = \frac{1}{2\Gamma} \left( 1 + \frac{1}{\omega_A} - \frac{\omega_A}{\omega_B} \frac{x_B^H}{x_A^H} \frac{1}{\omega_B} \frac{x_B^H}{x_A^H} \right).
$$

It is easy to see that $q_A \geq \frac{1}{2}$ if $\omega_A \geq \frac{x_B^H}{x_A^H + x_B^H}$ and $q_A \geq 1$ if $\omega_A \leq \frac{x_B^H}{(1-\Gamma)x_A^H + x_B^H}$. Then, we can use the expression for $E[V]$ given in the text to compute the manager’s best-response function and solve for $\omega_A^*$ and $q_A^*$. Plugging in the equilibrium values verifies the conjecture in Proposition 1 regarding the optimal trading policy.

A.2.4 Proof of Proposition 4

First, note that our measure of market efficiency is given by, $ME = \frac{\mathbb{E}[\text{Var}(V)|P]}{\text{Var}(P)}$. As we show in the text, we can show that market efficiency is equal to $\frac{\text{Var}(P)}{\text{Var}(V)} - 1$. Next, we compute the unconditional variance of the firm’s stock price and long-run value based on our results in Proposition 1:

$$
\text{Var}(P) = \frac{(3q_A - 1)(4 + \Gamma - 3q_A\Gamma(1 - \omega_A)^2 - (2 + \Gamma)(2 - \omega_A)\omega_A) \Gamma x_A^H x_A^H}{16} + \frac{(1 - q_A)(2 + (2 + \Gamma - 3q_A\Gamma(1 - \omega_A)^2 - (2 - \omega_A)\omega_A) \Gamma x_A^H x_B^H}{8} + \frac{(1 - q_A)(2 + (2 - \Gamma + q_A\Gamma)\omega_A^2) \Gamma x_B^H x_B^H}{16}.
$$
and

\[
\begin{align*}
\text{Var}(V) &= \frac{(4 + \Gamma - 3q_A\Gamma - (2 + \Gamma - 3q_A\Gamma)) \left(3q_A\Gamma - \Gamma + (2 + \Gamma - 3q_A\Gamma)\omega_A\right) x_A^H x_A^H}{16} \\
&\quad + \frac{(2(1 - q_A)\Gamma + (2 + \Gamma - 3q_A\Gamma)(\Gamma - 2 - q_A\Gamma)\omega_A - (2 + \Gamma - 3q_A\Gamma)(\Gamma - q_A\Gamma - 2)\omega_A^2) x_A^H x_A^H}{8} \\
&\quad + \frac{(4 - (2 - \Gamma + q_A\Gamma)^2\omega_A^2) x_B^H x_B^H}{16}.
\end{align*}
\]

Then we combine these two expressions and plug in the equilibrium values for \(q_A\) and \(\omega_A\) from Proposition 3.

Second, our measure of real efficiency is the expected long-term firm value. To compute this measure we take the unconditional expectation of \(V\) based on the results in Proposition 1.

\[
E[V] = \frac{(3q_A\Gamma - \Gamma + (2 + \Gamma - q_A\Gamma)\omega_A) x_A^H + (2 + (\Gamma - 2 - q_A\Gamma)\omega_A) x_B^H}{4}
\]

Then we plug in the equilibrium values for \(q_A\) and \(\omega_A\) from Proposition 3 to get the expression in Proposition 4.

A.2.5 Proof of Corollary 1

First, we consider market efficiency for a given level of \(\Gamma\). Based on our results in Proposition 4, we can see that the highest possible value is \(ME = 0\). It is easy to verify that \(\lim_{\kappa \to 0} ME = 0\).

Next, we consider real efficiency for a given level of \(\Gamma\). Based on our results in Proposition 4, we can see that the lowest possible value is \(RE = \frac{1}{2} x_A^H\), while the highest possible value is \(RE = \frac{5}{8} x_A^H\). It is easy to verify that \(\lim_{\kappa \to 0} RE = \frac{1}{2} x_A^H\) and \(\lim_{\kappa \to 1} RE = \frac{5}{8} x_A^H\).

A.2.6 Proof of Proposition 5

The proof follows the same steps as the proof for Proposition 3. The only difference is that \(\Gamma\) in \(E[V]\) is replaced by \(\tilde{\Gamma}\).
A.2.7 Proof of Proposition 6

Based on the definitions of $RE$ and $ME$, we can plug in the expressions for $\tilde{\omega}_A$ and $\tilde{q}_A$ in Proposition 5 to get (for $\tilde{\Gamma} > 1 - \kappa_x$)

$$
\mathcal{R} = \frac{\Gamma^2(1 - \kappa_x)(3 - \kappa_x(3 - \tilde{\Gamma}))(\tilde{\Gamma} - 4) + 2\kappa_x(3 + \kappa_x)^2\tilde{\Gamma}^2 + \Gamma(3 + \kappa_x)\tilde{\Gamma}(3 + \kappa_x(2\tilde{\Gamma} + \kappa_x(\kappa_x + 3) - 7))}{2(3 + \kappa_x)\tilde{\Gamma}((1 + \kappa_x)(3 + \kappa_x)\tilde{\Gamma} - (1 - \kappa_x)(4 - \tilde{\Gamma}))}\chi_A^H.
$$

The expression for $M$ follows the derivation in Appendix A.2.4. The only difference is that $\omega_A$ and $q_A$ are replaced by $\tilde{\omega}_A$ and $\tilde{q}_A$.

A.2.8 Proof of Corollary 2

Based on the results in Proposition 6, it can be shown that real efficiency is maximized at $\tilde{\Gamma}^*(\Gamma)$ with:

$$
\tilde{\Gamma}^*(\Gamma) = \begin{cases} 
0 & \text{if } \Gamma \leq 1 - \kappa_x \\
1 - \kappa_x & \text{if } 1 - \kappa_x < \Gamma \leq \frac{6+7\kappa_x-\kappa^2_x-\sqrt{97\kappa^2_x-2\kappa_x^2+\kappa_x^4}}{6} \\
\frac{12(1-\kappa_x)\Gamma}{3\Gamma(1-\kappa_x)+(3+\kappa_x)(3+3\kappa_x-\sqrt{3\kappa_x}\sqrt{\Gamma(1-\kappa_x)-1+\kappa_x(8+\kappa_x)})} & \text{if } \Gamma > \frac{6+7\kappa_x-\kappa^2_x-\sqrt{97\kappa^2_x-2\kappa_x^2+\kappa_x^4}}{6}
\end{cases}
$$

It follows that $\tilde{\Gamma}^*$ is always greater than $\Gamma$. 

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