Demand for safety, risky loans: 
A model of securitization

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Abstract

We build a competitive equilibrium model of securitization in the presence of demand for safety by some investors. Securitization allows to create safe assets by pooling idiosyncratic risks from loan originators, leading to higher aggregate loan issuance. Yet, the distribution of loan risks out of their originators creates a moral hazard problem. An increase in the demand for safety leads to a securitization boom and riskier originated loans. When demand for safety is high, welfare is Pareto higher than in an economy with no securitization despite the origination of riskier loans. Aggregate lending expansions driven by demand for safety may, paradoxically, lead to riskier loan issuance than expansions driven by standard credit supply shocks.

JEL Classification: G01, G20, G28

Keywords: securitization, originate-to-distribute, safety demand, diversification, moral hazard

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1 Introduction

The emergence of securitization in the last decades has changed financial intermediation from an originate-to-hold to an originate-to-distribute model. The increase in the demand for safe assets observed since the early 2000s is considered an important driver for this transformation (Bernanke (2005), Bernanke et al. (2011), Caballero and Krishnamurthy (2009)). Securitization consists in fact in the process of pooling loan cash-flows to diversify their idiosyncratic risks and create the safer securities demanded by investors. Yet, there is evidence that securitized loans perform worse than loans held by their originators (Loutskina and Strahan (2011), Bhattacharyya and Purnanandam (2011), Ashcraft et al. (2019)). How can an increase in demand for safety be met with the securitization of riskier loans? Are credit expansions driven by investors demanding safety different from those driven by investors willing to bear risk?

We provide an answer to these questions based on a novel competitive equilibrium model of securitization and the capital structure of the modern intermediation chain. The model features absolute demand for safety by some investors, loan originators exposed to idiosyncratic risks, and intermediaries that perform securitization by pooling risky securities from different originators. Securitization increases the supply of safe assets in the economy by diversifying idiosyncratic risks, allowing to expand aggregate loan issuance. Yet, securitization reduces originators’ exposure to the risks of the loans they issue, creating a moral hazard problem and the origination of riskier and lower quality loans. Increases in the wealth of investors demanding safety lead to lending booms fueled by securitization and to the issuance of riskier loans. In addition, credit expansions driven by investors that demand safety may, paradoxically, lead to riskier loan issuance than those driven by increases in the wealth of investors willing to bear risk. The reason is that the manufacturing of safe assets requires some of the scarce equity in the economy to be reallocated towards intermediaries that do securitization to provide loss absorption against aggregate risk, reducing the amount of equity held at loan origination where it reduces moral hazard problems. The paper also derives a rich set of additional predictions that are consistent with the saving glut narrative of the run-up to the crisis, assesses which agents benefit and which lose from the emergence of securitization, analyzes the constrained efficiency properties of the competitive equilibrium, and discusses how government guarantees to support safe asset creation can increase welfare.
and their implications for loan risk.

We model a two date competitive economy with two types of investors: experts and savers. Investors derive linear utility from consumption at either date and each of them is endowed with one unit of funds. Experts’ overall endowment is normalized to one. Experts are skilled agents that can set-up and invest their wealth in the equity of one out of two competitive financial firms: originators and intermediaries. Savers only invest in safe securities and their overall endowment determines the demand for safety in the economy.

Originators can issue loans under a constant return to scale technology. The loans are exposed to aggregate and institution-specific idiosyncratic risks, and have to be monitored to increase the likelihood that their payoff is high. Monitoring is not observable and involves a convex disutility cost for the expert managing each originator. Originators can expand lending by issuing in competitive markets safe and risky securities that are purchased by savers and intermediaries (the other type of financial firm), respectively. The issuance of safe securities can only be backed by the lowest return of the loans. The issuance of risky securities leads, as in Holmstrom and Tirole (1997), to a moral hazard problem because of the non-observability of loan monitoring.

Intermediaries engage in securitization. They purchase the risky securities issued by many originators, diversify away their idiosyncratic risks and “manufacture” additional safe securitized assets that can be sold to savers. Intermediaries’ leverage and size under this carry trade strategy is bounded because of aggregate risk: their equity must be sufficient to absorb their assets’ losses in the worst aggregate shock to ensure the safety of the securitized assets distributed to savers.

The capital structure of the financial firms in the intermediation chain, the risk of the originated loans, the returns of safe and risky securities and of financial firms’ equity, aggregate loan issuance and the size of the intermediary sector are all determined in equilibrium. In particular, the frictionless allocation of experts’ endowment between the equity of originators and intermediaries induces their equity returns to be equal in equilibrium. The equilibrium equity allocation trades off the gains from reducing moral hazard at origination (skin-in-the-game) and those from providing loss-absorption capacity against aggregate risk to support safe asset creation at intermediation (credit enhancement). The existence of competitive markets for safe and risky securities and (de facto) also for financial firms’ equity,
ensure that constrained versions of the Welfare Theorems hold in this economy.

The demand for safety in the economy determines the size of the securitization sector and the risk of originated loans. When the demand for safety is low, it is directly satisfied by originators. Safe assets are abundant, the return of safe securities equals that of equity, and there is no securitization. The equilibrium coincides with that of the traditional originate-to-hold economy. The risky part of the loans is entirely funded with originators’ equity, there are no moral hazard problems and loan risk is minimum (and coincides with its first-best level).

When the demand for safety is higher, safe securities become scarce, the safe rate falls and a positive equity spread arises. Experts set-up intermediaries to exploit the equity spread by creating safe assets through securitization, and a fraction of the originated loans is indirectly funded through securitization. The distribution of risk from originators to intermediaries creates a moral hazard problem that increases loan risk, and this is exacerbated by the reallocation of equity from origination to securitization. As the demand for safety keeps on increasing, the safe rate falls further. The widening equity spread allows intermediaries to increase leverage and to offer more attractive risky funding to originators, giving the later incentives to further increase leverage through the issuance of risky securities. A securitization boom fuels the aggregate lending expansion, the intermediation chain becomes “longer” as a larger fraction of aggregate lending is channelled through intermediaries, and loan risk increases. The model thus provides a rich set of predictions consistent with the saving glut narrative of the run-up to the crisis.

The differences in utilities of savers and experts in the modern originate-to-distribute economy relative to those in the traditional originate-to-hold economy result from the following quantity versus quality trade-off. On the one hand, securitization increases the quantity of safe payoffs in the economy relative to that in the traditional economy. This allows, firstly, savers to obtain a higher return on their investment in safe assets, and, secondly, aggregate loan issuance to increase. On the other hand, by reducing originators’ exposure to their loans’ risk, securitization leads to the issuance of riskier loans that have a lower expected return. When the demand for safety is sufficiently large, the available safe payoffs generated in the traditional economy are so scarce that savers end up consuming their funds instead of
allocating them into the origination of positive NPV loans. In such case, the quantity effect from the emergence of securitization dominates and securitization Pareto increases utility despite the origination of riskier loans. In contrast, when the demand for safety is medium, safe payoffs in the traditional economy are enough to offer savers a higher return than on their consumption and all savers funds are allocated into the origination of loans. Yet, there is scarcity of safe payoffs since the equilibrium safe rate is not as high as the return of originated loans and so there is a positive equity spread. In such case, securitization emerges but, in absence of an increase in overall lending, the loan quality effect dominates and aggregate surplus is lower than in the traditional economy. Nevertheless, the emergence of securitization has redistributive effects as savers’ utility increases due to the expansion of the supply of safe payoffs and in the equilibrium safe rate, while that of experts is reduced (by a larger amount). Interestingly, since securitization allows originators to indirectly raise additional funds from savers against the risky payoffs of their loans, this technology reduces financial frictions between originators and savers and, in equilibrium, deprives experts from some of the scarcity rents they enjoyed in the traditional economy. These results provide novel insights on the aggregate and distributional welfare effects associated with securitization.

The demand for safe securities by savers leads to the emergence of securitization and the issuance of riskier loans. What would be the risk of loans if, instead, savers were willing to buy risky securities? On the one hand, demand for safety constrains the amount of risky loan payoffs that originators can (indirectly) pledge to raise funds from savers. When savers are willing to buy risky securities, the pledgeability of originators’ loans increases and financing constraints get relaxed. This leads to an increase in originators’ demand for external risky funding and in the risky part of the loans promised to external creditors, which worsens the moral hazard problem and the quality of originated loans. On the other hand, when savers are willing to buy risky securities there is no need of securitization, and a fortiori no equity investment in intermediaries. Experts’ endowment is thus entirely invested in originators’ equity where it plays a skin-in-the-game role that reduces loan risk. In some cases the latter effect dominates and the following demand for safety paradox emerges: originated loans are riskier when savers only buy safe assets than when are willing to buy risky securities. While several papers have emphasized how demand for safety may increase financial sector fragility

\[1\text{Savers consumption, which is their alternative strategy to lending to financial firms, can be also interpreted as investment in low return but safe strategies like storage.} \]
due to its impact on maturity mismatch and roll-over risk (e.g., Caballero and Krishnamurthy (2009), Stein (2012), Moreira and Savov (2017)), this paper is, to the best of our knowledge, the first one highlighting credit risk as an additional source of fragility implied by safety demand.

A government with safe assets can use them to reduce safe asset scarcity in the economy by granting fiscally neutral guarantees to support the issuance of safe securitized assets. The guarantees require the government to inject funds into intermediaries following negative aggregate shocks, which expands intermediaries’ capability to create safe securities, and obliges intermediaries to reimburse the government following positive aggregate shocks. The guarantees thus provide a substitute for the loss absorption against aggregate risk role of equity in the economy, thereby expanding the Pareto frontier of the economy. When guarantees are suitably combined with lump sum transfers across agents at the initial date, they increase the utility of both savers and experts. These Pareto improvements in the economy are associated with reductions in loan risk when demand for safety is medium, and increases in loan risk when it is large.

The paper is organized as follows. Section 2 describes the related literature. Section 3 presents the model. Section 4 describes as a benchmark the equilibrium of the traditional originate-to-hold economy. Section 5 characterizes the equilibrium of the originate-to-distribute economy discusses the welfare effects from the emergence of securitization. Section 6 shows that constrained versions of the Welfare Theorems hold. Section 7 focuses on the demand for safety paradox. Section 8 analyzes government guarantees to financial firms. Section 9 concludes. All the proofs of the formal results in the paper are in the Appendix.

2 Related literature

This paper belongs to the literature that analyzes how the financial sector satisfies demand for safety in the economy. Our paper is mostly related to the contributions focusing on the manufacturing of safe assets through diversification (Gennaioli et al. (2013), Diamond (2016)). In Gennaioli et al. (2013) banks purchase pools of loans issued by other banks and create safe securitized assets. The paper assumes an exogenous loan risk and emphasizes how the neglect of tail risks leads to excessive securitization and financial crises. Our paper instead assumes rational expectations and focuses on the interplay between safe asset creation
through securitization and moral hazard problems at origination. Diamond (2016) shows that
the efficient creation by financial intermediaries of safe assets through diversification leads
to segmentation in the market for firms’ external funding: intermediaries invest in corporate
debt because its low exposure to aggregate risk reduces the intermediaries’ need of equity,
and households invest in corporate equity. The paper takes the real assets in the economy
as given, while we focus on how safe asset creation affects their risk. Another strand of the
literature analyzes how the financial sector satisfies demand for safety through the issuance
of short-term liabilities (Caballero and Krishnamurthy (2009), Stein (2012), Moreira and
Savov (2017), Ahnert and Perotti (2017)). These papers emphasize the fragility created by
the presence of roll-over risk, which is complementary to our focus on the implications of
safety demand on credit risk.

Moral hazard problems in the originate-to-distribute intermediation chain and how to
address them with endogenous risk retentions have been extensively studied in the literature
(Parlour and Plantin (2008), Chemla and Hennessy (2014), and Daley et al. (2017)). These
papers, though, abstract from the diversification benefits associated with securitization.²

In practice, the securitization process involves the pooling, tranching and distribution of
cash-flows generated by loans along an intermediation chain that exhibits different entities
(Ashcraft et al. (2008), and Pozsar et al. (2013)). Our paper provides a tractable equilibrium
framework of the financial architecture of the securitization process. A related paper is
DeMarzo (2004), in which a long intermediation chain with several rounds of pooling and
tranching emerges as the solution to a security design problem in presence of asymmetric
information. The paper exhibits endogenous risk retention along the chain but the risk of
the originated loans is exogenous.

Our paper is also related to a literature that analyzes how moral hazard problems shape
risk-taking by financial intermediaries. The equilibrium relationship between bank capital
requirements and risk-taking is analyzed in Repullo (2013) and in Martínez-Miera and
Repullo (2018). The implications of saving gluts or low interest rate environments for mon-
itoring and origination incentives are analyzed in Dell Ariccia et al. (2014), Martínez-Miera
and Repullo (2017) and Bolton et al. (2018).

²Another strand of the literature stresses the role of regulatory arbitrage for the the emergence of secu-
ritization (Calomiris and Mason (2004), Acharya et al. (2009), Acharya et al. (2013)). These aspects are
absent in our model.
Some recent papers analyze the endogenous capital structure of non-financial firms and banks (Allen et al. (2015), Gornall and Strebulaev (2018), Diamond (2016)). Although the focus of these papers is different from ours, we share the interest on how market forces shape the equity allocation in the economy. A contribution of our paper to this literature is to endogenize the risk of the real assets in the economy, which in those papers is taken as exogenous.

Finally, our paper also contributes to a large literature that studies the need and implications of public support to the financial sector following negative shocks (for recent contributions see, e.g., Diamond and Rajan (2012); Farhi and Tirole (2012); Keister (2015)). Most of the literature has highlighted a time consistency problem that makes public support optimal ex post but inefficient ex ante due to moral hazard. In contrast, in our model fiscally neutral guarantees guarantees to the issuance of safe assets are beneficial ex ante even though they may aggravate moral hazard and lead to higher loan risk.

3 The model

Consider an economy with two dates $t = 0, 1$ and two types of investors endowed at $t = 0$ with one unit of funds: experts and savers. The measure (and aggregate wealth) of savers is $w$ and that of experts is normalized to 1. Investors derive linear utility from consumption at either date and have a zero discount rate. At $t = 0$, each expert can set-up one out of two types of competitive financial firms, called originators and intermediaries. Both types of financial firm have access to some constant return to scale investment possibilities that are funded as described below. Each expert decides at $t = 0$ whether to set-up and invest its unit endowment as equity in his own firm, or consume. Savers are special in that they only invest in risk-free assets. Each saver decides at $t = 0$ whether to invest its unit endowment in safe securities issued by financial firms or consume.

We describe each of the financial firms that experts can create next.

Originators An originator is a financial firm that has access to a constant returns to scale loan issuance technology. The per unit return of loans, that we denote $A_z$, can be either high ($z = H$) or low ($z = L$), where $A_H > A_L \geq 0$. For each originator, the realization of $z$ depends on an institution-specific shock and an aggregate shock that are described next.
when we introduce intermediaries. We refer to $A_L$ as the safe return of the loan and to $\Delta \equiv A_H - A_L$ as its risky return. The probability that the high return is realized coincides with the unobservable monitoring intensity $p \in [0, p_{\text{max}}]$ exerted by the expert that sets up and manages the originator, where $p_{\text{max}} < 1$.\(^3\) We henceforth refer to $p$ as the loans’ risk, under the interpretation that high risk corresponds to a low value of $p$.\(^4\) The issuance of loans with risk $p$ entails the expert a disutility cost per loan unit given by a function $c(p) \geq 0$ satisfying:

Assumption 1 \(c(0) = 0, c'(0) = 0, c'(p_{\text{max}}) \geq \Delta, c''(p) > 0, \text{ and } c'''(p) \geq 0.\)

We denote with $\bar{p}$ the first-best loan risk, which is given by:

$$\bar{p} = \arg \max_p \{E[A_\varepsilon|p] - c(p)\}. \quad (1)$$

Assumption 1 implies that $\bar{p} \in (0, p_{\text{max}}]$ and is determined by the first order condition:

$$c'(\bar{p}) = \Delta. \quad (2)$$

We assume that:

Assumption 2 \(E[A_\varepsilon|\bar{p}] - c(\bar{p}) > 1.\)

Assumption 3 \(A_L < 1.\)

Assumption 2 states that loan issuance creates a surplus if first-best risk is chosen. Assumption 3 implies that loans cannot be funded exclusively with safe securities.

At $t = 0$, the originator issues $x$ units of loans that are financed with the unit of wealth of its expert (equity), and with the issuance of safe securities and risky securities in competitive markets in which the required expected market returns are $R_S$ and $R_I$, respectively.\(^5\) The amount of funds raised with safe (risky) securities is denoted with $x_S$ ($x_I$), and the overall notional promise on safe (risky) securities at $t = 1$ with $d_S x$ ($d_I x$). Notice that $d_S$ and $d_I$ are promises per unit of loan. For the sake of brevity we simply refer to them as the safe

\(^3\)Notice that since $p_{\text{max}} < 1$, the risky payoff $\Delta$ is in fact never realized with probability one.

\(^4\)This terminology is consistent with the interpretation of the return $A_L$ as the loan recovery value in case of default, so that $1 - p$ amounts to the probability of default.

\(^5\)We refer with subindex $I$ to risky securities because they are purchased by intermediaries.
and risky promise, respectively. We assume that the repayment of safe securities is senior to that of risky securities.

For given required expected returns $R_S, R_I$, the expert chooses at $t = 0$ a balance sheet tuple $(x, x_S, x_I, d_S, d_I, p)$ in order to solve the following maximization problem

$$\max_{(x, x_S, x_I, d_S, d_I, p)} R_{E,O} \equiv (E \max\{A_z - d_S - d_I, 0\}|p] - c(p)) x, \quad (3)$$

subject to the budget constraint

$$x = 1 + x_S + x_I, \quad (4)$$

the securities repayment constraints

$$d_S \leq A_L, \quad (5)$$
$$d_S + d_I \leq A_H, \quad (6)$$

the securities’ pricing constraints

$$R_S x_S = d_S x, \quad (7)$$
$$R_I x_I = E \min\{d_I, A_z - d_S\}|p] x, \quad (8)$$

and the optimal risk choice constraint

$$p = \arg \max_{p'} \{E \max\{A_z - d_S - d_I|p'| - c(p')\} \}.$$

The objective function $R_{E,O}$ in (3) is the expected utility the expert obtains from investing its wealth in the originator, which amounts to the value of the residual equity claim net of the monitoring costs. We will henceforth refer to $R_{E,O}$ as the originator’s equity return. The maximization of the equity return is subject to the following constraints. Constraint (4) states how the $x$ units of loans are financed. Constraint (5) ensures that safe securities are always repaid and constraint (6) ensures that risky securities are repaid in full in state $z = H$. Constraints (7) and (8) are the pricing equations that ensure that safe and risky securities yield the required expected market returns $R_S$ and $R_I$, respectively. In particular, the pricing equation for risky securities takes into account that these securities might not be repaid in full in state $z = L$, which happens with probability $1 - p$. Finally, constraint (9) characterizes the risk choice that maximizes the residual payoff of the expert taking into account the promises on safe and risky securities. Notice from the pricing constraint
and the optimal risk choice constraint (9), given a funding structure of the originator, investors form rational expectations on its unobservable risk-choice and price risky securities accordingly.

**Intermediaries** Intermediaries engage in securitization: they pool risky securities purchased from multiple originators, diversify their idiosyncratic risks and manufacture new safe assets. Intermediaries need to finance a fraction of their assets with own funds of their experts (equity) due to the presence of aggregate risk in the economy, which is described next.

At $t = 1$ an aggregate shock $\theta$ that affects the return of the originators’ loans is realized. Conditional on the realization of $\theta$, the high payoff of the loan of an originator with risk choice $p$ is $\theta p$. Hence, when $\theta > 1$ ($\theta < 1$) the conditional probability of a high payoff is larger (lower) than its unconditional value. In addition, conditional on $\theta$, the loan returns are independent across originators. The support of the shock is $[1 - \lambda, 1/p_{\text{max}}]$, with $\lambda \in (0, 1)$, and its distribution $F(\theta)$ has positive density in a neighborhood of $\theta = 1 - \lambda$ and satisfies $E[\theta] = 1$.

The aggregate risk parameter $\lambda$ determines the fraction of the expected return of a pool of originators’ loans in the economy that is destroyed under the worst aggregate shock. When $\lambda \to 1$, all the loans in the economy have a low return under the worst aggregate shock: no safe assets can be created by pooling idiosyncratic risks. When $\lambda \to 0$, all the risk in the originators’ loans is idiosyncratic: the return of a pool of originators’ loans is safe.

We next describe the intermediaries formally. For given market returns $R_S, R_I$, an intermediary invests $y$ units of funds at $t = 0$ into a pool of risky securities issued by many originators. The intermediary finances the asset purchases with the unit of equity provided by its expert and with $y_S$ units of funds obtained from the issuance of safe securities. Given the required rate $R_S$ on safe securities, the intermediary must issue safe securities with an overall notional promise of $R_S y_S$.

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6 Notice that the assumption $\theta \leq 1/p_{\text{max}}$ ensures that the conditional probability of the high return is upper bounded by 1. In addition, using that $E[\theta] = 1$, for an originator with risk choice $p$ we have:

$$
\Pr[A_z = A_H] = \int_{1-\lambda}^{1/p_{\text{max}}} \Pr[A_z = A_H|\theta]dF(\theta) = \int_{1-\lambda}^{1/p_{\text{max}}} \theta p dF(\theta) = p E[\theta] = p,
$$

as expected.
The return of the intermediary’s pool of risky securities depends on the return of each of the risky securities conditional on the realization $z \in \{H, L\}$ of their issuers’ loans and on the realization of the aggregate shock. For the sake of expository simplicity, we assume that all originators choose the same tuple $(x, x_S, x_I, d_S, d_I, p)$.

The return of a risky security contingent on the realization $z \in \{H, L\}$ of its issuer’s loans, that we denote with $R_{I,z}$, is thus given by:

$$R_{I,z} = \min\{d_I, A_z - d_S\} x_I.$$  
(10)

Thus, we can more compactly describe the risky securities in the market by a tuple $(R_{I,H}, R_{I,L}, p)$.

By definition of the expected return $R_I$, we have:

$$R_I = E [R_{I,z}|p].$$  
(11)

For given returns $R_S, R_I$ and risky securities described by the tuple $(R_{I,H}, R_{I,L}, p)$ satisfying (11), the expert chooses at $t = 0$ a balance sheet pair $(y, y_S)$ solving the maximization problem

$$\max_{(y, y_S)} R_{E,I} \equiv \int_{1-\lambda}^{1/p_{\max}} (E [R_{I,z}|p, \theta]) dF(\theta)y - R_S y_S = R_I y - R_S y_S,$$

subject to the budget constraint

$$y = 1 + y_S,$$

and the repayment constraint

$$R_S y_S \leq \min_\theta E [R_{I,z}|p, \theta] y = \min_\theta [\theta p R_{I,H} + (1 - \theta p) R_{I,L}] y.$$  
(14)

The objective function $R_{E,I}$ in (12) is the utility of the expert that sets-up an intermediary, which equals the expected residual payoff of the firm. We refer to $R_{E,I}$ as the intermediary’s equity return. Notice that the latter expression for $R_{E,I}$ in (12) immediately results from (11).

The maximization of the equity return is subject to the following constraints. Constraint (13) states how the intermediary finances its purchase of originators’ risky securities. Constraint (14) ensures that the safe securities issued by the intermediary are repaid always in full and takes into account that, by the law of large numbers, the payoff of the intermediary’s pool of risky securities at $t = 1$ is a function of the risk choice of the originators $p$ and the realization of the aggregate shock $\theta$.

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\footnote{This is the case in equilibrium because, as we will see, given market expected returns $R_S, R_I$, the maximization problem of the originator described in (3) - (9) has a unique solution.}
Equity allocation and aggregate lending  We denote $E_O, E_I$ the measures of experts that set-up at $t = 0$ an originator and an intermediary, respectively. The pair $E_O, E_I$ also describes the aggregate amounts of equity in each sector. Aggregate lending in the economy, which is denoted with $N$, amounts to $N = E_O x$. We say that the economy features full lending when all funds are allocated to originators’ loans, i.e., when $N = 1 + w$.

Equilibrium definition  A competitive equilibrium consists of choices for originators and intermediaries described by balance sheet tuples $(x^*, x^*_S, x^*_I, d^*_S, d^*_I, p^*), (y^*, y^*_S)$, respectively, overall amounts $E^*_O, E^*_I$ of equity in originators and intermediaries, respectively, and expected returns $R^*_S, R^*_I, R^*_E$ on safe securities, risky securities, and financial firms’ equity, respectively, such that:

1. The choices of originators and intermediaries satisfy the maximization problems in (3) - (9) and (12) - (14), respectively.

2. The return on equity obtained by an expert that sets-up any financial firm is $R^*_E$ and the experts’ decision to set-up a financial firm instead of consuming is optimal.

3. Savers’ investment and consumption decisions are optimal.

4. The markets for safe and risky securities clear.

Figure 1 graphically illustrates the funding structures, the financing and securities flows, and the market clearing conditions in the economy.

4 Benchmark: equilibrium without securitization

We consider in this section a benchmark economy in which experts cannot set-up intermediaries. Consider an expert that has set-up an originator and has to decide its loan size $x \geq 1$ at $t = 0$. Since the originator’s has one unit of equity, $x$ can be also interpreted as the firm’s leverage. In absence of intermediaries, the originator can raise external funds only with safe securities. For a given safe rate $R_S$, the originator’s problem is as described in (3) - (9) with the additional constraints $x_I = d_I = 0$. 
Using (1) and (5), the optimal loan risk condition in (9) implies that:

\[ p = \arg \max_{p'} \{ E[A_z - d_S|p'] - c(p') \} = \arg \max_{p'} \{ (E[A_z|p'] - c(p')) - d_S \} = \overline{p}. \]  

(15)

Loan risk is first-best because safe securities are totally repaid and the expert fully appropriates the marginal benefits from monitoring.

We denote with

\[ R_A(p) \equiv E[A_z|p] - c(p), \]  

(16)

the expected return of one unit of loans net of monitoring costs when risk choice is \( p \), and compactly refer to \( R_A(p) \) as the return of the originators’ assets. Using this definition and equations (4), (7) and (15), the originator’s equity return in (3) can be written as:

\[ R_{E,O} = R_S + (R_A(\overline{p}) - R_S)x. \]  

(17)

The expression states that the equity return of the originator exceeds the safe rate by an amount that is proportional to the spread between its return on assets and the safe rate \( (R_A(\overline{p}) - R_S) \) and its leverage \( (x) \).
From (17), we have that the originator only finds optimal to issue safe securities when 
\( R_A(\bar{p}) - R_S \geq 0 \). Moreover, savers find optimal to invest in safe securities only if \( R_S \geq 1 \), since they have the option to consume their endowment at \( t = 0 \). The next lemma results from the clearing of the safe securities market. (From here on we denote equilibrium variables in this benchmark economy with a \( b \) supraindex).

**Lemma 1** The equilibrium safe rate \( R_S^b \) satisfies

\[
1 \leq R_S^b \leq R_A(\bar{p}). 
\]  

(18)

We show next how the equilibrium is determined when the inequalities in Lemma 1 are strict. Suppose that:

\[
1 < R_S^b < R_A(\bar{p}). \tag{19}
\]

We have from (17) that the expert finds optimal to maximize leverage, which is achieved by issuing as many safe securities as possible, that is, constraint (5) binds. Due to the leverage of originators, the equilibrium returns satisfy \( R_{E,O}^b > R_S^b > 1 \). The entire wealth of experts is invested in originators’ equity and that of savers in safe securities. This in particular implies that there is full lending, that is, \( N^b = w + 1 \). The clearing of the market for safe securities can then be written as:

\[
w = \frac{A_L(w + 1)}{R_S^b}. 
\]  

(20)

The LHS in the expression above is the demand for safe securities and its RHS is their supply by originators. The latter takes into account that each unit of loans yields a safe pay-off \( A_L \), that overall lending is \( N^b = w + 1 \), and that investors discount safe payoffs at the rate \( R_S^b \).

Rewriting (20) we have that:

\[
R_S^b = \frac{A_L(w + 1)}{w}, \tag{21}
\]

which states that the equilibrium safe rate equals the ratio of the overall safe payoffs of originators’ loans and savers’ wealth \( w \). The equilibrium safe rate is decreasing on \( w \), and satisfies the conjectured inequalities in (19) if and only if \( w \) lays in an intermediate region.

The next formal result easily follows.

**Proposition 2** The equilibrium of the benchmark economy without securitization is unique up to Modigliani-Miller type of indifference when there is no spread between the return of
equity and that of safe securities. Let \( w \) be savers’ wealth, and define \( \bar{w} \equiv \frac{A_L}{R_A(p) - A_L} \). Let \( R^b_S \) and \( R^b_{E,O} \) be the equilibrium return on safe securities and equity, respectively, and \( N^b \) aggregate lending. We have:

(i) If \( w \leq \bar{w} \) then:
\[
R^b_S = R^b_{E,O} = R_A(p), \quad \text{and} \quad N^b = w + 1.
\]

(ii) If \( w \in (w, \bar{w}] \) then:
\[
R^b_S < R_A(p) < R^b_{E,O}, \quad \text{and} \quad N^b = w + 1.
\]

(iii) If \( w > \bar{w} \) then:
\[
1 = R^b_S < R_A(p) < R^b_{E,O}, \quad \text{and} \quad N^b = \frac{1}{1 - A_L} < w + 1.
\]

The proposition describes how the equilibrium of the economy depends on savers’ wealth, which measures the demand for safety in the economy. Figure 2 illustrates the results in the proposition. When the demand for safety is low (\( w \leq \bar{w} \)), the safe payoff of the originators’ loans is sufficiently large to deliver in equilibrium a high safe rate that equals the expected net return of the originators’ loans and there is no equity spread. As a result, there is full lending and a Modigliani-Miller indifference in the capital structure of originators. For an intermediate demand for safety (\( w \in (w, \bar{w}] \)), safe securities become scarce and the equilibrium safe rate falls. Since originators lever up with the issuance of safe securities, their return on equity increases, an experts obtain scarcity rents. In this region, a positive equity spread arises but still the entire \( w + 1 \) endowment of the economy is used to finance originators’ loans. When the demand for safety is large (\( w > \bar{w} \)), the safe rate falls to one, some savers opt to consume their endowment at the initial date and full lending is not achieved.

5 Equilibrium

In this section, we determine the equilibrium of the baseline economy in which experts can set up intermediaries. We start the analysis with the following lemma that provides the relevant range of values for the equilibrium returns.
Lemma 3 The equilibrium safe rate, $R_S^*$, satisfies

$$1 \leq R_S^* \leq R_A(\bar{p}).$$

Moreover, the equilibrium returns on risky securities, $R_I^*$, and equity, $R_E^*$, satisfy

$$R_S^* \leq R_I^* \leq R_E^*.$$ 

Finally, $R_S^* = R_I^* = R_E^*$ if and only if $R_S^* = R_A(\bar{p})$.

The lemma makes three intuitive statements. First, it provides bounds on the equilibrium safe rate that result from savers’ possibility to consume at $t = 0$ and the maximum expected return at $t = 1$ of the productive assets in the economy. Second, it states that the expected return on risky securities lays between the returns of safe securities and equity. This results from the fact that the assets of the intermediaries consist of risky securities, while their liabilities consist of safe securities and equity. Third, there is no spread between equity, risky securities and safe securities only when the safe rate is at its upper bound. In that case, safe securities are not scarce, a Modigliani-Miller type of capital structure indifference arises and the economy is payoff equivalent to the no securitization benchmark described in the previous section.
5.1 The five equilibrium equations

In this section, we derive five equilibrium equations that are sufficient to determine the equilibria of the economy in which \(1 < R_S < R_A(p)\), that is, equilibria with scarcity of safe securities but full lending. From Lemma 3, the returns of the three funding sources in such equilibria satisfy \(R_S^* < R_I^* < R_E^*\).

**Originators’ problem** Consider an originator’s optimal balance sheet tuple \((x, x_S, x_I, d_S, d_I, p)\) that solves the problem (3) - (9) for given returns \(R_S^* < R_A(p)\) and \(R_I^* > R_S^*\). Using the repayment constraints (5) and (6), and the pricing equations (7) and (8), the originator’s equity return in (3) can be written as:

\[
R_{E,O} = R_A(p) + (R_A(p) - R_S^*)x_S + (R_A(p) - R_I^*)x_I.
\]  

(22)

This expression extends that in (17) by including a third term that captures the spread the expert obtains by issuing risky securities to expand loan size. Notice that \(R_{E,O}\) depends on the originator’s risk choice \(p\), which we focus on next.

We have from the optimal risk choice condition in (9) that \(p\) is determined by the overall (per unit of loan) promise on the two types of securities issued by the originator, \(d_S + d_I\). Since \(R_S^* < R_A(p)\) and \(R_I^* > R_S^*\), it is easy to prove from (22) that the originator finds optimal to exhaust its capability to issue safe securities, that is, constraint (5) is binding:

\[
d_S^* = A_L.
\]  

(23)

The overall repayment constraint in (6) then implies that \(d_I \leq A_H - A_L = \Delta\), and we can rewrite the pricing constraint (8) as

\[
R_I^*x_I = pd_Ix.
\]  

(24)

Using (23), condition (9) takes the compact form

\[
p = \arg \max_{p'} \{p' (\Delta - d_I) - c(p')\},
\]  

(25)

and we have the following result.

**Lemma 4** For given \(R_S^* < R_A(p)\) and \(R_I^* > R_S^*\), the originators’ optimal risk choice is a function \(\hat{p}(d_I)\) of the risky promise \(d_I \in [0, \Delta]\) satisfying

\[
\frac{d\hat{p}(d_I)}{dd_I} < 0, \hat{p}(0) = p \text{ and } \hat{p}(\Delta) = 0.
\]  

(26)
The lemma states that as the risky promise \( d_I \) increases, the originator’s loans become riskier (\( p \) decreases). The reason is that when \( d_I \) is larger, the expert’s incentives to undertake the costly monitoring get reduced, since the value created by this action is to a larger extent appropriated by the holders of the risky securities. The non-observability of monitoring thus creates a moral hazard problem that increases loan risk when risky securities are issued.

Using (4), (7), (8), (22), (23) and Lemma 4, we can obtain the following expressions for the originator’s loan issuance and equity return as functions of the single choice variable \( d_I \):

\[
\begin{align*}
x(d_I) &= \frac{1}{1 - A_L/R_S^* - \hat{p}(d_I)d_I/R_I^*}, \quad (27) \\
R_{E,O}(d_I) &= \left[ R_A(\hat{p}(d_I)) - A_L - \hat{p}(d_I)d_I \right] x(d_I). \quad (28)
\end{align*}
\]

Equation (27) expresses the originator’s loan issuance, which can be interpreted as its leverage, as the inverse of the equity contribution to the funding of each unit of loans. Such equity downpayment equals the difference between the funds required per unit of loan and the amount of them provided by external investors (savers and intermediaries). Equation (28) in turn decomposes the originator’s equity return as the product of the expected residual cash-flow generated by each unit of loans after repayment of safe and risky securities and net of the monitoring costs (term in brackets) and leverage.

From the above discussion, we conclude that the originator’s problem (3) - (9) amounts to the following optimal choice of risky promise:

\[
\max_{d_I \in [0, \Delta]} R_{E,O}(d_I). \quad (29)
\]

In order to gain intuitions on the determinants of the optimal \( d_I \), we introduce the variable \( \chi^* \), defined as the equilibrium ratio between the return on equity and that of risky securities:

\[
\chi^* \equiv \frac{R_E^*}{R_I^*}. \quad (30)
\]

Notice from Lemma 3 that \( \chi^* \geq 1 \) and \( \chi^* > 1 \) when \( R_S^* < R_A(\hat{p}) \).

For equilibrium returns \( R_S^* < R_I^* \), the solution \( d_I^* \) to the problem (29) satisfies the FOC

\[
\left. \frac{dR_{E,O}(d_I)}{dd_I} \right|_{d_I = d_I^*} = 0. \quad (E1)
\]

Using (27) and (28), we can write this FOC as the following *optimal risky promise* equation:

\[
(\chi^* - 1) \left. \frac{d(\hat{p}(d_I)d_I)}{dd_I} \right|_{d_I = d_I^*} + \left. \frac{dR_A(\hat{p}(d_I))}{dd_I} \right|_{d_I = d_I^*} = 0.
\]

19
Equation \( E_1 \), which constitutes the first of the five equilibrium equations we derive, captures the trade-off faced by originators in their risky securities issuance decision. The first term captures the *leverage benefits* from augmenting the issuance of risky securities: a marginal increase \( d_d I \) in \( d_d I \) allows the originator to raise additional funds from risky securities amounting to \( dx_I = (1/R_I^*)d(\hat{p}(d_I) d_I) \) per unit of loan. The additional funds have a cost \( R_I^* \) for the originator, but free up an equal amount of equity that (levered up with external funds) allows to increase loan size and to obtain a return \( R_E^* \). The originator thus obtains a spread \( R_E^* - R_I^* \) on the additional funds raised with risky securities. The term then results from the identity \( (R_E^* - R_I^*)/R_I^* = \chi^* - 1 \).

The second term in \( E_1 \), which from (16) and Lemma 4 is negative, accounts for the *loan quality costs* from higher issuance of risky securities: a marginal increase \( d_d I \) in \( d_I \) weakens the originator’s incentives to monitor, which induces an increase in loan risk and a reduction in the net return of each unit of loans of \( d R_A < 0 \).

Equation \( E_1 \) determines the optimal risky promise \( d_I^* \) as a function of the endogenous equilibrium variable \( \chi^* = R_E^*/R_S^* \).\(^8\) This variable can in fact be interpreted as the discount offered by intermediaries to originators in the funding of the risky part of their loans relative to the equity return, which is the opportunity cost of funds of the experts that own originators. We henceforth refer to \( \chi^* \) as the equilibrium *intermediary funding discount*. When the intermediary funding discount is large the leverage benefits from issuing risky securities are substantial. The originator finds optimal to choose a large risky promise \( d_I^* \), despite the fact that this creates moral hazard problems and leads to high risk and low expected return loans. When \( \chi^* = 1 \), intermediaries do not offer a discount relative to equity funding, originators do not issue risky securities and loan risk is at its efficient (and minimum) level \((d_I^* = 0 \text{ and } p^* = \bar{p})\).

Combining (27), (28), (30), and using that in equilibrium \( R_E^* = R_{E,O}(d_I^*) \), we get the following equilibrium *equity return equation*

\[
R_E^* = \frac{R_A(\hat{p}(d_I^*)) - A_L + (\chi^* - 1)\hat{p}(d_I^*)d_I^*}{1 - A_L/R_S^*} .
\]

\( \text{E2} \)

\( \)The equation expresses the equilibrium equity return as a function of \( R_S^*, \chi^* \) and \( d_I^* \).

\(^8\)Notice from (27), (28) that \( R_{E,O}(d_I) \) depends on the two endogenous equilibrium returns \( R_S^*, R_I^* \). The variable \( \chi^* \) turns out to be very useful as it characterizes the FOC of the originator’s problem as a function of a single endogenous variable.
Intermediaries’ problem and the funding discount  We now turn to the intermediary’s problem for equilibrium returns $R_S^* < R_I^*$. Recall the definition of the realized return $R_{I,z}$ of risky securities in (10). Using (23), we have that

$$R_{I,L}^* = 0, \quad R_{I,H}^* = R_I^*/p^*.$$  \hfill (31)

An expert setting up and investing its wealth in an intermediary chooses at $t = 0$ a balance sheet tuple $(y, y_S)$ solving the maximization problem (12) - (14). Using (13), the intermediary’s return on equity $R_{E,I}$ can be written as the following function of its asset size, or leverage, $y$:

$$R_{E,I} = R_S^* + (R_I^* - R_S^*)y.$$  \hfill (32)

Since the intermediary earns a spread $R_I^* - R_S^* > 0$ on each unit of investment in risky securities, it chooses maximum leverage, which from (14) and (31) implies

$$R_{S,y_S}^* = \min_\theta E \left[R_{I,z}^*[p^*, \theta] \right] y = \min_\theta \left[ \theta p^* R_{I,H}^* + (1 - \theta p^*) R_{I,L}^* \right] y^* = (1 - \lambda) R_I^* y^*.$$  \hfill (33)

The expression highlights the benefits from diversification: while from (31) the lowest return of risky securities is zero, an intermediary holding a pool of them is able to pledge a fraction $1 - \lambda$ of their expected return for the issuance of safe securities.

Using (32) and (33), the equilibrium intermediary’s equity return can be rewritten as

$$R_{E,I}^* = \lambda R_I^* y^*.$$  \hfill (34)

Equations (33) and (34) capture how the safe and risky parts of the payoffs of the intermediary’s pool of securities are pledged to safe security investors and the expert who holds the intermediary’s equity, respectively.

From (13), (33), (34), and using that in equilibrium $R_{E,I}^* = R_E^*$, we get the following equilibrium intermediary funding discount pass-through equation:

$$\chi^* = (1 - \lambda) \frac{R_E^*}{R_S^*} + \lambda.$$  \hfill (E3)

The equation states that the discount $\chi^*$ originators obtain from intermediaries when financing the risky part of their loans with risky securities instead of equity amounts to the weighted average of the “discounts” with which the intermediaries finance their pools of risky
securities. In fact, in equilibrium a fraction \(1 - \lambda\) of the return of the intermediaries’ assets is financed with safe securities which have a cost advantage relative to equity of \(R_E^* / R_S^*\), while the complementary fraction \(\lambda\) is funded with equity at a discount of 1, that is, at no discount. Importantly, equation E3 implies that when in equilibrium the relative equity spread \(R_E^* / R_S^*\) is high, the intermediary offers a high funding discount to originators. Equation E1 then implies that originators make a large risky promise and their loan risk is high.

**The equity allocation**  Recall that \(E_O^*\) and \(E_I^*\) denote the equilibrium amounts of experts’ funds invested in the equity of originators and intermediaries, respectively. Since \(1 < R_S^* < R_E^*\), experts invest all their endowment in financial firms’ equity, that is:

\[
E_O^* + E_I^* = 1. \tag{35}
\]

The determination of the equity allocation across the two sectors results from two equilibrium conditions. First, the clearing of the market for risky securities, which can be written as:

\[
E_O^* x^*_I = E_I^* y^*, \tag{36}
\]

where the LHS captures the aggregate supply of risky securities by originators and the RHS accounts for its aggregate demand by intermediaries.

Second, the return experts obtain from investing in the equity of originators and intermediaries is the same, which from (28) and (34) implies that:

\[
[R_A(\hat{p}(d_I^*)) - A_L - \hat{p}(d_I^*)d_I^*] x^* = \lambda R_I^* y^*. \tag{37}
\]

We obtain from (24), (35), (36), and (37) the following equilibrium equity allocation equation:

\[
\frac{1 - E_O^*}{E_O^*} = \frac{\hat{p}(d_I^*)d_I^*}{R_A(\hat{p}(d_I^*)) - A_L - \hat{p}(d_I^*)d_I^*} \lambda. \tag{E4}
\]

The equation states that the ratio of equity invested in intermediaries relative to that in originators is the product of two factors. The first one captures how the expected risky payoff of the originators’ loans net of monitoring costs, \(R_A(\hat{p}(d_I^*)) - A_L\), is tranched into risky securities sold to intermediaries, \(\hat{p}(d_I^*)d_I^*\), and equity held by experts, \(R_A(\hat{p}(d_I^*)) - A_L - \hat{p}(d_I^*)d_I^*\). The second factor is the aggregate risk parameter \(\lambda\), that accounts for the fraction of the tranche placed to intermediaries that is funded with equity. Equation E4 implies that
when the expected payoff $\hat{p}(d^*_I) d^*_I$ of the risky securities sold to intermediaries is large, the amount of experts’ funds invested in the equity of intermediaries must also be large. From E1 and E3, that is the case when the relative equity spread $R^*_E/R^*_S$ is large.

**The safe rate** We conclude with the determination of the safe rate. Since $1 < R^*_S < R^*_I$, the entire endowment of the economy is used to invest in originators’ loans, and we have:

$$E^*_O x^* = N^* = w + 1.$$  \hfill (38)

In addition, the market clearing for safe securities can be written as:

$$w = E^*_O x^*_S + E^*_I y^*_S,$$  \hfill (39)

where the LHS includes the demand for safe securities, which coincides with savers’ endowment because $R^*_S < R^*_E$, and the RHS includes its supply by originators and intermediaries.

From (23), we have that the overall supply of safe securities by originators is

$$E^*_O x^*_S = \frac{A_L N^*}{R^*_S},$$  \hfill (40)

and from (33) and (36), we obtain that by intermediaries:

$$E^*_I y^*_S = \frac{(1 - \lambda)\hat{p}(d^*_I) d^*_IN^*}{R^*_S}. $$  \hfill (41)

The expressions above simply discount at the rate $R^*_S$ the overall safe part of the assets’ payoff of the two types of financial firms.

Combining conditions (38) - (41), we obtain the equilibrium *safe rate* equation:

$$R^*_S = \frac{[A_L + (1 - \lambda)\hat{p}(d^*_I) d^*_I](w + 1)}{w}. $$  \hfill (E5)

The equation, which extends that in (21) for the no securitization economy, states that the equilibrium safe rate equals the ratio of overall safe payoffs in the economy and savers’ wealth. The numerator in particular consists of the product of the sum of the per unit of loan safe payoffs pledged to savers directly by originators, $A_L$, and indirectly through intermediaries, $(1 - \lambda)\hat{p}(d^*_I) d^*_I$, times aggregate loan issuance, $w + 1$. Equation E5 shows that securitization, by increasing the supply of safe securities, increases the safe rate relative to that in the benchmark economy. In addition, the equation shows that, for a given $d^*_I$, the safe rate is decreasing in savers’ wealth. This is because as savers’ share of total wealth increases, the share of payoffs pledged to them decreases.
5.2 Equilibrium characterization

The conditions E1-E5 for an equilibrium with safe asset scarcity and full lending presented in the previous section provide a system of five equations in the five variables $d_I^*, R_S^*, R_E^*, \chi^*, E_O^*$. If a solution to such system with $1 < R_S^* < R_E^*$ exists, then the arguments in the previous section allow also to determine the remaining equilibrium variables. If such solution does not exist, then it can be proven that in equilibrium either there is no safe asset scarcity or not full lending. The next proposition follows.

Proposition 5 The equilibrium of the economy is unique up to a Modigliani-Miller type of indifference when there is no equity spread. Let $w$ be savers’ wealth and $\overline{w}$ the constants defined in Proposition 2. Let $R_S^*, R_E^*, p^*, N^*$ be the equilibrium safe rate, return on equity, originator’s risk choice, and aggregate lending, respectively. There exists a constant $\overline{w} \in R^+ \cup \{\infty\}$ satisfying $\overline{w} > \overline{w}$ such that:

(i) If $w \leq \overline{w}$, then there is no securitization and:

$$R_S^* = R_E^* = R_A(\overline{p}), p^* = \overline{p} \text{ and } N^* = w + 1.$$ 

(ii) If $w \in (\overline{w}, \overline{w}]$, then there is securitization and:

$$R_S^* < R_A(\overline{p}) < R_E^*, p^* < \overline{p} \text{ and } N^* = w + 1.$$ 

(iii) If $w > \overline{w}$, then there is securitization and:

$$1 = R_S^* < R_A(\overline{p}) < R_E^*, p^* < \overline{p} \text{ and } N^* \in (N^b, w + 1),$$

where $N^b$ is the equilibrium aggregate lending in the benchmark economy.

The proposition describes how the main equilibrium variables depend on the demand for safety in the economy. Figure 3 illustrates the results in the proposition and also exhibits some other equilibrium variables not discussed in the proposition. When the demand for safety is low ($w \leq \overline{w}$), the originators’ safe payoffs are enough to deliver a high return on safe securities. There is no equity spread and thus no securitization.

When demand for safety is medium ($w \in (\overline{w}, \overline{w}]$), the safe securities supplied by originators become scarce, which gives rise to a positive equity spread (panel a)), but the economy
still achieves full lending (panel b)). Intermediaries emerge to exploit the equity spread by creating safe securities through securitization. As originators pledge to intermediaries a fraction of their risky payoffs, the supply of safe securities increases but monitoring incentives at origination deteriorate, which leads to more loan risk ($p^* < \bar{p}$, panel c)). In this region, the equilibrium is determined by the system of equations E1-E5, which allows us to shed further light on the mechanisms driving the effects of an increase in demand for safety. When $w$ increases, the economy has a larger endowment and aggregate loan issuance increases (panel b)). The safe rate falls in order for the market for safe securities to clear (see E5, illustrated in panel a)). The fall in originators’ funding cost leads to an increase in the equity return (see E2, illustrated in panel a)). As a result, the relative equity spread widens, so that intermediaries are able to offer a larger funding discount to originators (see E3). This in turn leads originators to choose a higher risky promise that makes their loans riskier (see E1, illustrated in panel c)). The fall in the safe rate and the increase in the relative equity spread also lead to an expansion in leverage by originators and intermediaries, respectively (panel d)). In addition, since a larger fraction of the risky part of the originators’ loans is distributed to intermediaries, experts must in equilibrium reallocate some of their equity investments towards these financial firms, where loss-absorption capacity against aggregate risk is needed (see E4, illustrated in full line of panel e)). The equity reallocation exacerbates the increase in loan risk and makes the securitization sector grow faster than aggregate lending in the economy (dotted line of panel e)). The lending expansion is thus fueled by a securitization boom. In addition, the increase in demand for safety directly reduces the safe rate, and indirectly reduces the expected return of originators’ loans, $E[A_z|p^*]$, due to the lower monitoring by originators. Interestingly, the indirect effect may be stronger than the direct one and, following an increase in demand for safety, the spread $E[A_z|p^*] - R^*_S$ between the returns of originators’ loans and safe securities may fall (panel f)). Finally, when demand for safety becomes very large ($w > \bar{w}$), the safe rate falls to one and some savers opt to consume their endowment at the initial date.

**Saving gluts and the run-up to the crisis** The equilibrium results described above provide, to the best of our knowledge, the richest set of implications consistent with what some economists refer to as the “global saving glut” narrative of the run-up to the past
Figure 3: Equilibrium with intermediaries

Notes: Returns on equity $R^*_E$ (solid with dots) and safe securities $R^*_S$ (solid), aggregate lending $N^*$ (solid), risk choice $p^*$ (solid), leverage of originators $x^*$ (solid) and intermediaries $y^*$ (solid with dots), equity in intermediaries $E^*_I$ (solid) and size of securitization relative to aggregate lending $E^*_I y^*/N^*$ (solid with dots), and loan spread $E[A_z|p^*] - R^*_S$, as a function of the savers’ aggregate wealth $w$ for the equilibrium with intermediaries. In panels a), b) and c) dotted lines correspond to the benchmark economy without intermediaries.
According to this view, an increase in the demand for safe assets in the global economy exerted downward pressure on interest rates and led to a boom in credit fueled by securitization and to the deterioration of lending standards, thereby sowing the seeds for the financial crisis.\(^9\)

Specifically, the following list of empirical findings for the period 2002-2007 that support the saving glut hypotheses of the run-up to the crisis are consistent with the implications of an increase in demand for safety in our model:

i) Low real interest rate in safe assets (Bernanke et al. (2011), Caballero et al. (2017)).

ii) Rise in expected equity returns and expected equity premium (Duarte and Rosa (2015), Caballero et al. (2017), Loeys et al. (2005)).


iv) Expansion of financial institutions balance sheets and leverage (Adrian and Shin (2009), Adrian and Shin (2010)).

v) Securitization sector expansion at a higher rate than the overall economy (Adrian and Shin (2010), Merrill et al. (2017)).

vi) Compression of spread between risky loans and safe assets, which combined with the evidence in ii) cannot be due to a market-wide reduction in risk premia (Bernanke et al. (2011), Bolton et al. (2018), Loeys et al. (2005)).

\(^9\)The following excerpt from Bernanke et al. (2011) provides a more complete description of the saving glut hypotheses: “The strong demand for apparently safe assets by both domestic and foreign investors not only served to reduce yields on these assets but also provided additional incentives for the U.S. financial services industry to develop structured investment products that “transformed” risky loans into highly-rated securities. Finally, the demand for safe assets by investors, both domestic and foreign, appears to have engendered a strong supply response from U.S. financial firms. In particular, even though a large share of new U.S. mortgages during this period were of lower credit quality, such as subprime loans, agency guarantees and financial engineering in the private financial services industry resulted in the overwhelming share of mortgage-related securities being rated AAA.”
Equilibrium effects from emergence of securitization  To conclude this section, we analyze the impact of the emergence of securitization on the utilities of the two types of agents and on aggregate consumption by comparing these equilibrium variables in the baseline economy (with intermediaries) and in the benchmark economy (without intermediaries). Since investors have linear utilities with a zero discount rate, the equilibrium expected utility of savers and experts in the economy with intermediaries coincide with the expected return on safe securities, $R^*_S$, and equity, $R^*_E$, respectively.\(^{10}\)

Aggregate expected consumption in both periods net of monitoring costs, which we denote with $C^*$ and refer to as aggregate net expected consumption, satisfies

$$C^* = (w + 1 - N^*) + N^*E[A_z|p^*] - N^*c(p^*). \quad (42)$$

The expression takes into account that aggregate net expected consumption at $t = 0$ coincides with the amount of funds that are not invested in originators’ loans (first term in RHS) and at $t = 1$ coincides with the payoff of those loans net of the monitoring costs incurred by some experts (whose expected value amounts to the second term in RHS). Notice that by construction we have the following identity:

$$C^* = wR^*_S + R^*_E. \quad (43)$$

We denote with a superscript $b$ the analogous variables for the benchmark economy with no intermediaries studied in Section 4. The effects due to the emergence of securitization on savers and experts’ utility, and aggregate net consumption, can then be defined as the following differences:

$$\Delta R_S = R^*_S - R^*_S^b, \Delta R_E = R^*_E - R^*_E^b \text{ and } \Delta C = C^* - C^b.$$

Using (16), we can write $\Delta C$ as:

$$\Delta C = (N^* - N^b)(R_A(p^*) - 1) - N^b(R_A(\bar{p}) - R_A(p^*)). \quad (44)$$

The first term in the expression above accounts for the net consumption gains stemming from the expansion in aggregate lending allowed by the additional safe securities created by intermediaries ($N^* \geq N^b$, from Propositions 2 and 5). The second term accounts for the net

\(^{10}\)Notice that the statement is true also if $R^*_S = 1$, in which case some savers may consume their endowment at $t = 0$, or if $R^*_S = R^*_E$, in which some experts may invest in safe securities at $t = 0$.\]
consumption costs implied by the worsening of loans’ quality induced by the emergence of intermediaries ($p^* \leq p^b = \bar{p}$, from Proposition 5). We have the following formal result.

**Proposition 6** Let $w$ be savers’ wealth and $w, \bar{w}, \overline{\bar{w}}$ the constants defined in Propositions 2 and 5, which satisfy $w < \bar{w} < \overline{\bar{w}}$. Suppose that $w > w$. Let $\Delta R_S$, $\Delta R_E$, and $\Delta C$ be the effects from the emergence of securitization on savers and experts’ utility, and on aggregate net consumption. They satisfy:

(i) Savers’ utility: $\Delta R_S \geq 0$ for any $w$ and $\Delta R_S > 0$, if and only if $w \in (w, \bar{w})$.

(ii) Experts’ utility: there exists $w'_E \in (\bar{w}, \overline{\bar{w}}]$ such that $\Delta R_E < 0$ if $w < E'$ and $\Delta R_E > 0$ if $w > w'_E$. In addition, if $\overline{\bar{w}} < \infty$ then $w'_E < \overline{\bar{w}}$.

(iii) Aggregate net expected consumption: there exists $w' \in (\bar{w}, \overline{\bar{w}}]$ with $w' \leq w'_E$ such that $\Delta C < 0$ if $w < w'$ and $\Delta C > 0$ if $w > w'$. In addition, if $\overline{\bar{w}} < \infty$ then $w' < w'_E < \overline{\bar{w}}$.

The proposition describes the effects associated with the emergence of securitization. Panel a) in Figure 3 illustrates the results for the utility of savers and experts. Savers’ always weakly benefit from the entry of intermediaries and experts only benefit if the demand for safety is high. If demand for safety is not so low ($w > w$), a positive spread arises, intermediaries enter in the economy and the expansion of the supply of safe securities increases savers’ utility. However, this leads to the worsening in the quality of originators’ loans ($p^* < \bar{p}$) and the impact on experts’ utility and aggregate net consumption depend on how much additional loan issuance is allowed by securitization. For a medium demand for safety ($w < w < w'$), the increase in aggregate lending is not very large (if at all), the quality effect dominates and aggregate net consumption falls. The increase in savers’ utility is thus more than offset by a reduction in the utility of experts. The securitization technology increases competition among experts and deprives them of some of the scarcity rents they enjoyed in the benchmark economy. When the demand for safety is sufficiently large ($w > w'$), the quantity effect dominates and the entry of intermediaries increases aggregate net consumption. At higher values of the demand for safety ($w > w'_E > w'$), the quantity effect is so important that experts’ welfare increases with the entry of intermediaries. In this

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11The proposition focuses on the case $w > w$, because otherwise there is no safe asset scarcity and the equilibria of the two economies coincide.
region, securitization leads to a Pareto improvement in the economy despite the increase in originators’ loan risk it induces.

6 Constrained efficiency of the equilibrium

In this section, we describe the problem of a constrained Social Planner (SP) and show that constrained versions of the Welfare Theorems hold in this economy. We consider a SP that at \( t = 0 \): i) decides which experts set up, manage and hold the residual claim of originators and intermediaries, and which experts remain passive, ii) allocates agents’ funds into originators’ loans and initial date consumption, iii) decides the safe securities that originators issue and distributes them between savers and passive experts, iv) decides the risky securities that originators issue and distributes diversified pools of them to intermediaries, and v) decides the safe securities that intermediaries issue and distributes them between savers and passive experts. Notice that a) the residual claims held by the experts managing financial firms, and b) the distribution of safe securities between savers and passive experts, totally describe how the originators’ loan pay-offs at \( t = 1 \) are allocated for consumption across agents. The SP is constrained insofar as she cannot choose the originators’ loans risk and she can only distribute riskless securities to savers. For the sake of brevity, we refer to the three expert groups created by the SP decisions as originators, intermediaries, and passive. Notice that due to constant returns to scale in the financial firms’ technologies and to the linearity in utility from consumption, the actual measures of experts in each group is irrelevant for the aggregate welfare of this class of agents.  

Formally, a SP allocation, which we denote with \( \gamma \), is described by: the aggregate loan issuance by originators \( N \in [0, w + 1] \), their loan risk choice \( p \), the per unit of loan promises \( d_S, d_I \) made by originators on safe and risky securities, the per unit of loan promise \( b_S \) made by intermediaries on safe securities, the aggregate consumption at \( t = 0 \) of savers, passive experts, originators and intermediaries, \( (C_{S,0}, C_{P,0}, C_{O,0}, C_{I,0}) \), the aggregate consumption at \( t = 1 \) of savers, \( C_{S,1} \), and passive experts, \( C_{P,1} \), and, for each aggregate shock \( \theta \), the aggregate consumption at \( t = 1 \) net of monitoring costs of originators, \( C_{O,1}(\theta) \), and of intermediaries, \( C_{I,1}(\theta) \).

\(^{12}\)The only restriction is that there is a continuum of originators and at least one intermediary to take advantage of the possibility to diversify idiosyncratic loan risks.
An allocation \( \gamma = (N, p, d_S, d_I, b_S, C_{S,0}, C_{P,0}, C_{O,0}, C_{I,0}, C_{S,1}, C_{P,1}, (C_{O,1}(\theta))_\theta, (C_{I,1}(\theta))_\theta) \) is *constrained feasible* if it satisfies the following properties:

- Originators always totally repay safe promises and totally repay risky promises if their loans’ return is high:
  \[
  d_S \leq A_L, \tag{45}
  \]
  \[
  d_S + d_I \leq A_H. \tag{46}
  \]

- The originators’ risk choice \( p \) coincides with that maximizing their residual claim:
  \[
  p = \arg \max_{p'} \{ E [\max\{ A_z - d_S - d_I, 0\}] | p' - c(p') \}. \tag{47}
  \]

- Intermediaries always repay safe promises:
  \[
  b_S \leq \min_\theta (\theta p d_I + (1 - \theta p) \min\{d_I, A_L - d_S\}) , \tag{48}
  \]
  which takes into account that intermediaries hold diversified portfolios of risky securities issued by originators.

- Aggregate consumption at \( t = 0 \) equals the amount of funds that are not invested in originators’ loans:
  \[
  C_{S,0} + C_{P,0} + C_{O,0} + C_{I,0} = w + 1 - N. \tag{49}
  \]

- Aggregate consumption at \( t = 1 \) of savers and passive experts equals the overall payoff of safe securities:
  \[
  C_{S,1} + C_{P,1} = (d_S + b_S) N. \tag{50}
  \]

- For each \( \theta \), aggregate net consumption of originators at \( t = 1 \) equals their residual claim net of monitoring costs:
  \[
  C_{O,1}(\theta) = [\theta p (A_h - d_I - d_S) + (1 - \theta p) (\max\{A_L - d_I - d_S, 0\}) - c(p)] N. \tag{51}
  \]

- For each \( \theta \), aggregate consumption of intermediaries at \( t = 1 \) equals their residual claim:
  \[
  C_{I,1}(\theta) = [\theta p d_I + (1 - \theta p) \min\{d_I, A_L - d_S\} - b_S] N. \tag{52}
  \]
We next define the constrained Pareto frontier of the economy. In order to do so, we compactly denote with $\Gamma$ the set of constrained feasible allocations. For given weights $\omega_S \geq 0$ assigned by the SP to the utility of savers and $\omega_E \geq 0$ to that of experts, with $\omega_S + \omega_E > 0$, we define the weighted welfare induced by an allocation $\gamma \in \Gamma$ as:

$$W_{\omega_S, \omega_E}(\gamma) \equiv \omega_S(C_{S,0} + C_{S,1}) + \omega_E(C_{P,0} + C_{O,0} + C_{I,0} + C_{P,1} + E[C_{O,1}(\theta) + C_{I,1}(\theta)|p]).$$ (53)

We say that $\gamma \in \Gamma$ is constrained efficient if it solves the problem

$$\max_{\gamma \in \Gamma} W_{\omega_S, \omega_E}(\gamma).$$ (54)

Finally, the constrained Pareto frontier of the economy is defined as the set of allocations that are constrained efficient for some weights $\omega_S, \omega_E \geq 0$, with $\omega_S + \omega_E > 0$.

For given weights $\omega_S, \omega_E$, it can be easily proven from the definition of constrained feasibility of an allocation in (45) - (52) and of the weighted welfare function in (53), that for any $\gamma \in \Gamma$ there exists $\gamma' \in \Gamma$ such that $W_{\omega_S, \omega_E}(\gamma') = W_{\omega_S, \omega_E}(\gamma)$ and the safe promise constraints (45) and (48) are binding.\(^{13}\) We can hence, with no loss of generality, focus the characterization of the constrained Pareto frontier to constrained feasible allocations in which the safe promise constraints are binding. Notice that for those allocations, we have from (47) that the incentive compatible risk of originators’ loans is given by the function $\hat{p}(\cdot)$ defined in Lemma 4.

Finally, using the aggregate consumption constraints (49) - (52), the definition of the weighted welfare in (53) and the definition of $R_A(p)$ in (16), a constrained efficient allocation for given weights $\omega_S, \omega_E$ can be simply described by a choice of aggregate loan issuance, $N$, risky promise by originators, $d_I$, and aggregate consumption of savers at each of the dates, $C_{S,0}, C_{S,1}$, that solves the following problem:

$$\max_{(N, d_I, C_{S,0}, C_{S,1})} \omega_E(w + 1 - N + R_A(\hat{p}(d_I))N) + (\omega_S - \omega_E)(C_{S,0} + C_{S,1}),$$ (55)

\(^{13}\)The intuition is that the SP has flexibility on how to allocate safe consumption at $t = 1$ to experts. Allocations with binding safe promise constraints (45) and (48) leave no safe part on the residual claim of originators and intermediaries, so that all the safe consumption of experts at $t = 1$ is allocated through the safe securities distributed to passive experts.
subject to

\[ N \leq w + 1, \quad (56) \]
\[ d_I \leq \Delta, \quad (57) \]
\[ C_{S,0} \leq w + 1 - N, \quad (58) \]
\[ C_{S,1} \leq [A_L + (1 - \lambda)\hat{p}(d_I)d_I]N. \quad (59) \]

The objective function of the SP in (55) is weighted welfare rewritten as the sum of aggregate net consumption of the two agent types, loaded with weight $\omega_E$, and the aggregate consumption of savers, loaded with (positive or negative) weight $\omega_S - \omega_E$. The first two constraints in the SP problem simply capture the upper bounds on $N$ and $d_I$. The last two constraints state that savers’ consumption at each date is bounded by the availability of safe pay-offs. In addition, it is easy to prove that constraint (58) is binding in any solution to the SP problem. The reason is that experts can issue positive NPV loans, so that allocating them consumption at the initial date reduces (weighted) welfare.

We next describe how the efficient allocations depend on the SP weights $\omega_S, \omega_E$.$^{14}$ Figure 4 graphically exhibits the aggregate net consumption of the two agent types in the Pareto constrained efficient set of the economy. When the SP gives a lower weight to savers than experts ($\omega_S \leq \omega_E$), the second term in (55) is (weakly) negative and we trivially have that constrained efficient allocations maximize (unweighted) aggregate expected net consumption and thus satisfy $N = w + 1, d_I = 0$. $^{15}$ In this case, there is no securitization, full lending is achieved and loan risk is efficient. This part of the Pareto frontier is labeled as region I in Figure 4. Notice that it consists of a straight line with slope minus one, since increases in savers’ consumption are achieved by allocating them a larger fraction of the safe return $A_L$ of the loans and reducing by the same amount that allocated to experts.

When the SP weights savers more than experts ($\omega_S > \omega_E$), constraint (59) must be necessarily binding in any constrained efficient allocation. The SP thus faces a trade-off between maximizing aggregate net consumption (first term in (55)) and maximizing the available safe pay-offs that are consumed by savers (second term in (55) after plugging in the binding constraints (58) and (59)). In fact, we have the following FOC for an optimal

$^{14}$For a formal proof of some of the statements made in this discussion see the proof of Proposition 7.
$^{15}$In addition, if $\omega_S < \omega_E$ savers’ consumption is zero under the constrained efficient allocation.
choice of risky promise $d_I$:

$$\left( \frac{\omega_S}{\omega_E} - 1 \right) (1 - \lambda) \frac{d (\hat{p}(d_I)d_I)}{dd_I} + \frac{dR_A(\hat{p}(d_I))}{dd_I} = 0. \quad (60)$$

The FOC above shows that the SP faces a quantity versus quality trade-off in its risky promise choice $d_I$ whenever an allocation exhibits some loan issuance. First, an increase in $d_I$ increases the expected pay-off of the risky securities distributed to intermediaries. Due to diversification, a fraction $1 - \lambda$ of the expected pay-off of those securities becomes safe. The quantity of safe pay-offs increases, which allows to increase the consumption of savers at $t = 1$. Second, an increase in $d_I$ reduces the monitoring incentives of the originators, whose loans become of worse quality and have lower net expected return. The larger the ratio $\omega_S/\omega_E$ the more important the quantity effect becomes relative to the quality effect, and the higher the risky promise $d_I$ associated with the SP efficient allocation. The economy creates more safe pay-offs at the cost of worse quality (riskier) loans.

For an intermediate range of values of $\omega_S/\omega_E > 1$, it is optimal to allocate all the endowment of the economy to loan issuance. This part of the Pareto frontier is labeled as region II in Figure 4. As the ratio $\omega_S/\omega_E$ increases, aggregate net consumption of the two agent types under the constrained efficient allocation moves rightwards along the frontier. Notice that the absolute value of the slope of the frontier in this region is bigger than one and increasing as we move rightwards and more consumption is allocated to savers. This is because the expansion of safe payoffs requires an increase in the the risky promise $d_I$, which reduces loan quality and expected total payoffs net of monitoring costs. The increase in consumption by savers thus implies a larger reduction in net consumption by experts. When $\omega_S/\omega_E$ becomes sufficiently large, allocating consumption to savers through the manufacturing of safe pay-offs might lead to such a bad loan quality, that the SP find optimal to allocate some of the economy’s endowment to savers’ consumption at $t = 0$ instead of issuing loans. This part of the Pareto frontier is labeled as region III in Figure 4.\footnote{This region, in which there is no full lending, exists if and only if $A_L + (1 - \lambda) \max_{d_i} \hat{p}(d_I) < 1$, that is, if and only if the maximum amount of safe payoffs that can be created per unit of loan is strictly below one.}
The quantity of safe pay-offs versus loan quality trade-off faced by the SP in its $d_I$ choice presented in (60) is analogous to the leverage versus loan quality trade-off faced by the originator in its $d_I$ choice presented in E1. In fact, using the equilibrium funding discount pass-through equation E3, it is immediate to check that equations (60) and E1 are equivalent provided that the SP weights $\omega_S, \omega_E$ and the equilibrium returns of the baseline economy $R^*_S, R^*_E$ satisfy

$$\frac{\omega_S}{\omega_E} = \frac{R^*_E}{R^*_S}. \quad (61)$$

The above property suggests that the competitive equilibrium outcome is Pareto constrained efficient. Conversely, if after some initial date transfers across investors any possible equity spread $R^*_E/R^*_S$ can be induced, then all the Pareto constrained efficient allocations would be achieved as an equilibrium outcome of the economy. Building on these intuitions we obtain the following result.

**Proposition 7** The equilibrium of the economy belongs to the Pareto constrained efficient set. Any allocation in the Pareto constrained efficient set can be achieved as the equilibrium of the economy following some transfers across investors at the initial date.

The reason why Welfare Theorems hold in this economy is that experts can freely set-up and invest in originators and intermediaries, which issue safe and risky securities in competitive markets, so that in equilibrium the returns of the funding sources lead experts internalize the same trade-off between increasing safe pay-offs and worsening loan quality as the as the SP does. For any distribution of wealth across agents, the competitive equilibrium is thus Pareto constrained efficient. And conversely, by making sufficiently large lump-sum transfers from savers to experts (experts to savers) at the initial date the ensuing competitive equilibrium spans the left (right) part of the Pareto frontier illustrated in Figure 4. \(^{17}\)

7 **The demand for safety paradox**

In this Section, we further study the relationship between the demand for safety in the economy and the risk of originated loans by comparing the equilibrium of our baseline economy, in which savers demand safety, with another economy in which savers are willing to buy risky

\(^{17}\)In contrast, it can be proved that when experts’ equity allocation is exogenously fixed the resulting equilibrium is not necessarily constrained efficient.
securities. We henceforth refer to each of the economies as demand for safety economy and risky funding economy. The analysis shows that a \textit{demand for safety paradox} may emerge, namely that equilibrium loan risk in the demand for safety economy might be higher than in the risky funding economy. In other words, when savers demand safety the economy might originate worse quality loans than when savers are willing to bear risk. This result shows that credit expansions fueled by demand for safe assets might create more financial stability risks than standard credit booms, and contribute to a growing literature emphasizing the macroeconomic from safe assets shortages (see Caballero et al. (2017)).

Consider an alternative version of the model in which savers are willing to buy risky securities and derive linear utility from their returns. The rest of the model is left invariant. In particular, originators’ are exposed to the same moral hazard problem when they issue risky securities, which in this economy can be directly placed to savers. There is thus no need for the manufacturing of safe assets through securitization and intermediaries do not emerge.

We are interested on how the equilibrium loan risk in this economy compares to that in the demand for safety economy. In order to get intuitions of the forces driving the comparison it is convenient to do the following conceptual exercise. Consider the equilibrium of the

Figure 4: Constrained Pareto frontier
safety demand economy for some savers’ wealth \( w \in (w, \bar{w}) \) such that safe securities are scarce, securitization emerges and the economy features full lending. Notice that since there is aggregate risk in the economy, an amount \( E^*_I > 0 \) of the experts’ endowment is invested in intermediaries where it provides the necessary loss-absorption capacity to create safe securities. Suppose that savers become willing to purchase risky securities. We split the transition to the equilibrium of the risky funding economy into two steps: i) holding fixed experts’ investments \( E^*_I \) in the equity of the financial firms in the equilibrium of the safety demand economy, and ii) allowing afterwards experts to reallocate their investments among financial firms.

Consider thus a first step in which the equilibrium equity allocation of the demand for safety economy is fixed. In the demand for safety economy, originators’ inability to directly pledge risky payoffs to savers constrains their demand for external funds. The limited amount of loan payoffs that originators can (indirectly) pledge to raise funds from savers, leads to a low return on the savers’ investments \( R^*_S \). The overall promise made by originators to obtain the \( w + E^*_I \) units of funds they raise from outside investors is not very large, and hence loan risk is not very large either. When savers are willing to buy risky securities, the pledgeability of originators’ loans increases and financing constraints get relaxed. This leads to an increase in originators’ demand for external risky funding and so in the return paid on external funding. So that, in the new equilibrium, originators make a higher overall promise to obtain the \( w + E^*_I \) from external creditors—savers and the experts locked in intermediaries—whose only investment opportunity is to provide external funding to the set-up originators. The higher overall risky promise implies higher loan risk due to the moral hazard problem in monitoring. At the end of this first step, savers’ willingness to buy risky securities has led to higher loan risk. Notice that the driving force is an increase in originators’ demand for external funds, which is reminiscent of the intuition for why the introduction of the securitization technology in the region \( w \in (w, \bar{w}) \), by relaxing the financial constraints in the economy, leads to lower experts’ rents, higher external promises and more loan risk.

Consider a second step in which experts can freely allocate their equity investments in financial firms. Since intermediaries do not create any more value in the economy, their return on equity equals that of safe and risky securities, and is below the equity return of originators. Experts thus reallocate their equity funds on intermediaries towards originators.
Since equity provides skin-in-the-game incentives at origination, this effect reduces loan risk in the risky funding economy relative to that in the demand for safety economy. The following proposition shows that the equity reallocation effect might dominate the increase in competition effect, and the safety paradox emerge.

**Proposition 8** There exist pairs of aggregate risk $\lambda$ and savers’ wealth $w$ such that the safety demand paradox emerges, that is, equilibrium loan risk in the baseline economy is higher than equilibrium loan risk in an economy in which savers are willing to invest in risky securities.

The possibility that a demand for safety paradox emerges for some values of the parameters of the model is illustrated in Figure 5. The figure exhibits how loan risk in the safety demand and risky funding economies, which are denoted with $p^*$ and $p^r$, respectively, depend on savers’ wealth $w$. When savers’ wealth exceeds the level $\underline{w}$ above which safe assets become scarce, a positive spread between equity and both safe and risky securities arises in the two economies. As a result, originators find optimal to issue risky securities and loan risk increases in the two economies ($p^* < \overline{p}, p^r < \overline{p}$). Due to the reallocation of equity from originators to intermediaries required for securitization, the increase in loan risk is initially higher in the demand for safety economy and the demand for safety paradox emerges ($p^* < p^r$). Only for very high values of savers’ wealth, the risky funding economy induces higher loan risk than the baseline economy.

This section highlights that credit expansions driven by demand for safety are different from other credit expansions and may, paradoxically, lead to higher credit risk. These results complement the emphasis in the extant literature that demand for safety may increase financial sector fragility due to refinancing risk (e.g., Caballero and Krishnamurthy (2009), Stein (2012), Moreira and Savov (2017)).

**8 Government guarantees and loan risk**

In this section, we consider a government with safe resources at $t = 1$ and analyze how it can use them to provide fiscally neutral guarantees to support the issuance of safe securities. We find that guarantees, if suitably combined with lump-sum transfers across agents at $t = 0$,
increase the utility of both savers and experts. Interestingly, their impact on originators’ loan risk is ambiguous.

We assume throughout the section that savers’ wealth satisfies \( w > \bar{w} \), which from Proposition 5 implies that safe securities are scarce in the baseline economy. We extend the baseline model to include a risk-neutral government with some assets whose pay-offs at \( t = 1 \) are \( X > 0 \). The government can use its assets to provide at the initial date guarantees to support the issuance of safe securities by financial firms. The guarantees are fiscally neutral, that is they are repaid by financial firms in expectation. For the sake of brevity, we focus only on guarantees to intermediaries but it is possible to prove that guarantees to originators would constitute an equivalent policy.

A guarantee to intermediaries is described by an aggregate shock threshold \( \overline{\theta} \in [1 - \lambda, 1] \), and transfers \( T_{\overline{\theta}}(\theta, y|R_I) \) at \( t = 1 \) from the government to each intermediary conditional on the realization \( \theta \) of the aggregate shock, the intermediary’s size \( y \), and the market return \( R_I \), given by

\[
T_{\overline{\theta}}(\theta, y|R_I) = \min(\overline{\theta} - \theta, 0)R_Iy. \tag{62}
\]

The after guarantees safe payoff of an intermediary of size \( y \) amounts to \( \theta R_Iy + T_{\overline{\theta}}(\theta, y|R_I) \) and from (62) satisfies

\[
\min_{\theta \in [1 - \lambda, 1/p_{\text{max}}]} (\theta R_Iy + T_{\overline{\theta}}(\theta, y|R_I)) = \overline{\theta} R_Iy. \tag{63}
\]
A guarantee with threshold $\theta$ thus allows the intermediary to pledge a fraction $\bar{\theta} \geq 1 - \lambda$ of the return of its assets to issue safe securities, instead of the fraction $1 - \lambda$ that the intermediary can pledge in the baseline model. The guarantees with threshold $\bar{\theta} = 1 - \lambda$ and $\bar{\theta} = 1$, correspond to the cases of no guarantees and full guarantees, respectively.

Intermediaries repay in expected terms the guarantee of the government out of their residual claim when the aggregate shock satisfies $\theta \geq \bar{\theta}$. Since by construction, for a given balance sheet tuple $(y, y_S)$ and returns $R_S, R_I$, the introduction of a fiscally neutral guarantee does not affect the value of the intermediary’s expected residual claim, we have that the only change to the intermediary’s problem analyzed in Section 5.1 is the replacement of the maximum safe debt constraint in (33) with

$$R_S y_S \leq \bar{\theta} R_I y.$$  \hfill (64)

The guarantee can thus be interpreted as a technological change that reduces the aggregate risk parameter to which the intermediaries are exposed from $\lambda$ to $1 - \bar{\theta}$.

The government can also conduct lump-sum transfers across agent types at $t = 0$ that are described by the (positive or negative) amount of funds $\tau \in [-1, w]$ transferred from savers to experts. We refer to a pair $(\bar{\theta}, \tau)$ of guarantees and lump-sum transfers as a government policy, and say that a policy is feasible if the competitive equilibrium of the economy it induces satisfies the following government’s resource constraint:

$$E^*_I (\theta - 1 + \lambda) R^*_I y^* \leq X.$$  \hfill (65)

The LHS of this inequality accounts for the overall disbursement made by the government to satisfy the guarantees to intermediaries conditional on the worst aggregate shock, $\theta = 1 - \lambda$. The RHS includes the government safe payoffs at $t = 1$. Since the government disbursement is decreasing in the realization of the aggregate shock $\theta$, the government satisfies guarantees for any $\theta$ if and only if (65) holds.

Finally, a feasible government policy is optimal if it induces an equilibrium outcome that:  
1) weakly increases the utility of each agent type relative to the equilibrium of the no intervention policy, and  
2) is not Pareto improved by the equilibrium induced by any other feasible policy.

\footnote{It is easy to check that the expected residual claim of intermediaries conditional on $\theta \geq \bar{\theta}$ is above the expected cost for the government of any guarantee with threshold $\bar{\theta} \leq 1$.}
We next discuss how government policies increase the utility of the two agent types. In absence of a government intervention, the economy exhibits scarcity of safe securities or, equivalently, of experts’ funds. This scarcity leads to loan risk above its first-best level (due to the emergence of securitization) and may also avoid full lending. The two effects reduce the overall amount of net consumption the economy can allocate to savers and experts. A government that has safe resources and grants guarantees to intermediaries provides additional loss absorption capacity in the economy, mitigating the scarcity of experts’ funds. From our discussion above, government guarantees can be interpreted as a technological improvement that reduces the exposure of intermediaries to aggregate risk. In fact, the constrained SP problem in (55) - (59) implies that a reduction on the aggregate risk parameter expands the Pareto frontier of the economy. Taking into account that a constrained version of the Second Welfare Theorem holds (Proposition 7), guarantees combined with a suitable range of lump-sum transfers across agent types at $t = 0$ allow to increase the utility of both savers and experts relative to that in the baseline economy. The next result formalizes this statement and provides additional properties of the optimal policies.

**Proposition 9** Consider a demand for safety $w$ satisfying $w > w$, where $w$ is defined in Proposition 2. Optimal policies Pareto improve the equilibrium of the baseline economy. Besides:

- **If the baseline economy exhibits full lending (that is, if $w \leq \bar{w}$), optimal policies reduce loan risk, but they never induce its efficient level.**

- **If the baseline economy does not exhibit full lending (that is, if $w > \bar{w}$), optimal policies increase lending and, for $X$ small, increase loan risk.**

The proposition states that optimal policies Pareto improve utility in the economy. This is because they provide a substitute for the loss absorption role of intermediary’s equity, which reduces equity scarcity and expands the Pareto frontier of the economy. The expansion of the Pareto frontier is illustrated in Figure XX.

Proposition 9 also provides some results on how optimal policies affect originators’ loan risk. When the baseline economy exhibits full lending, the only way of increasing utility of the two agent types is by reducing loan risk, so that optimal policies must necessarily achieve
a reduction in loan risk. The efficient loan risk is nevertheless never induced, because some securitization must be conducted under optimal policies. Otherwise, no guarantees would \textit{de facto} be granted in equilibrium, and the economy cannot be Pareto improved by means of lump-sum transfers only (Proposition 7). When the baseline economy does not exhibit full lending, the equilibrium safe rate equals one. Guarantees on intermediaries with a zero lump-sum transfer increase aggregate loan issuance by expanding the supply of safe securities but are not able to increase the safe rate above one when government resources are small. In this case, experts fully appropriate the benefits from the increase in aggregate lending stemming from the guarantees. This implies that the equilibrium return on equity and the intermediary funding discount increase, giving incentives to originators to issue more risky securities and inducing higher loan risk at origination.\textsuperscript{19} Pareto optimal policies in this case must thus include a weakly positive lump-sum transfer from experts to savers, which further increases loan risk.

9 Conclusion

We present an equilibrium model of the capital structure and risk-taking in the originate-to-distribute intermediation chain in presence of absolute demand for safety by some investors and limited endowment by equity investors. Loan originators can finance the risky part of their assets through equity or by obtaining funding from intermediaries. The latter implies the off-balance sheet transfer of risk and worsens originators’ risk-taking incentives. Intermediaries can pool the acquired idiosyncratic risks to issue safe securities and expand their balance sheets. Yet, the presence of aggregate risk implies that intermediaries rely on equity to do securitization. Equity investment in the intermediation chain serves two different purposes. At origination, it provides experts skin-in-the game that increases their incentives to monitor the loans. At intermediation, equity is a cushion for aggregate risk losses.

Following an increase in the demand for safety, the model predicts a securitization boom. The demand for safe assets leads to the reallocation equity from originators to intermediaries and implies an increase in leverage along the intermediation chain, the relative size of the

\textsuperscript{19}Originators’ loan risk increases because the intermediary funding discount, which from E1 determines it, increases in equilibrium. The latter is the result of two effects that can be observed in the intermediary funding discount pass through equation in E3: first, the increase in $R^*_E/R^*_S$ increases $\chi^*$; second, the guarantee on the intermediary is equivalent to a reduction in $\lambda$, which also increases $\chi^*$. 
intermediary sector and risk-taking at origination. We thus provide a single framework that captures the main features emphasized by the saving glut narrative of the run-up to the crisis.

We show that the frictionless capability to allocate equity between originators and intermediaries and the existence of competitive markets for safe and risky securities ensure the validity of constrained versions of the welfare theorems. The competitive equilibrium of the economy is constrained Pareto efficient and any allocation in the constrained Pareto frontier can be achieved as the competitive equilibrium of the economy following some redistribution of wealth across investors’ types at the initial date.

We analyze the welfare implications of the emergence of securitization by comparing the originate-to-distribute economy relative to a traditional originate-to-hold benchmark. Securitization leads to the following general quantity versus quality trade-off. On the one hand, the distribution of risks out of originators leads to more risk-taking and reduces aggregate surplus. On the other hand, the expansion of safe securities supply increases aggregate lending when in the traditional economy all endowment cannot be channeled to finance loans, which increases aggregate surplus. We find that the aggregate lending effect on total surplus dominates if and only if the demand for safety is sufficiently large. Instead, if the demand for safety is not large enough, excessive risk-taking leads to aggregate losses and implies redistributive effects. Safety investors always benefit from the increased supply of safe assets. In contrast, the possibility to engage in securitization increases competition in the supply of safe securities and ends up depriving equity investors of some of the scarcity rents they enjoyed in the traditional financial sector.

We also show that when a government has safe resources, fiscally neutral public guarantees to the issuance of securitized assets can reduce the scarcity of safe securities in the economy, and lead to Pareto improvements in welfare if properly combined with lump-sum transfers across investors’ types. Besides, these policies are preferrable to the introduction of guarantees to originators because of the higher exposure of intermediaries’ assets to aggregate risk. Despite Pareto improving welfare in the economy, the effect of these policies on risk-taking at origination is ambiguous. These results shed new light on the equilibrium effects of public guarantees to the financial sector, the need to combine them with other redistributive policies and their interplay with risk-taking at origination.
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This appendix contains the proofs of the formal results included in the body of the paper. We start defining the share of savers’ wealth $\mu = w/(1+w)$. For simplicity, in this appendix, we express some equilibrium results as function of $\mu$ instead of $w$. We thus define: $\mu = w/(1+w)$ and $\bar{\mu} = \bar{w}/(1+\bar{w})$.

**Proof of Lemma 1** Recall that $R_A(\bar{p}) > 1$ from Assumption 2. Suppose that $R_b < 1$. The demand for safe securities would be zero. Since $R_A(\bar{p}) > R_b$, we have from (17) that originators would borrow as much as possible and $R_{E,O} > R_A(\bar{p})$. Hence, all experts would find optimal to set-up originators and invest in them, so that the supply of safe securities would be strictly positive. The market for safe securities would not clear.

Suppose that $R_b > R_A(\bar{p})$. Since $R_A(\bar{p}) > 1$ savers would invest their entire endowment in safe securities and the demand for these assets would be strictly positive. From (17) originators would not find optimal to issue safe securities, so that their supply would be zero. The market for safe securities would not clear.

**Proof of Proposition 2** We proceed in a sequence of steps. Define $w = \frac{A_L}{R_A(\bar{p}) - A_L}$, which implies $\mu = A_L/R_A(\bar{p})$.

a) $R_S^b \in (1, R_A(\bar{p}))$ is the safe rate of an equilibrium if and only if $R_S^b = \frac{A_L}{\mu}$, and in that case $N^b = 1 + w$.

Suppose $R^b_S \in (1, R_A(\bar{p}))$. If $R^b_S$ is the safe rate in an equilibrium then the arguments in the main text preceding the proposition show that $R^b_S$ satisfies (21), that is, $R^b_S = \frac{A_L}{\mu}$, and that $N^b = 1$.

If $R^b_S = \frac{A_L}{\mu}$, then those arguments can be reverted and $R^b_S$ is the safe rate of an equilibrium in which $N^b = 1 + w$.

b) If $w \in (\underline{w}, \bar{w}]$, which implies $\mu \in (\underline{\mu}, A_L]$, then the equilibrium is unique and satisfies the properties in statement ii) in the Proposition.

Suppose first that $\mu \in (\underline{\mu}, A_L]$. Define $R^b_S$ as $R^b_S = \frac{A_L}{\mu}$. By the definition of $\mu$, we have that $R^b_S \in (1, R_A(\bar{p}))$ and a) shows the existence of an equilibrium.

Suppose there exists another equilibrium and denote $R^b_S'$ its safe rate. Using Lemma 1, it must be the case that $R^b_S' = 1$ or $R^b_S = R_A(\bar{p})$.

If $R^b_S' = 1$ then reproducing the arguments in the main text preceding the proposition we have that the supply of safe assets amounts to $\frac{A_L(1-\mu)}{1-A_L}$, which satisfies

$$\frac{A_L(1-\mu)}{1-A_L} > \mu.$$
This implies that the market for safe assets does not clear because their demand is upper bounded by \( \mu \).

If \( R_b^b = R_A(\bar{p}) \), then we have from (17) that \( R_{b,O}^b = R_A(\bar{p}) \). Experts would be indifferent between investing in originators and in safe securities. This implies that the supply of safe assets is upper bounded by \( \frac{\mu(1-\mu)}{1-\mu} \), which satisfies

\[
\frac{\mu (1-\mu)}{1-\mu} < \mu.
\]

This implies that the market for safe assets does not clear because their demand is lower bounded by \( \mu \).

Suppose that \( \mu = A_L \). It suffices to reproduce arguments done above to show that the equilibrium is unique and satisfies \( R_S^b = 1 \) and \( N^b = 1 + w \).

\( c) \) If \( w > \bar{w} \), which implies \( \mu > A_L \) then the equilibrium is unique and satisfies the properties in statement iii) in the Proposition.

It suffices to reproduce arguments done in the proof of b).

\( d) \) If \( \mu \leq \mu \) then there exist Modigliani-Miller equilibria satisfying the properties in statement i) in the Proposition and all the equilibria are of this type.

It suffices to reproduce arguments done in the proof of b).■

**Proof of Lemma 3** In the proof we will make use of some expressions and results that are presented in the main text of the paper after Lemma 3. Each time we do so we include a footnote in which we explain why the arguments are not subject to circularity problems. The superscript \( * \) will throughout the proof denote equilibrium variables.

We have to prove three results

\( i) \) If an equilibrium exists, then \( 1 \leq R_S^* \leq R_A(\bar{p}) \)

The inequality \( R_S^* \geq 1 \) is proven as in Lemma 1.

Suppose that \( R_S^* > R_A(\bar{p}) \) and an equilibrium exists. Since \( R_A(\bar{p}) > 1 \) savers would invest their entire endowment in safe securities. Since experts have the option to invest in safe securities, then \( R_E^* \geq R_S^* \), because otherwise there would be no investment at all in the economy to back the repayment of safe securities. Suppose that aggregate investment by originators is \( N^* \) and their risk choice is \( p^* \). At \( t = 1 \) all the payoffs in the economy are distributed to the measure \( N^* \) of savers and experts that have provided funding either directly or indirectly (through intermediaries) to originators. Since \( R_E^* \geq R_S^* \), necessarily have that \( R_A(p^*) \geq R_S^* \) which implies \( R_A(p^*) > R_A(\bar{p}) \), which contradicts (1).

\( ii) \) For given \( R_S \leq R_A(\bar{p}) \), if a partial equilibrium exists then \( R_S \leq R_E^* \leq R_S^* \)

Suppose \( R_S \leq R_A(\bar{p}) \) and a partial equilibrium exists. Suppose that \( R_E^* > R_S^* \). From the
expression for $R_{E,I}$ in (32),\(^{20}\) we have that an expert that sets up an intermediary obtains a return on equity $R_{E,I}$ satisfying $R_{E,I} \geq R^*_I$, so that in equilibrium $R^*_E \geq R_{E,I} \geq R^*_I$.

Suppose that $R_S > R^*_I$. From (32), we have that an expert that sets up an intermediary can obtain a return on equity $R_{E,I}$ satisfying $R_{E,I} = R^*_I$. Besides, an expert that sets-up an originator obtains a return $R_{E,O}$ satisfying $R_{E,O} \geq R_A(p) \geq R_S > R^*_I$. All experts would thus find optimal to set-up originators, and the demand for risky securities (whose potential only buyers are intermediaries) would be zero. Market clearing in the market for risky securities then implies that originators do not issue risky securities. Yet, since $R^*_I < R_S \leq R_A(p)$, from (17) we have that originators would find optimal to issue safe securities in the market for risky securities.

\(\text{i)}\) For given $R_S < R_A(p)$, if a partial equilibrium exists then $R_S < R^*_S < R^*_E$, and for $R_S = R_A(p)$, if a partial equilibrium exists and $R_S = R^*_S = R^*_E$.

Suppose that $R_S < R_A(p)$ and a partial equilibrium exists. Since $R_S < R_A(p)$, the arguments in the main text preceding Proposition 2 imply that $R_S < R^*_E$ because an originator has the possibility not to issue risky securities. Suppose that $R^*_S = R^*_E$, which implies that $R_S < R^*_I$. Then using expression (E3), we would have that $R_S = R_{E,I} = R^*_E = R^*_I$. Hence, we must have $R^*_I < R^*_E$. Suppose that $R_S = R^*_S$ from (32), we would have that $R_S = R_{E,I} = R^*_E$. Hence, we must have $R_S < R^*_I$.

Finally, suppose that $R_S = R_A(p)$. The same argument as at the end of \(\text{i)}\) and \(\text{ii)}\) implies that $R_S = R^*_S = R^*_E$.\(\blacksquare\)

**Proof of Lemma 4** The first order condition of (25), which characterizes the optimal risk-choice $p$ as a function of $d_I$, is

$$\Delta - d_I = c'(p).$$

(66)

The lemma is a direct implication of the optimality condition (66) and Assumption 1.\(\blacksquare\)

---

\(^{20}\)Equation (32) is presented in Section ?? but is derived directly from the objects and conditions in the maximization problem (12) presented in Section 3.
Proof of Proposition 5  We start by showing some preliminar results about the existence and uniqueness of the equilibrium with exogenous safe rate $R_S$, in which the safe securities market doesn’t clear. Then, we prove Proposition 5 in which $R^*_S$ is endogenous.

Equilibrium with exogenous safe rate $R_S$. The equilibrium is determined by E1 - E4. Here we denote equilibrium variables with the superscript * and show their dependence on the exogenous $R_S$. The following lemmas characterize the equilibrium with exogenous safe rate.

We start describing the solution to the originator’s problem. First, Lemma A.1 shows the admissible range for $R_I$ and proves the uniqueness of the optimal choice $d_I$ for such range. Next, building on Lemma A.1, Lemma A.2 shows that the equilibrium originator’s choices $d_I^*(R_S), p^*(R_S)$ are determined by the equilibrium discount $\chi^*(R_S)$ and describes their dependence, which follows from E1.

Lemma A. 1 For given $R_S < R_A(\bar{p})$, let $\bar{R}_I > R_I$ be the positive constants given by

$$\bar{R}_I = \frac{R_A(\bar{p}) - A_L}{1 - A_L/R_S}$$

and $R_I = \max_{d_I} (\hat{p}(d_I)d_I) / (1 - A_L/R_S)$.

Suppose that $R_I > R_S$. Then if $R_I \in (R_I, \bar{R}_I)$, the solution $d_I^*$ to (29) is unique, satisfies

$$(R_{E,O}(d_I) - R_I) \left(\frac{1}{R_I} \frac{d(\hat{p}(d_I)d_I)}{dd_I}\right) + \frac{dR_A(\hat{p}(d_I))}{dd_I} = 0,$$

and leads to $R_{E,O}(d_I^*) > R_I$. Besides, if $R_I \geq \bar{R}_I$ then $d_I^* = 0$ is the unique solution to (29), while if $R_I \leq R_I$, then $R_{E,O}(d_I)$ can grow unboundedly.

Proof of Lemma A. 1  We first present the following results which are an immediate consequence of (16), Lemma 4 and (66), and will be used without explicit reference throughout the proof of this lemma and the next proposition:

$$\frac{d(\hat{p}(d_I))}{dd_I} = -\frac{1}{c''(\hat{p}(d_I))},$$

and

$$\frac{dR_A(\hat{p}(d_I))}{dd_I} \leq 0$$

Consider an exogenous $R_S < R_A(\bar{p})$ and $R_I > R_S$. Let $\bar{R}_I > R_I$ be the constants defined in the Lemma. By definition we have that $\bar{R}_I = R_{E,O}(0)$. The originator’s problem is described by (29). Denote with $d_I^*$ any of its solutions in case they exist. After some algebra, we have
from (28) that:
\[
\frac{dR_{E,O}(d_I)}{dd_I} = \frac{(R_{E,O}(d_I) - R_I) \left( \frac{1}{R_I} \frac{d(\hat{p}(d_I))}{dd_I} \right) + \frac{dR_A(\hat{p}(d_I))}{dd_I}}{1 - A_L/R_S - \hat{p}(d_I)d_I/R_I},
\]
(70)
\[
\frac{dR_{E,O}(d_I)}{dd_I} \bigg|_{d_I=0} = \frac{(R_I - R_I) \frac{p}{R_I}}{1 - A_L/R_S}.
\]
(71)

We proceed in a sequence of steps.

i) If \( R_I \geq R_{\bar{I}} \) then \( d_I^* = 0 \) is the unique solution to (29).

If \( R_I \geq R_{\bar{I}} \), then consider the function \( G(a) = \frac{R_A(\hat{p}) - A_L}{R_I} - (1 - A_L/R_S) \). We have:
\[
G'(a) = \frac{(R_A(\hat{p}) - A_L - \hat{p}(d_I)d_I) - (1 - A_L/R_S) - A_L/R_S - a/R_I}{(1 - A_L/R_S - a/R_I)^2}.
\]
Using the definition of \( R_{\bar{I}} \) and \( R_I \geq R_{\bar{I}} \), we have from the expression above that \( G'(a) \leq 0 \).
The following sequence of inequalities follows immediately for \( d_I > 0 \):
\[
R_{E,O}(d_I) = \frac{R_A(\hat{p}) - A_L - \hat{p}(d_I)d_I}{1 - A_L/R_S - \hat{p}(d_I)d_I/R_I} = G(\hat{p}(d_I)d_I) \leq G(0) = R_{E,O}(0),
\]
which proves the claim.

ii) If \( R_I \leq R_{\bar{I}} \) then a solution to (29) does not exist, because \( R_{E,O}(d_I) \) can grow unboundedly.

We have from Assumption 1 and (66) that for any \( d_I \in [0, \Delta] \):
\[
R_A(\hat{p}(d_I)) - A_L - \hat{p}(d_I)d_I = \hat{p}(d_I)c'(\hat{p}(d_I)) - c(\hat{p}(d_I)) > 0.
\]
(72)

By definition of \( R_{\bar{I}} \) we have that \( 1 = A_L/R_S + \max d_I (\hat{p}(d_I)d_I)/R_{\bar{I}} \). As a result, if \( R_I \leq R_{\bar{I}} \) for \( d_I \) sufficiently close to \( \arg \max d_I (\hat{p}(d_I)d_I) \) the originator could lever up unboundedly and from (72) its equity return would also do so.

iii) If \( R_I \in (R_{\bar{I}}, R_{\bar{I}}) \) then any \( d_I^* \) satisfies (67).

If \( R_I \in (R_{\bar{I}}, R_{\bar{I}}) \) then \( R_{E,O}(d_I) \) is bounded in the compact interval [0, \( \Delta \)] and some \( d_I^* \) exists. From (71) we have that \( \frac{dR_{E,O}(d_I)}{dd_I} \bigg|_{d_I=0} > 0 \). Besides, since \( \hat{p}(\Delta) = 0 \), we have that \( R_{E,O}(\Delta) = 0 < R_{E,O}(0) \). Hence any \( d_I^* \) must be interior and satisfy \( \frac{dR_{E,O}(d_I)}{dd_I} \bigg|_{d_I=d_I^*} = 0 \), which from (70) is equivalent to (67).

iv) For given \( \chi \geq 1 \), the following equation in \( d_I \) has a unique solution in the interval [0, \( \Delta \)]:
\[
(\chi - 1) \frac{d(\hat{p}(d_I)d_I)}{dd_I} + \frac{dR_A(\hat{p}(d_I))}{dd_I} = 0.
\]
(73)
Using (16) and (66), (73) can be rewritten as

\[ d_I = \left( \frac{\chi - 1}{\chi} \right) \hat{p}(d_I) c''(\hat{p}(d_I)), \]  

so that it is sufficient to prove that this equation has a unique solution. From Assumption 1, we have that

\[ \frac{d(\hat{p}(d_I) c''(\hat{p}(d_I)))}{dd_I} \leq -1. \]  

(75)

If \( \chi > 1 \), from (75) we have that the RHS in (74) is decreasing in \( d_I \). Besides from Assumption 1 and Lemma 4 it is strictly positive for \( d_I = 0 \) and is zero for \( d_I = \Delta \). Hence it has a unique intersection with the line \( d_I \) in the interval \((0, \Delta)\), and (74) has a unique solution. If \( \chi = 1 \), we trivially have that \( d_I = 0 \) is the unique solution of (74).

v) If \( R_I \in (\hat{R}_I, \bar{R}_I) \) then \( d^*_I \) is unique.

Suppose \( R_I \in (\hat{R}_I, \bar{R}_I) \) and there exist two solutions. From \( iii \), they must satisfy (67). Let \( R^*_{E,O} \) the originator’s equity return they lead to. Define \( \chi = R^*_{E,O}/R_I \). Since \( \frac{dR_{E,O}(d_I)}{dd_I} \bigg|_{d_I=0} > 0 \) we have \( R^*_{E,O} > \bar{R}_I > R_I \) and \( \chi > 1 \). By definition of equation (73) and \( \chi \), any solution to (67) is also a solution to (73), and conversely. From iv) the latter equation has a unique solution, which contradicts that the former has at least two.

\[ \text{Lemma A. 2} \quad \text{There exists a function } \hat{d}_I(\chi) \text{ defined for any } \chi \geq 1, \text{ such that if an equilibrium exists for a given safe rate } R_S < R_A(\bar{p}) \text{ then the equilibrium variables } d^*_I(R_S), p^*(R_S) \text{ and } \chi^*(R_S) \text{ satisfy} \]

\[ d^*_I(R_S) = \hat{d}_I(\chi^*(R_S)) \text{ and } p^*(R_S) = \hat{p} \left( \hat{d}_I(\chi^*(R_S)) \right). \]  

Besides, the function \( \hat{d}_I(\chi) \) satisfies

\[ \frac{d\hat{d}_I(\chi)}{d\chi} > 0, \quad \frac{d\hat{p} \left( \hat{d}_I(\chi) \right)}{d\chi} < 0, \quad \frac{d \left( \hat{p} \left( \hat{d}_I(\chi) \right) \hat{d}_I(\chi) \right)}{d\chi} > 0, \text{ and } \hat{d}_I(1) = 0. \]  

(77)

\[ \text{Proof of Lemma A. 2} \quad \text{Recall partial result } iv \text{) in the proof of Lemma A. 1. For given } \chi \geq 1, \text{ denote } \hat{d}_I(\chi) \text{ the unique solution to (73), or equivalently to (74). We proceed in two steps:} \]

i) \( \text{The function } \hat{d}_I(\chi) \text{ satisfies the properties in (77)} \)

We have that \( \frac{(\chi-1)}{\chi} \) is increasing in \( \chi \) for \( \chi \geq 1 \). From (75) we immediately have that \( \frac{d\hat{d}_I(\chi)}{d\chi} > 0 \), and hence from Lemma 4 that \( \frac{d\hat{p}(\hat{d}_I(\chi))}{d\chi} < 0 \). Besides, from (74) we have after some immediate algebra that

\[ \frac{d \left( \hat{p} \left( \hat{d}_I(\chi) \right) \hat{d}_I(\chi) \right)}{d\chi} = \hat{p} \left( \hat{d}_I(\chi) \right) \frac{d\hat{d}_I(\chi)}{d\chi}. \]  

(78)
Moreover, from (74) and \( \hat{p}(d_I) = 0 \) if and only if \( d_I = \Delta \), we have that \( \hat{p} \left( \hat{d}_I(\chi) \right) > 0 \) for all \( \chi \geq 1 \). We hence have from (78) and \( \frac{d\hat{d}_I(\chi)}{d\chi} > 0 \) that \( \frac{d(\hat{p}(\hat{d}_I(\chi))\hat{d}_I(\chi))}{d\chi} > 0 \).

Finally, (74) implies that \( \hat{d}_I(1) = 0 \) and a continuity argument leads to \( \lim_{\chi \to 1} \hat{d}_I(\chi) = 0 \).

ii) The function \( \hat{d}_I(\chi) \) corresponds to that defined in the proposition.

For given \( R_S < R_A(\bar{p}) \), suppose an equilibrium exists and let \( d^*_I, p^*, \chi^* \) denote the associated equilibrium variables. From (E1) we have that

\[
(\chi^* - 1) \frac{d(\hat{p}(d^*_I)d^*_I)}{dI} + \frac{dR_A(\hat{p}(d^*_I))}{dI} = 0.
\]

Comparing with (73), we conclude that \( d^*_I = \hat{d}_I(\chi^*) \) and hence from Lemma 4 that \( p^* = \hat{p} \left( \hat{d}_I(\chi^*) \right) \).■

**Lemma A. 3** For a given safe rate \( R_S < R_A(\bar{p}) \), suppose an equilibrium exists and let \( R^*_E(R_S) \) and \( \chi^*(R_S) \) denote the associated equilibrium variables. They satisfy:

\[
\chi^*(R_S) = (1 - \lambda) \frac{\hat{R}_E(R_S, \chi^*(R_S))}{R_S} + \lambda.
\]

with

\[
\hat{R}_E(R_S, \chi) = \frac{R_A(\hat{p}(\hat{d}_I(\chi))) - A_L + (\chi - 1)\hat{p}(\hat{d}_I(\chi))\hat{d}_I(\chi)}{1 - A_L/R_S},
\]

and \( R^*_E(R_S) = \hat{R}_E(R_S, \chi^*(R_S)) \)

**Proof of Lemma A. 3** The Lemma directly follows from the steps in the main text. Equations (79) and (80) are analogous to E3 and E2, but explicitly show the dependence on \( R_S \). ■

Notice that (79) and (80) determine the equilibrium \( \chi^*(R_S) \), which, in turn, determines all other equilibrium variables. Lemma A.4 builds on these two equation to show the existence and uniqueness of the equilibrium \( \chi^*(R_S) \), while Lemma A.5 and A.6 describe the other equilibrium variables.

**Lemma A. 4** There exists \( R_S \in (A_L, R_A(\bar{p})) \), such that for a given safe rate \( R_S < R_A(\bar{p}) \) an equilibrium exists if and only if \( R_S > R_S \), in which case the equilibrium is unique. For \( R_S > R_S \), the functions \( R^*_E(R_S), \chi^*(R_S) \) describing the equilibrium return on equity and intermediary funding discount, respectively, satisfy

\[
\frac{dR^*_E(R_S)}{dR_S} < 0, \lim_{R_S \to R_S^*} R^*_E(R_S) = \infty, \text{ and } \lim_{R_S \to R_A(\bar{p})} R^*_E(R_S) = R_A(\bar{p}),
\]

\[
\frac{d\chi^*(R_S)}{dR_S} < 0, \lim_{R_S \to R_S^*} \chi^*(R_S) = \infty, \text{ and } \lim_{R_S \to R_A(\bar{p})} \chi^* = 1.
\]
Proof of Lemma A. 4  We first present the following partial derivatives of the function  
\( \hat{R}_E(R_S, \chi) \) defined in (80):

\[
\frac{\partial \hat{R}_E(R_S, \chi)}{\partial R_S} < 0 \quad \text{and} \quad \frac{\partial \hat{R}_E(R_S, \chi)}{\partial \chi} = \frac{\hat{p}(\hat{d}_I(\chi))\hat{d}_I(\chi)}{1 - A_L/R_S} > 0,
\]

(81)

where for the partial derivative with respect to \( \chi \) we have used the optimality condition in (E1) and that \( \frac{dR_A(p)}{dp} = \Delta - c'(p) \).

Let \( R_S < R_A(\overline{p}) \). Any equilibrium intermediary funding discount \( \chi^* \) satisfies \( \chi^* \geq 1 \) and (79), and conversely. We denote with \( G(\chi^*, R_S) \) the function of \( \chi^* \) and \( R_S \) in the RHS of (79). Notice that we do not make explicit the dependence of \( G(\chi^*) \) on \( R_S \) for the sake of notacional simplicity. Using (81), we have that

\[
\frac{\partial G(\chi^*, R_S)}{\partial \chi^*} = \frac{(1 - \lambda)\hat{p}(\hat{d}_I(\chi^*))\hat{d}_I(\chi^*)}{R_S - A_L}.
\]

(82)

We proceed in a sequence of steps:

i) For any \( R_S \), any solution \( \chi^* \geq 1 \) to (79) satisfies \( \partial G(\chi^*, R_S) / \partial \chi^* < 1 \).

Suppose that there exists a solution \( \chi^* \geq 1 \) to (79) with \( \partial G(\chi^*, R_S) / \partial \chi^* \geq 1 \). Let \( R^*_E \) and \( R^*_I \) denote the equilibrium returns in the economy with equilibrium intermediary funding discount \( \chi^* \). From (82) we have

\[
\frac{\hat{p}(\hat{d}_I(\chi^*))\hat{d}_I(\chi^*)}{R_S - A_L} \geq \frac{1}{1 - \lambda}.
\]

(83)

Recall from Lemma 1 that if an equilibrium exists we must have \( R^*_I > R_I \), otherwise \( R^*_E \) would be infinity and \( \chi^* \) as well. From the definition of \( R_I \) and (83), we have that

\[
R_I = \max_{dI} \hat{p}(dI) \hat{d}_I \geq \frac{\hat{p}(\hat{d}_I(\chi^*))\hat{d}_I(\chi^*)}{1 - A_L/R_S} \geq \frac{R_S}{1 - \lambda}.
\]

The equilibrium condition (E3) and the inequality above imply that

\[
\frac{1}{R^*_I} = \frac{1}{R^*_E} + \frac{1 - \lambda}{R_S} \geq \frac{1}{R_I} > \frac{1}{R^*_E} \geq \frac{1}{R^*_I},
\]

which contradicts that \( R^*_I < R_I \).

ii) Equation (79) has at most one solution \( \chi^* \geq 1 \)

Suppose that there exist two solutions \( \chi^*_1 < \chi^*_2 \). Notice that the derivative with respect to \( \chi^* \) of the LHS of (79) is equal to one. From Proposition 2 and (82) we have that \( \frac{\partial G(\chi^*, R_S)}{\partial \chi^*} > 0 \). And then the existence of two solutions \( \chi^*_1 < \chi^*_2 \), implies that

\[
\frac{\partial G(\chi^*_1, R_S)}{\partial \chi^*} < 1 < \frac{\partial G(\chi^*_2, R_S)}{\partial \chi^*}.
\]

55
The second inequality contradicts \( i \).

Before stating the next partial results, we denote \( \Gamma = \{ R_S < R_A(\overline{p}) \} \) st (79) has a solution \( \chi^* \geq 1 \).

From \( ii \) we can define for any \( R_S \in \Gamma \) the unique solution to (79) as \( \chi^*(R_S) \). We also introduce the function \( F^*(R_S) = \frac{\partial G(\chi^*, R_S)}{\partial \chi^*} \bigg|_{\chi^* = \chi^*(R_S)} \).

\( iii \) \( \Gamma \) is non empty

We have from (82) and Proposition 2 that \( \frac{\partial G(\chi^*, R_S)}{\partial \chi^*} = 0 \). In addition, from (E2) we have

\[
\lim_{R_S \to R_A(\overline{p})} R_E^*(R_S, \chi^* = 1) = R_A(\overline{p}),
\]

so that as \( R_S \to R_A(\overline{p}) \), we have that \( G(1, R_S) \) tends to 1. Then equation (79) necessarily has a solution for \( R_S \) sufficiently close to \( R_A(\overline{p}) \).

\( iv \) If \( R_{S, 1}, R_{S, 2} < R_A(\overline{p}) \) with \( R_{S, 1} < R_{S, 2} \) and \( R_{S, 1} \in \Gamma \), then \( R_{S, 2} \in \Gamma \)

This simply results from the fact that \( G(\chi^*, R_S) \) is decreasing in \( R_S \) and that for all \( R_S < R_A(\overline{p}) \) we have \( G(\chi^* = 1, R_S) > 1 \).

\( v \) There exists \( R_S < R_A(\overline{p}) \) such that \( \Gamma = (R_S, R_A(\overline{p})) \)

Let \( R_S = \inf(\Gamma) \). It suffices to prove that \( R_S \notin \Gamma \). Suppose that \( R_S \in \Gamma \). Then \( i \) implies that \( F^*(R_S) < 1 \). By definition this implies that for all \( \varepsilon > 0 \), we have that \( \chi^* \in (\chi^*(R_S), \chi^*(R_S) + \varepsilon) \) implies \( \chi^* > G(\chi^*, R_S) \). And thus for small \( \delta > 0 \), we have that \( R_S' \in (R_S - \delta, R_S) \) implies that \( \chi^* > G(\chi^*, R_S') \). Since we have that \( 1 < G(\chi^* = 1, R_S') \), we conclude that \( R_S' \notin \Gamma \). But we have that \( R_S' < R_S = \inf(\Gamma) \leq R_S' \).

\( vi \) \( \chi^*(R_S) \) is strictly decreasing in \( R_S \), with \( \lim_{R_S \to R_A(\overline{p})} \chi^*(R_S) = 1 \)

The monotonicity of \( \chi^*(R_S) \) can be obtained by derivating implicitly equation (79) and using \( i \), and \( \frac{\partial G(\chi^*, R_S)}{\partial R_S} < 0 \). The other statement results from \( \lim_{R_S \to R_A(\overline{p})} G(\chi^* = 1, R_S) = 1 \) and \( ii \).

\( vii \) \( R_E^*(R_S) \) is strictly decreasing in \( R_S \), with \( \lim_{R_S \to R_A(\overline{p})} R_E^*(R_S) = R_A(\overline{p}) \)

By definition we have \( R_E^*(R_S) = R_E^*(R_S, \chi^*(R_S)) \). The monotonicity of \( R_E^*(R_S) \) is immediately obtained from (81) and \( vii \).

Lemma A. 5 For a given safe rate \( R_S \in (\overline{R}_S, R_A(\overline{p})) \), the functions \( d^*_I(R_S), p^*(R_S), x^*(R_S) \) describing the originator’s equilibrium risky security promise, risk choice, and leverage, respectively, satisfy

\[
\frac{dd^*_I(R_S)}{dR_S} < 0, \quad \frac{dp^*(R_S)}{dR_S} > 0, \quad \frac{d(p^*(R_S)d^*_I(R_S))}{dR_S} < 0 \quad \text{and} \quad (84)
\]

\[
\lim_{R_S \to R_A(\overline{p})} d^*_I(R_S) = 0, \quad \lim_{R_S \to R_A(\overline{p})} p^*(R_S) = \overline{p}R
\]

Proof of Lemma A. 5 The results on \( d^*_I(R_S), p^*(R_S) \) and \( p^*(R_S)d^*_I(R_S) \) are an immediate consequence of Lemma A. 2 and Lemma A. 4.
Lemma A. 6 For a given safe rate \( R_S \in (R_S, R_A(\bar{p})) \), the equilibrium aggregate investment \( N^*(R_S) \) is given by

\[
N^*(R_S) = \frac{1 - \mu}{1 - (A_L + (1 - \lambda)p^*(R_S)d_I^*(R_S)) / R_S},
\]

where \( \mu = w/(1 + w) \) denotes the share of savers’ wealth, and satisfies

\[
\frac{dN^*(R_S)}{dR_S} < 0, \lim_{R_S \to \bar{R}_S} N^*(R_S) = \infty.
\]

Proof of Lemma A. 6 The expression in (85) is obtained from (38) - (41) and the remaining results are a consequence of Lemma A. 4.

Proof of Proposition 5 We now use the previous characterization of the equilibrium for given \( R_S \) together the market clearing condition for safe securities, which is represented in \( E5 \) when \( 1 < R^*_S < R_A(\bar{p}) \), to prove Proposition 5.

We first prove existence and then uniqueness of equilibrium. Each of the two claims is proven in a sequence of steps.

Recall that for a given exogenous \( R_S \in (R_S, R_A(\bar{p})) \) the partial equilibrium of the economy exists, is unique and described by the previous lemmas.

Existence of general equilibrium

a) If \( R_S \in (R_S, R_A(\bar{p})) \) and \( R_S > 1 \) (\( R_S = 1 \), then \( N^*(R_S) = 1 + w \) \( N^*(R_S) \leq 1 + w \)) if and only if \( R_S \) is the safe rate in some general equilibrium

Suppose a given safe rate \( R_S \) satisfying \( R_S \in (R_S, R_A(\bar{p})) \) and \( R_S > 1 \). We know that a unique partial equilibrium of the economy exists for such \( R_S \). From Lemma A.4, it satisfies \( R^*_E > R_S > 1 \) which implies that savers find strictly optimal to invest in safe securities and experts in financial firms’ equity. In order to prove the existence of a general equilibrium for the given \( R_S \), it suffices to prove that the market for safe securities clears. Taking into account that the existence of a partial equilibrium implies the clearing of the market for equity and risky securities, and that the two type of investors fully invest their endowment in financial firms, the clearing of the market for safe securities is equivalent to the entire endowment of the economy being invested (directly or indirectly) into originators’ loans, that is, \( N^*(R_S) = 1 + w \). The result then follows.

The statement for \( R_S = 1 \) is proven analogously after noticing that savers are indifferent between investing in safe securities or consuming.

b) If \( w \in (w, \bar{w}] \) there exists an equilibrium satisfying \( R^*_S < R_A(\bar{p}) < R^*_E, p^* < \bar{p} \) and \( N^* = 1 + w \). Moreover, the equilibrium is unique within the class of equilibria with \( R^*_S < R_A(\bar{p}) \).
Expressing it in terms of the share of wealth: \( w \in (\underline{w}, \overline{w}] \) is equivalent to \( \mu \in (\underline{\mu}, \overline{\mu}] \). Suppose that \( \mu \in (\underline{\mu}, \overline{\mu}] \). From (85), we have that

\[
\lim_{R_S \to R_A(\overline{\mu})} N^*(R_S) < 1 + w \iff \mu > \underline{\mu}.
\]  

(86)

Using Lemma A.6 and the definition of \( \mu \), we conclude that there exists a solution \( R_S^* < R_A(\overline{\mu}) \) such that \( N^*(R_S^*) = 1 + w \iff \mu > \underline{\mu} \), and in such a case the solution is unique.\(^{21}\) In addition, the solution \( R_S^* \) satisfies \( R_S^* \geq 1 \) iff \( R_S < 1 \) and \( N^*(1) \geq 1 + w \), which from the definition of \( \overline{\mu} \) is equivalent to \( \mu \leq \overline{\mu} \). The result is then a consequence of \( a) \), Lemma A.4 and Lemma A.5.

c) If \( w > \overline{w} \) there exists an equilibrium satisfying \( 1 = R_S^* < R_A(\overline{\mu}) < R_E^* \), \( p^* < \overline{\mu} \) and \( N^* = \left( \frac{1 - \underline{\mu}}{\overline{\mu}} \right) (1 + w) \in (N^b, 1 + w) \). Moreover, the equilibrium is unique within the class of equilibria with \( R_S^* < R_A(\overline{\mu}) \).

Suppose that \( w > \overline{w} \), which is equivalent to \( \mu > \overline{\mu} \). From the definition of \( \overline{\mu} \), this implies that \( R_S < 1 \). From (85), we have also that \( N^*(1) = \frac{1 - \mu}{\overline{\mu}} (1 + w) < 1 + w \). Then \( a) \) implies that \( R_S^* = 1 \) is the safe rate of a general equilibrium. The results for the associated equilibrium variables, except from \( N^* > N^b \), are then a consequence of Lemma A.4 and Lemma A.5. The inequality \( N^* > N^b \) results from Proposition 2. Finally, for any \( R_S \in (1, R_A(\overline{\mu})) \) we have from Lemma A.6 that \( N^*(R_S) < 1 + w \) and \( a) \) implies that \( R_S \) is not the safe rate in some general equilibrium.

d) If \( w \leq \underline{w} \) there exists M-M equilibria with \( R_S^* = R_E^* = R_A(\overline{\mu}), p^* = \overline{\mu} \) and \( N^* = 1 \)

Suppose that \( w \leq \underline{w} \), which is equivalent to \( \mu \leq \underline{\mu} \). The equilibria of the economy with no intermediaries are in the M-M indifference region and satisfy \( R_S^* = R_E^* = R_A(\overline{\mu}) \). Consider one such equilibrium and suppose the return of the risky securities is \( R_{S}^* = R_A(\overline{\mu}) \). It is easy to directly prove from the originator’s problem (3) - (9) that for the pair of returns \( R_S^* = R_{S}^* = R_A(\overline{\mu}) \) it is weakly optimal for the originator to choose \( d_t = 0 \). If originators do not issue risky securities, then market clearing implies that the supply of risky securities is zero which means that intermediaries do not enter. This proves that the equilibrium of the no intermediary benchmark economy can be sustained when experts can set up intermediaries and they expect a return for risky securities \( R_{S}^* = R_A(\overline{\mu}) \) in that market.

e) An equilibrium exists.

Immediate from \( b) \), \( c) \) and \( d) \).

Uniqueness of equilibrium

f) If \( w > \underline{w} \) the equilibrium is unique.

Suppose that \( w > \underline{w} \), which is equivalent to \( \mu > \underline{\mu} \). We have from \( b) \) and \( c) \) that the economy has a unique equilibrium with a safe rate \( R_S^* = R_A(\overline{\mu}) \). Suppose that \( R_S^* = R_A(\overline{\mu}) \)

\(^{21}\)Notice that \( N^*(R_S^*) = 1 + w \) is equivalent to \( E5 \).
is the safe rate in some general equilibrium. Then Lemma 3 implies that $R^*_S = R^*_I = R^*_E = R_A(\bar{p})$ and the arguments made in the proof of that lemma imply that originator’s risk choice is $p^* = \bar{p}$. Besides, aggregate investment must be $N^* = 1 + w$. Let $d^*_S, d^*_I$ be the equilibrium safe and risky promises per unit of loan made by originators. From (9) we have that $p^* = \bar{p}$ implies that $d^*_S + d^*_I \leq A_L$, and thus risky securities are in fact safe. This means that intermediaries, in case they enter in the economy, they do not expand the supply of safe securities by diversifying idiosyncratic risks. Formally, the supply of safe securities in this economy is necessarily upper bounded by

$$\frac{A_LN^*}{R^*_S} = \frac{A_L}{R_A(\bar{p})} = \mu.$$ 

Besides, since $R^*_S > 1$, savers find strictly optimal to invest in safe securities and the demand for safe securities is at least $\mu$. But then $\mu > \mu$ implies that this market does not clear. We conclude that $R^*_S = R_A(\bar{p})$ cannot be the safe rate in some general equilibrium.

**g)** If $w \leq \bar{w}$ all the equilibria are M-M type with $R^*_S = R^*_E = R_A(\bar{p}), p^* = \bar{p}$ and $N^* = 1 + w$.

Suppose there exists an equilibrium with $R^*_S < R_A(\bar{p})$. Then a) implies that $N^*(R^*_S) \leq 1+w$. From Lemma A.6, we have that $\lim_{R_S \to R_A(\bar{p})} N^*(R_S) < 1+w$ and (86) states that $\mu > \mu$ or, equivalently, that $w > \bar{w}$. We conclude that any equilibrium must have $R^*_S = R_A(\bar{p})$. Reproducing arguments made in f) we get the result.

**h)** The equilibrium is unique up to M-M indifference if and only if $w \leq \bar{w}$.

Immediate from f) and g).
Proof of Proposition 6  We start with a preliminary observation. If the equilibrium returns in either of the economies are denoted with $R'_E, R'_S$, then aggregate welfare in that economy can be written as

$$W' = R'_E + (1 + w)R'_S. \tag{87}$$

Notice the expression holds also if $R'_S = 1$ or if $R'_E = R'_S$.

We prove sequentially each of the statements in the proof.

For any given $w$, we refer in this proof to equilibrium variables in the no intermediaries economy with $b$ superscript and to equilibrium variables in the baseline economy with a * superscript. Moreover we will make explicit the dependence of these variables on $w$.

Statement i)

It follows immediately from Proposition 2, Proposition 5, and the expressions for $R^b_S(w)$ in (21) in the region $w \in (\underline{w}, \overline{w}]$ and for $R^*_S(w)$ in (E5) in the region $w \in (\underline{w}, \overline{w}]$.

Statement ii)

Let us consider three regions

First, $w > \overline{w}$. Notice that this requires that $\overline{w} < \infty$. From Proposition 2 and Proposition 5, we have that $R^*_S(w) = R_b^S(w) = 1$ and $\chi^*(w) > 1$ we have from Proposition 2 that in the baseline economy the originator finds strictly optimal to issues a positive amount of risky securities, that is $d^*_I(w) > 0$. Notice that since $R^*_S(w) = R_b^S(w) = 1$ and the originator finds strictly suboptimal to set $d_I = 0$ in which case its return on equity would be equal to that in the no intermediaries economy, $R^b_E(w)$, we must have that $R^*_E(w) < R^b_E(w)$.

Second, $w \in (\underline{w}, \overline{w}]$. From Proposition 2 and Proposition 5, we have that $N^*(w) = N_b^*(w) = 1$ and $p^*(w) < p^b(w) = \bar{p}$. Since all the consumption in the two economies is at the final date, all funds in the two economies is invested (directly or indirectly) in originator’s loans, and their payoffs are consumed by savers and experts, from (87), we have that

$$R^*_E(w) + wR^*_S(w) = W^*(w) = R_A(p^*(w)) < R_A(\bar{p}) = W^b(w) = R^b_E(w) + wR^b_S(w).$$

Using from i) that $R^b_S(w) > R^*_S(w)$ we conclude from the inequality above that $R^*_E(w) < R^b_E(w)$.

Third, $w \in (\underline{w}, \overline{w}]$. From Proposition 2 and Proposition 5 we have that $R^b_E(w)$ is constant in all this region while $R^*_E(\mu)$ is strictly increasing.

The statement in ii) then results immediately from our results in the three regions.

Statement iii)

Using (87), it follows from the two previous statements.

Proof of Proposition 7  Recall the variables $R_S \in (A_L, R_A(\bar{p}))$ defined in Lemma A.4, and $\underline{w}, \overline{w}, \overline{w}$ defined in Proposition 5. We rely extensively in this proof without explicit
reference to the results in Proposition 5 and to the discussion in the main text preceding Proposition 7, in particular the equivalence between the FOC in (60) and that in (E1) after plugging in the equilibrium equation (E3).

We first describe the set of Pareto efficient allocations. For SP weights $\omega_S, \omega_E$ with $\omega_E > 0$, we define $\omega \equiv \omega_S/\omega_E$ and adopt the convention that $\omega = \infty$ when $\omega_E = 0$. We have from (55) that if $\omega_E > 0$ then the associated optimal allocations depend only on $\omega$. Besides, we have from the main text that optimal allocations are described by a tuple $(N, d_I, C_{S,0}, C_{S,1})$. For each value of $\omega$, the optimal allocations are denoted with a superscript $SP$, can be obtained from (55), and are presented next (Details of the derivations are ommitted):

**I-** For $\omega < 1$: $N^{SP} = 1 + w, d_I^{SP} = 0, C_{S,0}^{SP} = 0, C_{S,1}^{SP} = 0$

**II-** For $\omega = 1$: $N^{SP} = 1 + w, d_I^{SP} = 0, C_{S,0}^{SP} = 0, C_{S,1}^{SP}$ is any value satisfying (59)

For the rest of the allocation Pareto frontier, we distinguish two cases:

**Case $R_S \geq 1 (\iff \overline{w} = \infty)$:**

**III-** For $\omega > 1$: $N^{SP} = 1 + w, d_I^{SP} \in (0, \Delta)$ is the unique solution to (60), $C_{S,0}^{SP} = 0, C_{S,1}^{SP}$ satisfies (59) with equality.

**Case $R_S < 1 (\iff \overline{w} = \infty)$:** Let $\chi^*(R_S = 1)$ denote the intermediary funding discount in the equilibrium of the economy with an exogenous $R_S = 1$. Define $\overline{w} = (1 - \lambda)\chi^*(R_S = 1)$.

**III.a-** For $\omega \in (1, \overline{w})$: $N^{SP} = 1 + w, d_I^{SP} \in (0, \Delta)$ is the unique solution to (60), $C_{S,0}^{SP} = 0, C_{S,1}^{SP}$ satisfies (59) with equality.

**III.b-** For $\omega = \overline{w}$: $N^{SP} = 1 + w, d_I^{SP} \in (0, \Delta)$ is the unique solution to (60), $C_{S,0}^{SP} = 1 - N^{SP}, C_{S,1}^{SP}$ satisfies (59) with equality.

**III.c-** For $\omega > \overline{w}$: $N^{SP} = 1 + w, d_I^{SP}$ is irrelevant since there is no investment, $C_{S,0}^{SP} = 1, C_{S,1}^{SP} = 0$.

We now proceed to the proof of the two constrained Welfare Theorems in the proposition. For the sake of brevity we restrict to the slightly more involved case of $R_S < 1 \iff \overline{w} < \infty$.

**First Welfare Theorem:**

For given $w$, we need to prove that the (general) equilibrium of the economy is a Pareto efficient allocation. We distinguish three cases:

i) $w \leq w$: The equilibrium is of the M-M type and thus belongs to the efficient allocation region I if $w = 0$ and II if $w > 0$.

ii) $w \in (w, \overline{w})$: Let $\omega = (1 - \lambda)\chi^*$ where $\chi^*$ denotes the general equilibrium value of this variable for the given $w$. We have by construction that $\omega \leq \overline{w}$ and the equilibrium coincides with the efficient allocation in region **III.a** if $\omega < \overline{w}$ and the unique efficient allocation in the region **III.b** with $N^{SP} = 1 + w$ if $\omega = \overline{w}$.

iii) $w > \overline{w}$: The equilibrium coincides with the unique efficient allocation in the region **III.b** with $N^{SP} = 1 + \overline{w}$.
Second Welfare Theorem: We now consider initial date transfers between experts and savers. Such transfers would modify the initial wealth of each kind of investors, while maintaining the overall wealth in the economy \( 1 + w \). Therefore, we express the economy with transfers in terms of \( \mu \) the share the total wealth owned by savers after transfers. Recall the thresholds \( \underline{\mu}, \overline{\mu} \) defined in the proof of Proposition 5.

For given Pareto efficient allocation \((N^{SP}, d^{SP}_I, C^{SP}_S, 0, C^{SP}_S, 1)\), we need to prove that there exists \( \mu \) such that the allocation coincides with that induced by the equilibrium of the economy for such value of \( \mu \). We distinguish three cases:

i) \((N^{SP}, d^{SP}_I, C^{SP}_S, 0, C^{SP}_S, 1)\) in regions I or II: Define \( \mu = \frac{C^{SP}_S}{R_A(\bar{p})} \). Then we have by construction that \( \mu \leq \overline{\mu} \) and the equilibrium of the economy for this value of \( \mu \) induces the allocation.

ii) \((N^{SP}, d^{SP}_I, C^{SP}_S, 0, C^{SP}_S, 1)\) in regions III.a or III.b with \( N^{SP} = 1 + w \): Let \( \omega \leq \overline{\omega} \) be the SP weight ratio associated with the allocation. Taking into account the properties of the partial equilibrium function \( \chi^*(R_S) \) described in Lemma A.4, we have that there exists a unique \( R'_S \in [1, R_A(\bar{p})] \) such that \( \chi^*(R'_S) = \omega/(1 - \lambda) \). Define \( \mu = C^{SP}_S/R'_S \). Then we have by construction that \( \mu \in (\underline{\mu}, \overline{\mu}] \) and the equilibrium of the economy for this value of \( \mu \) induces the efficient allocation.

iii) \((N^{SP}, d^{SP}_I, C^{SP}_S, 0, C^{SP}_S, 1)\) in regions III.b with \( N^{SP} < 1 + w \) or III.c: Define \( \mu \) to be the unique solution to \( N^{SP} = \frac{1 - \mu}{1 - \overline{\mu}} \). Then, we have by construction that \( \mu < \overline{\mu} \) and the equilibrium of the economy for this value of \( \mu \) induces the efficient allocation.

\[ \square \]

**Proof Proposition 8** We prove the proposition in two steps. First, we show the equivalence between the risky economy and the baseline economy when \( \lambda = 0 \). Next, we prove that there exist \( \lambda \) such that for \( 0 < \lambda < \lambda^* \) there exist values of savers’ endowment \( w \) such that \( p^*(\lambda; w) < p^*(0; w) \).

Step (i). The equilibrium in the risky economy is equivalent to the the one in the baseline economy when \( \lambda = 0 \).

Originators’ problem is the same in the risky economy than in the baseline. The only difference is that market clearing conditions change since savers buy directly the risky securities. In the risky economy, savers are indifferent between holding safe or risky securities so their returns must be the same: \( R_S = R_I \). Intermediaries don’t enter the market. Thus, we have that all equity is allocated to originators \( E_O = 1 \). While the market clearing of total originators external funding implies:

\[ R_Iw = (A_L + pd_I)N. \]

It is easy to see from E1-E5 that this is the case in the baseline economy with \( \lambda = 0 \).

\[ \square \]
Step (ii). There exist $\tilde{\lambda}$ such that for $0 < \lambda < \tilde{\lambda}$ there exist values of savers’ endowment $w$ such that $p^*(\lambda; w) < p^*(0; w)$.

Recall that from E1 we have that the equilibrium $d_i^*(\lambda; w)$ and so $p^*(\lambda; w)$ are determined only by the equilibrium intermediary discount $\chi^*(\lambda; w)$. So, we need to prove that $\chi^*(\lambda; w) > \chi^*(0; w)$ for $\lambda < \tilde{\lambda}$.

From E3 it follows that

$$\frac{d\chi^*(\lambda; w)}{d\lambda} = -\left(\frac{R_E^*(\lambda; w)}{R_S^*(\lambda; w)} - 1\right) + \frac{d}{d\lambda}\left(\frac{R_E^*(\lambda; w)}{R_S^*(\lambda; w)}\right).$$

(88)

The first term in the RHS of (88) is negative, and represents the direct effect of a change in $\lambda$ in E3: for given equity spread a higher $\lambda$ reduces the leverage of intermediaries and the discount they can offer. The second term in the RHS of (88) represent the indirect effect in E3 through the change in the equity spread: higher $\lambda$ reduces the amount of safe pay-offs, thus increases the rents obtained by experts and the equity spread.

For the case of full investment ($w \in [w, \bar{w}]$), we have from E5, (43) and (42)

$$\frac{R_E^*(\lambda; w)}{R_S^*(\lambda; w)} = \frac{R_A^*(\lambda; w) - A_L - (1 - \lambda)p^*(\lambda; w)d_i^*(\lambda; w)}{(A_L + (1 - \lambda)p^*(\lambda; w)d_i^*(\lambda; w)) \frac{1}{w}}.$$  

(89)

From (88) and (89), we have that

$$\text{sgn}\left(\frac{d\chi^*(\lambda; w)}{d\lambda}\right) = \text{sgn}(H(\lambda, w))$$

(90)

with

$$H(\lambda, w) = -\left(\frac{R_E^*(\lambda, w)}{R_S^*(\lambda, w)} - 1\right) + (1 - \lambda)p^*(\lambda, w)d_i^*(\lambda, w) \frac{(1 + w)}{R_S^*(\lambda, w)} \left(1 + \frac{R_E^*(\lambda, w) \frac{1}{R_S^*(\lambda, w)}}{w}\right).$$

(91)

Using (74), which is an implication of E1, we have that (after some manipulations)

$$\text{sgn}(H(\lambda, w)) = \text{sgn}\left(p\tilde{c}''(p)(1 - \lambda)^2p \frac{1}{R_S^*(\lambda, w)} \left(1 + \frac{R_A^*}{R_S^*} \frac{1}{w}\right) - \chi\right).$$

(92)

For $w \to \bar{w}$, we have

$$\lim_{w \to \bar{w}} \text{sgn}(H(\lambda, w)) = \text{sgn}\left(\tilde{p}\tilde{c}''(\tilde{p})\left(\frac{R_A^*}{R_A^* - A_L \frac{1}{A_L}} - \frac{1}{(1 - \lambda)^2}\right)\right).$$

(93)

Since from Assumption 1 we have that the first term in the RHS of (93) is greater than 1, we can see that there exist $\tilde{\lambda}$ such that, for $\lambda \in (0, \tilde{\lambda})$, $\lim_{w \to \bar{w}} \text{sgn}(H(\lambda, w)) > 0$. Therefore, from (90) we have that $\lim_{w \to \bar{w}} \chi^*(\lambda, w)$ is increasing in $\lambda \in (0, \tilde{\lambda})$, which immediately proves the statement.■
Proof Proposition 9  We denote equilibrium variables of the economy with no intervention with a * superscript. We focus on feasible policies \((\bar{\theta}, \tau)\) and denote with \(p(\bar{\theta}, \tau), N(\bar{\theta}, \tau), W(\bar{\theta}, \tau)\) the values of these variables induced by the policy \((\bar{\theta}, \tau)\). We denote with \(\lambda_0\) the exogenous aggregate risk parameter in the baseline economy and refer to an economy with generic aggregate risk parameter \(\lambda\) as a \(\lambda\)-economy.

We prove the statements in the proposition in a sequence of steps.

i) The equilibrium of the economy with no intervention is not a first-best allocation.

This results from \(w > w\) and Proposition 5.

ii) The first-best allocations in the Pareto frontier of a \(\lambda\)-economy are independent of \(\lambda\).

From the proof of Proposition 7, we have that first-best allocations in the Pareto frontier of the economy correspond to regions I and II, which are independent from \(\lambda\).

iii) The non first-best part of the Pareto frontier of a \(\lambda\)-economy with positive investment is strictly shifted rightwards as \(\lambda\) decreases.

From the proof of Proposition 7, an allocation of the non first-best part of the Pareto frontier of a \(\lambda\)-economy can be described by a pair \((N, d_I)\) satisfying the properties in III or in III-a-b-c, which in particular imply that \(d_I > 0\). In either case, the overall welfare for savers and experts, \(W_S, W_E\) is given by:

\[
W_S(N, d_I|\lambda) = (A_L + (1 - \lambda)p(d_I)d_I)N + (1 - N),
\]

\[
W_E(N, d_I|\lambda) = (R_A(p(d_I)) - A_L - (1 - \lambda)p(d_I)d_I)N.
\]

Notice in addition from (66) that the function \(\hat{p}(d_I)\) does not depend on \(\lambda\).

Let \(\lambda_1 < \lambda_2\) and consider an allocation in the non-first best Pareto frontier of the \(\lambda_2\)-economy with positive investment. It is thus described by a pair \((N_2, d_{I,2})\) with \(d_2 > 0\) and induces welfare for savers and experts amounting to \(W_S(N_2, d_{I,2}|\lambda_2), W_E(N_2, d_{I,2}|\lambda_2)\), respectively. Besides, we must have that

\[
\hat{p}(d_I)d_I < \hat{p}(d_{I,2})d_{I,2} \text{ for any } d_I < d_{I,2}.
\]

This is because if the inequality were not satisfied for some \(d_I < d_{I,2}\), the allocation in the \(\lambda_2\)-economy described by \((N_2, d_{I,2})\) would Pareto dominate that described by \((N_2, d_{I,2})\).

Now, consider the allocation of the \(\lambda_1\)-economy described by \((N_1, d_{I,1})\), where \(N_1 = N_2\) and \(d_{I,1}\) is such that \((1 - \lambda_1)p(d_{I,1})d_{I,1} = (1 - \lambda_2)p(d_{I,2})d_{I,2}\). Using that \(\lambda_1 < \lambda_2\) and (96), we have that \(d_{I,1} < d_{I,2}\), \(R_A(\hat{p}(d_{I,2})) < R_A(\hat{p}(d_{I,1}))\) and:

\[
W_S(N_1, d_{I,1}|\lambda_1) = W_S(N_2, d_{I,2}|\lambda_2) \text{ and } W_E(N_1, d_{I,1}|\lambda_1) > W_E(N_2, d_{I,2}|\lambda_2),
\]

which shows that the allocation induced by \((N_2, d_{I,2})\) in the \(\lambda_2\)-economy does not belong to the Pareto frontier of the \(\lambda_1\)-economy.
iv) Let \((\tilde{\theta}, \tau)\) be a Pareto optimal policy, then \(W(\tilde{\theta}, \tau) > W^*\).

Let \(\bar{\theta} = 1 - \lambda_0 + \varepsilon\) with \(\varepsilon > 0\) sufficiently small to ensure that any policy \((\bar{\theta}, \tau')\) is feasible, and \(\bar{\lambda} = 1 - \bar{\theta}\). From Proposition 7, the equilibrium of the economy with no intervention is in the Pareto frontier of the \(\lambda_0\)-economy. Using that by construction \(\bar{\lambda} < \lambda_0\), claims i) and iii) imply that the equilibrium of the economy with no intervention does not belong to the Pareto frontier of the \(\bar{\lambda}\)-economy. Choose an allocation of such an economy that Pareto improves the equilibrium with no intervention. Using Proposition 7 for the \(\bar{\lambda}\)-economy, such allocation is the equilibrium of a \(\lambda_0\)-economy after sum lump-sum transfers \(\tau'\). By construction, the policy \((\bar{\theta}, \tau')\) is feasible, induces the just described allocation and thus satisfies \(W(\bar{\theta}, \tau') > W^*\). A fortiori, any Pareto optimal policy \((\theta, \tau)\) satisfies \(W(\theta, \tau) > W^*\).

v) Let \((\theta, \tau)\) be a Pareto optimal policy, then \(\theta > 1 - \lambda_0\).

This can be proved by contradiction using the definition of a Pareto optimal policy, Proposition 7, and iv).

vi) Pareto optimal policies cannot induce first-best allocations

Suppose a Pareto optimal policy \((\bar{\theta}, \tau)\) induces a first-best allocation. From ii) we have that \((\bar{\theta}, \tau)\) induces an allocation in the Pareto frontier of the \(\lambda_0\)-economy. From here we can reproduce the arguments in v) to get a contradiction.

vii) For a feasible policy \((\bar{\theta}, \tau)\) we have that:

\[
W(\bar{\theta}, \tau) - W^* = (N(\bar{\theta}, \tau) - N^*)(RA(p(\bar{\theta}, \tau)) - 1) - N^*(RA(p^*) - RA(p(\bar{\theta}, \tau))).
\]

(97)

Taking into account that the government breaks-even by construction under the allocation induced by a feasible policy, the equation above is analogous to (44) and can be derived in the same manner.

viii) If \(w \leq \bar{w}\), then Pareto optimal policies induce less risk-taking.

We have from Proposition 5 that \(N^* = 1\). Let \((\bar{\theta}, \tau)\) be a Pareto optimal policy. From iv) and (97) we must necessarily have that \(p(\bar{\theta}, \tau) > p^*\).

ix) For any \(\lambda\), let \(\chi^*(R_S, \lambda)\) denote the variable defined in Proposition 4. We have that \(\partial \chi^*(R_S, \lambda) / \partial \lambda < 0\).

The partial equilibrium variable \(\chi^*(R_S, \lambda)\) satisfies (79). Notice that the expression for \(R^*_E(R_S, \chi^*)\) in (E2) and its partial derivatives satisfy (81). The property \(\partial \chi^*(R_S, \lambda) / \partial \lambda < 0\) then immediately results.

x) If \(w > \bar{w}\), then Pareto optimal policies increase investment.

We have from Proposition 5 that \(N^* < 1\) and \(R^*_S = 1\). Let \((\bar{\theta}, \tau)\) be a Pareto optimal policy and suppose that \(N(\bar{\theta}, \tau) \leq N^*\). From iv) and (97) we must necessarily have that \(p(\bar{\theta}, \tau) > p^*\).

Denote \(\bar{\lambda} = 1 - \bar{\theta}\). From v) we have that \(\bar{\lambda} < \lambda_0\). We have in addition that the equilibrium induced by \((\bar{\theta}, \tau)\) is an equilibrium of the \(\bar{\lambda}\)-economy and since \(N(\bar{\theta}, \tau) < 1\) we must have
that its equilibrium safe rate is \( R_S = 1 \). We thus have from Proposition 2 that
\[
p(\bar{\theta}, \tau) = \hat{p} \left( \hat{d}_I(\chi^*(R_S = 1, \bar{\lambda})) \right).
\]

Notice in addition from the proof of Lemma A.2 that the function \( \hat{d}_I(\chi) \) does not depend on \( \lambda \) and that from (66) that the function \( \hat{p}(d_I) \) neither depends on \( \lambda \). Using ix) and the monotonicity properties of \( \hat{d}_I(\chi) \) and \( \hat{p}(d_I) \), described in Lemma A.2 and Lemma 4, respectively, we have that \( \bar{\lambda} < \lambda_0 \) implies that
\[
p(\bar{\theta}, \tau) = \hat{p} \left( \hat{d}_I(\chi^*(R_S = 1, \bar{\lambda})) \right) < \hat{p} \left( \hat{d}_I(\chi^*(R_S = 1, \lambda_0)) \right) = p^*,
\]
which contradicts that \( p(\bar{\theta}, \tau) > p^* \).

ix) If \( w > \bar{w} \) and \( X \) sufficiently small, then Pareto optimal policies increase risk-taking
We have from Proposition 5 that \( N^* < 1 \) and \( R_S^* = 1 \). For \( X \) sufficiently small, due to continuity arguments we have that a Pareto optimal policy \( (\bar{\theta}, \tau) \) cannot induce full investment, that is \( N(\bar{\theta}, \tau) < 1 \) and \( R_S(\bar{\theta}, \tau) = 1 \). We can reproduce the arguments in the proof of claim x) to prove that \( p(\bar{\theta}, \tau) < p^* \).