End-of-Day Trading via Banks

October 1, 2019

Abstract

Investment banks like Goldman Sachs have started a “guaranteed close” business where investors looking to buy or sell shares of a certain stock can get a guarantee from the investment bank to honor their orders at the closing price set on the primary exchange. Using the TAQ data from 2012 to 2018, we find that when the fraction of off-exchange at-close-price trades increases, the informativeness of closing price increases. We develop a model featuring three trading venues: regular session, close auction and bank’s close service, and three types of investors: index funds, liquidity traders, and informed traders, who endogenously choose trading venues. A bank conducting “guaranteed close” business competes with the exchange on transaction fees, and gains profit from both transaction fees and trading strategically utilizing the order flow information. Our model shows that the introduction of the bank as an alternative closing venue improves price discovery at the market close by concentrating the price-relevant information into the exchange, absorbing the uninformed noise orders and amplifying the informed orders.
1 Introduction

Over recent years, the total volume executed in the primary listing markets’ closing auctions has increased more than 70%, from 200 million shares per day in 2012 to almost 350 million shares per day in 2016\(^1\). In the meantime, continuous trading volume has increased only 13%. Closing auctions on the primary listing markets reached nearly 5% of the total executed volume in 2016.

Despite the drastically increasing trading volume at closing auctions, they have been only conducted on the primary exchanges. The lack of competition has driven up the fees charged by the primary exchanges. In particular, both NYSE and Nasdaq have increased their fees considerably over the last few years. NYSE’s base rate has gone up by 16% and Nasdaq increased its fee by 60\(^2\).

Starting from late 2016, however, Goldman Sachs leads a pack of investment banks who start elbowing into the crucial exchange business of end-of-day trading. Specifically, they build up a business called the “guaranteed close”. Investors looking to buy or sell shares of a stock can get a guarantee from an investment bank to honor their orders at the closing price set on the exchange where the stock is listed. As it comes close to 4 pm, the bank pairs buyers with sellers. It can send unmatched orders to the exchange or take the other side of the trade itself, storing the extra stock shares or short interest on its books overnight. Figure 1 shows that, the share of closing-price trades executed outside of exchanges, by broker-dealers like Goldman, doubled from mid-2015 through the end of last year, from 16% to 32%. The “guaranteed close” products are reported to be used by index-fund managers including Vanguard Group and BlackRock Inc., as well as smaller broker-dealers\(^3\).

Analysis of this phenomenon helps us determine what is a good market design for end-of-day trading. Specifically, how will the introduction of the banks as an alternative closing venue impact the venue choice of different types of investors? How do the banks optimally set the transaction fees to maximize the profits they can make through the “guaranteed close” business? Will the introduction of the banks help to reduce the fees charged on the closing auctions by the primary exchanges? More importantly, will the emergence of the bank-intermediated guaranteed close services improve the price discovery and the close price informativeness, or the other way around? Given that the close price is commonly regarded as the benchmark to track stock performances, understanding its informativeness is of great importance for the market participants as well as policy makers and regulators. Our paper takes a first step to shed light on those questions.

\(^1\)Securities Exchange Act Release No. 80683, supra note 2


\(^3\)Goldman Cashes In on Passive-Investing Boom With Big 4 P.M. Trade, August 26, 2018, Wall Street Journal
First, we empirically investigate how the trading activities via banks affect the informativeness of the close price. We utilize the NYSE TAQ (trade and quote) millisecond level data, which contain detailed information on the volume, execution prices, trade sale conditions and so on for each trade. We proxy the trade volume via banks by the off-exchange trades between 4:00 pm –4:10 pm EST and executed at the official close price\(^4\). To measure the informativeness of the close price, we consider how well the close price predicts next day’s open price. We take two approaches to investigating this. First, we create a measure for the distance between the close price and the next-day open price that is grounded on our theory. Then we demonstrate that when the fraction of off-exchange at-close-price trades increases, the distance between the two prices shrinks. The results suggest that more information is incorporated in the close price when we have more off-exchange trades. As our second approach, we regress the next-day open price on the close price, both differenced by the average price before the market closes, and we control for the market fundamentals, return volatility and stock and trading day fixed-effects. We find that when a higher fraction of at-close-price daily trade volume is executed off exchange, the return from last several minutes before market close to market close predicts better the return from last several minutes before market close to the next

\(^4\)The same definition is used in the SEC analysis report: The Division of Economic and Risk Analysis (2017), “Bats Market Close: Off-Exchange Closing Volume and Price Discovery”.
day’s market open. The results from both approaches are robust during different time period and in sub-samples, are and stronger for S&P500 firms, which appear more frequently in index funds’ portfolios. These empirical findings altogether suggest that the introduction of the banks as a new venue to trade improves price discovery.

Next, we develop a static three-period model (Period 0, 1, and 2) with three trading venues: regular session, close auction, and the “guaranteed close” service through a bank. There is one single type of asset in the market and its true value will be realized only in Period 2. Our model features three types of investors: for-profit traders who can learn the asset value at cost, liquidity traders who have to trade a given quantity due to liquidity need, and index funds who want to adjust their holdings at the close price to match their benchmark; plus a group of competitive market makers and a bank. In Period 0, market makers on regular session announce the bid and ask prices and the bank announce the transaction fee it charges, then for-profit traders determine whether to get informed and all the three types of investors determine which venues they will trade at and submit their orders. Note that the fee charged by the bank is chosen to maximize its expected total profit from both charging fees and trading based on rational expectations of all the possible equilibria, and the bid and ask prices of the market makers are chosen to make them break-even unconditionally. Later in Period 0, the regular session executes orders at the bid and ask prices as announced at the beginning of the period and ends. In Period 1, the bank observes the order flow it receives and identifies the index funds knowing they are uninformed, then submit both the imbalance orders and a strategical position for itself to the close auction. Then the market makers of the close auction receive all the orders simultaneously and determine the close price \( p^c \) which equals its expectation of the asset value based on the order flow. Then the orders submitted to the close auction and the bank are executed at the close price. In Period 2, the asset value \( v \) is revealed and paid. Our key objective is to analyze the informativeness of \( p^c \), for the fundamental value \( v \) of the asset.

The equilibrium in our model has the following elements: the quoting strategy of the exchange market makers, the fee charged by the bank, the information acquisition decision of the for-profit traders, and the trading venue choices of the informed traders, liquidity traders, index funds and the bank. In equilibrium, the competitive market makers break even in expectation both in the regular session and in the close auction. All the investors and the bank maximize their expected net profits. We solve for the equilibrium in two steps. First, we derive the equilibrium strategies of all market participants given the bank’s fee along with all the exogenous variables. Second, we derive the profit of the bank charging a given fee, and find the optimal fee to charge. Following Kyle (1985), we are able to obtain a linear Nash Equilibrium in terms of the close price and the proprietary trading orders of the bank. The equilibrium results show that introduction of bank’s close service improves the informativeness of the close price. Our results are driven by two simple forces. On the one hand, the bank will strategically learn information about the asset value from the orders it receives, then amplifies informed orders and absorbs noise orders. On the other hand, the bank can
identify index funds for sure knowing that they are not informed, thus take an opposite position that reduces the noisy price effect of index funds. The interaction of the two determines the equilibrium structure we receive. In particular, the bank’s participation into the closing business increases the fraction of useful information in the close auction, which improves the informativeness of the close price. Numerical solutions and a dynamic version of our model are still in development.

The remainder of this paper is organized as follows. Section 2 presents the institutional background and summarizes the relevant literature. Section 3 presents our empirical analysis. Section 4 illustrates the static model and characterizes the equilibrium. Section 5 concludes.

2 Literature Review

To the best of our knowledge, this paper is the first to empirically and theoretically show that off-exchange MOC activities can improve price discovery. The finding stands in contrast to the SEC report The Division of Economic and Risk Analysis (2017) and various reports by the investment banks that conduct the market closing service\(^5\), where they empirically find that the off-exchange MOC activities do not affect price discovery. They use different empirical methods from ours, and none of them provides a theoretical model.

Our paper is related to a couple of existing models of dark pools and alternative trading venues. Our model is similar to that of Zhu (2014), who shows that adding a dark pool along the exchange can improve price discovery. However, the mechanism in his model is different from ours. He focused on the fact that dark pools do not guarantee execution and informed traders tend to trade in the same direction, crowd on the heavy side of the market, and face a higher execution risk in the dark pool. This incentivizes the informed traders to remain in the exchange. In our model, informed traders tend to remain in the exchange because they want to avoid their order flow information from being exploited by the bank.

Other models about trading venue choice include Hendershott and Mendelson (2000), Buti et al. (2017), Ye (2010). However, their models either do not model asymmetric information, or do not allow all the agents to freely select venues. Besides, they do not consider the endogenous transaction fee competition among different venues. As described in Zhu (2014), “analyzing endogenous trading fees, which can potentially interact with the self-selection mechanism, is left for future research.”. Our model attempts to fill the gap, by incorporating all the three elements, asymmetric information, all agents freely choosing venues, and endogenous fee competition.

The existing models of dark pools and alternative trading venues do not feature that the trading

\(^5\)For example, Credit Suisse market commentary report “Off-Exchange Closing Volumes Rise”, 30 April 2019, Victor Lin
venues strategically use the order flow information and gains informational advantage. In that sense, our model is close to Röell (1990). In her model, dual-capacity dealers can judge the motives behind their customers’ orders, and they can trade profitably on their own account. He finds that as a result of dual-capacity dealing, transaction costs for liquidity-motivated traders in the aggregate fall, but they rise for those traders who are unable to convince any dealer that they have no inside information. However, in his model informed traders do not choose which dealer to trade with, which is different from our model. Fishman and Longstaff (1992) model dual trading in the futures market, featuring both information asymmetry and endogenous commission fee, however their model is under a very simplified setting with only two traders, and do not feature venue choice.

3 Institutional Background

3.1 Closing Auction

Each listing exchange has unique closing auction mechanisms, however, most of them are in large similar. Now we introduce the closing auction mechanism in NYSE, whose characteristics are largely shared by other exchanges like NYSE Arca and Nasdaq.

There are several order types that can be used in NYSE closing auction, with the most common being Market-On-Close (MOC) and Limit-On-Close (LOC) orders. An MOC order is an unpriced order to buy or sell a security at the closing price and is guaranteed to receive an execution in the NYSE closing auction. An LOC order sets the maximum price an investor is willing to pay, or the minimum price for which an investor is willing to sell, in the closing auction. An LOC order priced better than the final closing auction price is guaranteed to receive an execution in the NYSE closing auction.

The timeline of the close auction is as follows. From 6:30am in the morning, MOC and LOC orders can be entered and existing MOC and LOC orders can be canceled until 3:50pm. At 4:00pm, the regular session trading ends and the close auction commences. The method for determining the closing prices is to follow two principles: (1) maximize the number of shares that can be executed in the closing auction (2) minimize the difference between the closing price and a reference price is multiple closing price satisfy principle (1). Effectively the auction aggregates the supply and demand curve constituted by the MOC and LOC orders, and the transaction price and trade volume is determined by the intersection of the two curves.

Market makers play an important role in the closing auction. The Designated Market Makers (DMMs) recognized by the NYSE set the closing price at a level that satisfies all interest that is willing to participate at a price better than the closing auction price, and supplying liquidity as needed to offset any remaining auction imbalances that exist at the closing bell. Market-on-Close orders are essentially guaranteed to be executed thanks to the Designated Market Makers.

### 3.2 Bank “Guaranteed Close” Service

A group of investment banks including Goldman Sachs, Morgan Stanley, Credit Suisse Group AG, and UBS Group AG, have started the “guaranteed close” business over the past few years. Investors looking to buy or sell shares of a stock can get a guarantee from an investment bank to honor their orders at the closing price set on the corresponding primary exchange, where the stock is listed. At 4:00 p.m., the bank pairs the buyers with the sellers of the stock. For the unmatched orders, it can either send them to the exchange or take the other side of the trade itself, storing the extra shares or short interest on its books overnight. According to the WSJ article we cite, the “guaranteed close” venue is used by index-fund managers including Vanguard Group and BlackRock Inc., as well as some smaller broker-dealers. People familiar with the matter report that recently Goldman Sachs cut the fees it charges other broker-dealers for trading in its MOC mechanism to zero (buy-side clients still pay a fee). This suggests that the bank can make profits besides charging the fees.

### 4 Empirical Analysis

#### 4.1 Data and Methodology

We empirically analyze the relation between closing price informativeness and off-exchange trading activity at the close. To construct our sample, we utilized the NYSE TAQ millisecond level trade and quote data spanning from 2012 to 2018 to determine the amount of closing auction volume, and the amount of off-exchange Market-on-Close (MOC) activity. Firstly, we exclude invalid or erroneous trades that were later cancelled or changed. Then we exclude the days when there is no closing print in TAQ, which are the days when no closing auction were held.

As for the close auction activities, for each stock in each day, we measure the trading volumes that happen in the primary exchanges’ close auctions by the total volume of the TAQ trades with the sale condition of 6 (Closing Print). These are primary listing auction volumes, according to the SEC report (*The Division of Economic and Risk Analysis, 2017*). The market close prices are the transaction price of these closing trades. Similarly, for each stock in each day, we measure the open auction volume by the trades occurred in the open auctions, which in TAQ are trades with the
sale condition of O (Market Center Opening Trade) or Q (Market Center Official Open), and the market open price by the transaction price of these trades.

To measure the off-exchange market-on-close activity, we follow the approach described in The Division of Economic and Risk Analysis (2017). Specifically, we consider all the trades from TAQ that are not cancelled or corrected and occur between 4:00–4:10 PM EST at the official market close price determined by the close auction. Although these trades are executed at the market close price, they may not all come from market-on-close orders. In the SEC report however, the authors validate this approach using two other regulatory datasets with more detailed information—the FINRA Trade Reporting Facility data, and the FINRA-provided Audit Trail data. These two datasets identify off-exchange executions by venue and trace the executions back to the original orders. They measured the off-exchange executed volume of MOC orders using these two datasets, and find that the measure is almost identical to the measure based on TAQ data. Indeed, some of these MOC orders may be executed via market makers other than banks. But since banks anecdotally are the major players in providing the closing services, and that other market makers who can trade based on the MOC order flow information essentially play the same role as banks in our model, we consider our measure of the MOC activities a good proxy for the trades executed by the bank’s guaranteed close service.

Based on the TAQ data, we also measure the total trading volume of each stock in each day by aggregating all the trades in the regular trading session between 9:30am and 4:00pm, together with the trades in the close and open auctions. The fraction of trades that occurred in the close auction and off-exchange at the close is hence calculated respectively as the corresponding trade volumes divided by the total trading volumes. We also calculated the intraday return volatility based on TAQ data. Specifically, we firstly aggregate the execution prices of trades to get the volume-weighted-average-price at every 1-minute interval, then calculate the log return in every minute, and calculate the standard deviation of these log returns.

Figure 2 plots the fraction of daily total trade volumes of all the stocks in TAQ data that occurred in the close auctions from 2012 to 2018. We see that the closing auction activities steadily increase. In 2012, the close auction takes up 4% of the total trade volume, yet in 2018, the number increases to over 10%. Figure 3 plots the fraction of daily total trade volumes of all the stocks in TAQ data that occurred off exchange using the market-on-close orders. We see the off-exchange MOC activity was negligibly low before 2016. But after that it experienced a significant and rapid growth after that. In recent days, the off-exchange market-on-close orders take almost 4% of the daily trade volume. These patterns are in accordance to existing observations.

\footnote{For example, see the Wall Street article: Goldman Cashes In on Passive-Investing Boom With Big 4 P.M. Trade, August 26, 2018, and SEC report: Securities Exchange Act Release No. 34-80683 (May 6, 2017)}
Our aim is to study how the off-exchange market-on-close activities affect the informativeness of market close price determined by the close auction. To be more concrete, we consider how well the close price incorporates the overnight information relevant to a stock’s value. We use two simple measures of the informativeness of the close price. The first measure is the “closeness” between the close price and next day’s open price. The second measure is the “predictability” of the close price on the next day’s open price.

We firstly measure the “closeness” between the close price and next day’s open price by the differ-
ence in the close and next day’s open price scaled by the last-5-minute volume-weighted-average-price or the last-15-minute volume-weighted-average-price. We also consider a measure of the “closeness” that is the log difference between close and next day’s open price. Then we take the absolute value or the squared value of the differences and the log difference. The reason to use scaled difference and log difference is to make sure the close price of stocks with lower price is not mechanically closer to the open price. As shown in the following regression equations – Equations (1)- (3), we regress the “closeness” measures for each stock on the fraction of trades occurred using the off-exchange market-on-close orders, controlling for a set of variables \( X_{i,t} \) including market capitalization, trade volume and intraday return volatility, etc.

These measures have natural interpretations, for example, when use the squared value of the differences, the expected value of the “closeness” measure is simply the mean squared error between close and open price. The regressions essentially study how can off-exchange market-on-close trading activities affect the mean squared error between close and open price, controlling for other stock-date characteristics. This establishes a tight connection to our model, where we also use the mean squared error as a price informativeness measure.

\[
\text{dist}(\frac{\text{Open}_{i,t+1} - \text{Close}_{i,t}}{\text{Last-5-min VWAP}}) = \beta_1 \text{Frac}_{Off} \text{Ex}_{i,t} + \gamma \text{X}_{i,t} + a_i + b_t + \varepsilon_{i,t} \tag{1}
\]
\[
\text{dist}(\frac{\text{Open}_{i,t+1} - \text{Close}_{i,t}}{\text{Last-15-min VWAP}}) = \beta_1 \text{Frac}_{Off} \text{Ex}_{i,t} + \gamma \text{X}_{i,t} + a_i + b_t + \varepsilon_{i,t} \tag{2}
\]
\[
\text{dist}(\log \text{Open}_{i,t+1} - \log \text{Close}_{i,t}) = \beta_1 \text{Frac}_{Off} \text{Ex}_{i,t} + \gamma \text{X}_{i,t} + a_i + b_t + \varepsilon_{i,t} \tag{3}
\]

where \( \text{dist} \) is a distance function that we use either absolute distance or squared distance. For stock \( i \) at date \( t \), \( \text{Open}_{i,t} \) is market open price, \( \text{Close}_{i,t} \) is market close price. \( \text{Frac}_{Off} \text{Ex}_{i,t} \) is the fraction of trade volume occurred off exchange using market-on-close orders. \( \text{X}_{i,t} \) is a vector of other variables that we control for, including log market capitalization, log trade volume, the fraction of trade volume occurred in close auction, and intraday return volatility. \( a_i \) and \( b_t \) are stock and date fixed effects that we controlled for.

The alternative measure of the close price informativeness is how well the close price predicts next day’s open price. If the close price fully and accurately incorporates all the relevant overnight information, we would expect the close price to be perfectly correlated with next day’s open price. So a change in close price implies a one-for-one change in the open price. In reality, this correlation is lower than one, and our measure is based on such correlation. The closer is the correlation to one, we think the close price is more informative. We examine the predictability of the return from last 5 minutes in the regular session to the market close (Last5min_Close) on the return from last 5 minutes of the regular session to the next day’s open price (Last5min_Open) by running the regression specified by Equation (4). Similarly as in Equation (5) we run the regression where we calculate the return from the last 15 minutes of the regular session. We run the regression using returns essentially in order to normalize the close and open price by the last-5-minute or
last-15-minute VWAP.

\[
\text{Last5min}_\text{Open}_{i,t} = \beta_0 \text{Last5min}_\text{Close}_{i,t} + \beta_1 \text{Last5min}_\text{Close}_{i,t} \times \text{Frac}_\text{Off Ex}_{i,t} \\
+ \gamma X_{i,t} + a_i + b_t + \varepsilon_{i,t}
\]  \hspace{1cm} (4)

\[
\text{Last15min}_\text{Open}_{i,t} = \beta_0 \text{Last15min}_\text{Close}_{i,t} + \beta_1 \text{Last15min}_\text{Close}_{i,t} \times \text{Frac}_\text{Off Ex}_{i,t} \\
+ \gamma X_{i,t} + a_i + b_t + \varepsilon_{i,t}
\]  \hspace{1cm} (5)

where for stock \(i\) on date \(t\), \text{Last5min}_\text{Open}_{i,t} is the return from last 5 minutes in the regular session to market open, calculated based on the last-5-minutes value-weighted-average-price and next day’s official market open price. \text{Last5min}_\text{Close}_{i,t}, is the return from last 5 minutes in the regular session to market close. And similarly are \text{Last15min}_\text{Open}_{i,t} and \text{Last15min}_\text{Close}_{i,t} defined. \text{Frac}_\text{Off Ex} is the fraction of trade volume occurred off exchange using market-on-close orders. \(X_{i,t}\) is a vector of other variables that we control for, including log market capitalization, log trade volume, the fraction of trade volume occurred in close auction, and intraday return volatility. \(a_i\) and \(b_t\) are stock and date fixed effects that we controlled for.

The coefficient that indicates the effect of off-exchange activity on the price informativeness is \(\beta_1\) in front of the interaction term. As we will see in the estimation results below, \(\beta_0\) is usually about 0.6. A positive \(\beta_1\) suggests that an increase in the off-exchange activity will strengthen the correlation between close price and open price and make it closer to one. We interpret this as the off-exchange trading activity improves the “predictability” of the close price on the open price, hence improves the close price informativeness.

Table 1 reports the summary statistics of all the variables used in the regression from 2016 to 2018, since our main empirical results use the sample from 2016 to 2018. It is worthwhile to note that, for robustness, we trimmed the outliers in any variables except for market capitalization at the 1% tails before we conduct any analysis.
Table 1: Summary Statistics

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<th>min</th>
<th>p25</th>
<th>p50</th>
<th>p75</th>
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<td>Last15m_Close</td>
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<td>log(Open)-log(Close)</td>
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<td>0.4734</td>
<td>0.4570</td>
<td>0.0000</td>
<td>0.1289</td>
<td>0.3373</td>
<td>0.6798</td>
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<td>Frac_Close Auction</td>
<td>5931900</td>
<td>0.0365</td>
<td>0.0553</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0089</td>
<td>0.0542</td>
<td>0.3270</td>
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<td>Frac_Off Ex</td>
<td>5899023</td>
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<td>0.0000</td>
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<td>0.0330</td>
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<td>1.5718</td>
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Notes: Table reports the summary statistics of the key variables used in our analysis. We use TAQ trade data from 2916 to 2018, and aggregated the transaction-level data to variables at daily frequency. Last5m_Open is the return calculated by last 5 minutes VWAP and next day’s open price. Last5m_Close is the return calculated by last 5 minutes VWAP and close price. Similarly are Last15m_Open and Last15m_Close calculated, using instead the last 15 minutes VWAP. |log(Open)-log(Close)| is the absolute log difference between close price and next day’s open price. Frac_Close Auction is each individual stock’s fraction of daily trades that occurred in the close auction. Frac_Off Ex is each individual stock’s fraction of daily trades that occurred off-exchange yet at close price. log(Market Cap) is log market capitalization based on CRSP data. log(Trade Volume) is log total daily trading volume for each individual stock. Intraday Return Volatility is the volatility of the log VWAP return calculated at every 1 minute interval.

4.2 Estimation Results

We firstly show the set of estimation results based on our first measure of price informativeness—the “closeness” between close price and next day’s open price.

Table 2 reports the estimation results for Equation (1), where we regress the absolute difference and squared difference between close and open price, scaled by the last-5-minute volume-weighted-average-price, on the fraction of trade volumes executed off exchange using market-on-close orders. We see that as the fraction of trade volumes executed off exchange using market-on-close orders increases, the absolute and squared difference between the close and open prices significantly decreases. The result is robust after we controlled for a rich set of control variables, stock fixed effects and date fixed effects.

Table 3 reports estimation results for Equation (2), where the outcome variable now is the absolute and squared difference between close and open price, scaled by the last-15-minute volume-weighted-average-price. Table 4 reports estimation results for Equation (3), where the outcome variable is the absolute and squared log difference between close and open price. The results in these two tables are similar to that in Table 2. As the fraction of trade volumes executed off exchange using market-on-close orders increases, the close price gets significantly “closer” to the next day’s open price.
Table 2: Closeness between Close Price and Next Day’s Open Price: price difference scaled by last 5 minutes VWAP

<table>
<thead>
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<th></th>
<th>(1) (ΔNormalized Price)$^2$</th>
<th>(2)</th>
<th>(3) (ΔNormalized Price)$^2$</th>
<th>(4)</th>
<th>(ΔNormalized Price) $^2$</th>
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</thead>
<tbody>
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<td>Frac Off Ex</td>
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<td>0.058***</td>
<td>-0.427***</td>
<td>0.035***</td>
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<tr>
<td>log(Total Volume)</td>
<td></td>
<td>0.058***</td>
<td></td>
<td>0.035***</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(11.560)</td>
<td></td>
<td>(11.994)</td>
<td></td>
</tr>
<tr>
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<td>-0.066*</td>
<td>-0.066*</td>
<td>-0.032</td>
<td>(-1.318)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-1.709)</td>
<td>(-1.318)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>log(Market Cap)</td>
<td>0.003</td>
<td>0.004</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.129)</td>
<td>(0.359)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intraday Return Volatility</td>
<td>2.612***</td>
<td>1.767***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(10.999)</td>
<td>(13.249)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>357162</td>
<td>357162</td>
<td>335283</td>
<td>335283</td>
<td></td>
</tr>
<tr>
<td>R$^2$</td>
<td>0.282</td>
<td>0.347</td>
<td>0.284</td>
<td>0.348</td>
<td></td>
</tr>
<tr>
<td>Stock FE</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td></td>
</tr>
<tr>
<td>Date FE</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td></td>
</tr>
</tbody>
</table>

$t$ statistics in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table reports the regression results of the closeness between close price and next day’s open price on the fraction of shares traded off exchange yet at close price. For the outcome variable, $\Delta$ Return is (close price/last 5 min VWAP - next-day open price / last 5 min VWAP), and we used both the absolute value and squared value of $\Delta$Return to measure the closeness between close price and next day’s open price. Frac Off Ex is each individual stock’s fraction of daily trades that occurred off exchange yet at the close price. The rest are control variables. log (Total Volume) is log total daily trading volume for each individual stock. Frac Close Auction is each individual stock’s fraction of daily trades that occurred in the close auction. log(Market Cap) is log market capitalization based on CRSP data. Intraday Return Volatility is the volatility of the log VWAP return calculated at every 1-minute interval. The sample includes TAQ transaction-level trade data for S&P 500 firms from 2016-2018, and variables were aggregate to daily frequency. Stock fixed effects and date fixed effects are added to all regressions. Standard errors are double clustered standard errors at stock and at date level. For robustness, outliers in each variable are trimmed at 1% tails.
Table 3: Closeness between Close Price and Next Day’s Open Price: price difference scaled by last 15 minutes VWAP

<table>
<thead>
<tr>
<th></th>
<th>(1) (ΔNormalized Price)^2</th>
<th>(2)</th>
<th>(3) (ΔNormalized Price)^2</th>
<th>(4)</th>
<th>(ΔNormalized Price)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frac_Off Ex</td>
<td>-0.823***</td>
<td>-0.433***</td>
<td>-0.410***</td>
<td>-0.193***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-6.698)</td>
<td>(-6.276)</td>
<td>(-3.681)</td>
<td>(-3.003)</td>
<td></td>
</tr>
<tr>
<td>log(Total Volume)</td>
<td></td>
<td>0.062***</td>
<td>0.036***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(11.883)</td>
<td>(12.160)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Frac_Close Auction</td>
<td></td>
<td>-0.081**</td>
<td>-0.037</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-2.023)</td>
<td>(-1.483)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>log(Market Cap)</td>
<td></td>
<td>0.001</td>
<td>0.003</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.032)</td>
<td>(0.299)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intraday Return Volatility</td>
<td></td>
<td>2.519***</td>
<td>1.696***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(10.306)</td>
<td>(12.332)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>359083</td>
<td>359083</td>
<td>337141</td>
<td>337141</td>
<td></td>
</tr>
<tr>
<td>R^2</td>
<td>0.277</td>
<td>0.345</td>
<td>0.278</td>
<td>0.347</td>
<td></td>
</tr>
<tr>
<td>Stock FE</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td></td>
</tr>
<tr>
<td>Date FE</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td></td>
</tr>
</tbody>
</table>

t statistics in parentheses
* p < 0.1, ** p < 0.05, *** p < 0.01

Table reports the regression results of the closeness between close price and next day’s open price on the fraction of shares traded off exchange yet at close price. For the outcome variable, Δ Return is (close price/last-15-min VWAP - next-day open price / last-15-min VWAP), and we used both the absolute value and squared value of ΔReturn to measure the closeness between close price and next day’s open price. Frac_Off Ex is each individual stock’s fraction of daily trades that occurred off exchange yet at the close price. The rest are control variables. log (Total Volume) is log total daily trading volume for each individual stock. Frac_Close Auction is each individual stock’s fraction of daily trades that occurred in the close auction. log(Market Cap) is log market capitalization based on CRSP data. Intraday Return Volatility is the volatility of the log VWAP return calculated at every 1-minute interval. The sample includes TAQ transaction-level trade data for S&P 500 firms from 2016-2018, and variables were aggregate to daily frequency. Stock fixed effects and date fixed effects are added to all regressions. Standard errors are double clustered standard errors at stock and at date level. For robustness, outliers in each variable are trimmed at 1% tails.
Table 4: Closeness between Close Price and Next Day’s Open Price: log price difference

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(log Open − log Close)^2</td>
<td></td>
<td>(log Open − log Close)</td>
<td></td>
</tr>
<tr>
<td>Frac_Off Ex</td>
<td>-0.007***</td>
<td>-0.370***</td>
<td>-0.004***</td>
<td>-0.167***</td>
</tr>
<tr>
<td></td>
<td>(-6.406)</td>
<td>(-5.613)</td>
<td>(-3.735)</td>
<td>(-2.693)</td>
</tr>
<tr>
<td>log(Total Volume)</td>
<td>0.000***</td>
<td>0.029***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(10.396)</td>
<td>(10.576)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Frac_Close Auction</td>
<td>-0.000</td>
<td>-0.019</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-1.046)</td>
<td>(-0.785)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>log(Market Cap)</td>
<td>-0.000</td>
<td>-0.011</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-1.421)</td>
<td>(-1.013)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intraday Return Volatility</td>
<td>0.020***</td>
<td>1.487***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(10.176)</td>
<td>(11.866)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

N 336771 336771 315535 315535
R^2 0.297 0.345 0.299 0.347
Stock FE YES YES YES YES
Date FE YES YES YES YES

t statistics in parentheses
*p < 0.1, **p < 0.05, ***p < 0.01

Table reports the regression results of the closeness between close price and next day’s open price on the fraction of shares traded off exchange yet at close price. For the outcome variable, we used both the absolute value and squared value of log(close price) - log(next-day open price), to measure the closeness between close price and next day’s open price. Frac_Off Ex is each individual stock’s fraction of daily trades that occurred off exchange yet at the close price. The rest are control variables. log (Total Volume) is log total daily trading volume for each individual stock. Frac_Close Auction is each individual stock’s fraction of daily trades that occurred in the close auction. Intraday Return Volatility is the volatility of the log VWAP return calculated at every 1-minute interval. log(Market Cap) is log market capitalization based on CRSP data. The sample includes TAQ transaction-level trade data for S&P 500 firms from 2016-2018, and variables were aggregate to daily frequency. Stock fixed effects and date fixed effects are added to all regressions. Standard errors are double clustered standard errors at stock and at date level. For robustness, outliers in each variable are trimmed at 1% tails.

Next, we show the set of estimation results based on our second measure of price informativeness—the “predictability” of close price on next day’s open price.

Table 5 reports the results of the regression in Equation 4 and 5. In Column (1) and (2), we regress the last-5-minutes to next day’s open returns on the last-5-minutes to the market close returns in two subsamples— one with the fraction of trade volumes executed off exchange greater than 2%, and the other one with such fraction lower than 2%. Comparing the two columns, we see that the close price predicts the next day’s open price better in the sample where higher fraction of trade volumes are executed off exchange. In Column (3) and (4), we add the interaction term of last-5-minutes to close returns and the fraction of trade volumes executed off exchange using market-on-close orders, and conduct the regression in the subsample of S&P 500 firms and the full sample. We see that the coefficient in front of the interaction term is statistically significantly positive and economically large, meaning that the off-exchange market-on-close activities can strengthen the predictability of close price on next day’s open price. Comparing the results between S&P 500 firms and the full sample, we see the effect is estimated to be stronger within the S&P 500 firms sample. This corresponds well to our model and proposed mechanism, where the bank’s guaranteed close service attracts uninformed orders from index funds and improves price discovery. Since S&P 500 firms are more widely held by index funds and other liquidity traders, the effect of bank’s service on price
discovery should manifest to be stronger for their stocks. In Column (5) and (6), we simply change the outcome variables and main explanatory variables to be based on last-15-minutes to close and open returns. We see the results remain quantitatively the same.

Table 5: Predictability of Close Price on Next Day’s Open Price

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Frac Off Ex &gt;0.02</td>
<td>Frac Off Ex &lt;0.02</td>
<td>S&amp;P 500 All Firms</td>
<td>S&amp;P 500 All Firms</td>
<td>S&amp;P 500 All Firms</td>
<td></td>
</tr>
<tr>
<td>Last5m Close</td>
<td>0.658***</td>
<td>0.506***</td>
<td>0.501***</td>
<td>0.565***</td>
<td>0.551***</td>
<td>0.585***</td>
</tr>
<tr>
<td>Last5m Close × Frac Off Ex</td>
<td>3.232**</td>
<td>1.967**</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.120)</td>
<td>(2.540)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Last15m Close</td>
<td>0.551***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(18.659)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Last15m Close × Frac Off Ex</td>
<td>3.031**</td>
<td>2.080**</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.486)</td>
<td>(2.402)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>log(Market Cap)</td>
<td>0.001***</td>
<td>0.000**</td>
<td>0.000***</td>
<td>0.000</td>
<td>0.000***</td>
<td>0.000***</td>
</tr>
<tr>
<td></td>
<td>(4.293)</td>
<td>(2.285)</td>
<td>(3.058)</td>
<td>(1.081)</td>
<td>(3.079)</td>
<td>(1.106)</td>
</tr>
<tr>
<td>log(Trade Volume)</td>
<td>0.000*</td>
<td>0.000***</td>
<td>0.000***</td>
<td>0.000***</td>
<td>0.000***</td>
<td>0.000***</td>
</tr>
<tr>
<td>Frac Close Auction</td>
<td>0.000</td>
<td>0.001</td>
<td>0.001</td>
<td>-0.000</td>
<td>0.001</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>(1.191)</td>
<td>(1.572)</td>
<td>(1.606)</td>
<td>(-0.002)</td>
<td>(1.595)</td>
<td>(0.187)</td>
</tr>
<tr>
<td>Intraday Return</td>
<td>0.006</td>
<td>-0.000</td>
<td>-0.000</td>
<td>0.001***</td>
<td>-0.000</td>
<td>0.001***</td>
</tr>
<tr>
<td>Volatility</td>
<td>(0.728)</td>
<td>(-0.069)</td>
<td>(-0.012)</td>
<td>(4.629)</td>
<td>(-0.114)</td>
<td>(4.509)</td>
</tr>
</tbody>
</table>

|                  | 91616       | 268703      | 338464      | 319426      | 340135      | 3225526     |
|                  | 0.360       | 0.301       | 0.308       | 0.145       | 0.309       | 0.152       |
| N                | Stock FE    | YES         | YES         | YES         | YES         | YES         |
|                  | Date FE     | YES         | YES         | YES         | YES         | YES         |

* p < 0.1, ** p < 0.05, *** p < 0.01

In the Table 6 and Table 7 in the Appendix, we provide robustness test results, where we extend our sample to be in 2012-2018. And we used different time periods to fully exploit the variations in the data and provide basic test for external validity across different time periods. Some estimations use data from 2012 to 2018, while others use data only from 2012 to 2015, and the estimation is conducted for both the S&P 500 firms sample and the full sample. The results remain similar. This address the concern that the spike of off-exchange closing activity starting from early 2017 implies some structural changes. But it is worthwhile to note the caveat that we haven’t controlled for...
intraday return volatility in those regressions, as we are still preparing the volatility data for the pre-2016 period.

5 Model

In this section, we present a three-period model with three trading venues: regular session, close auction and bank close service. Then we solve for the equilibrium strategies of market participants and analyze the effect of bank close service on the informativeness of close price.

5.1 Markets and traders

There are three trading periods, denoted by t=0,1,2. At the end of period 2, an asset pays an uncertain dividend \( v \) that is equally likely to be \(+\sigma\) or \(-\sigma\). The asset value \( v \) is publicly revealed at the beginning of period 2. For example, this revelation of private information may represent an earnings announcement that happens overnight.

Three trading venues operate in parallel: a regular session, a close auction and a bank’s guaranteed close service. Each charge a transaction fee of \( \mu_r, \mu_c, \mu_b \) per unit of transaction. \( \mu_r \) and \( \mu_c \) are exogenous, \( \mu_r > \mu_c \), while \( \mu_b \) can be chosen strategically by the bank but announced at the beginning of period 0. Market orders sent to the regular session or the close auction arrive simultaneously. In the regular session, exchange buy orders are executed at the ask; exchange sell orders are executed at the bid. For simplicity, we assume bid and ask prices are announced at the beginning of period 0 unconditional on orders. In the close auction, orders are executed at a single close price. At the end of the close auction, a competitive market maker announces the close price \( p_c \), which is the expected asset value, conditional on the net orders \( y \) received. Our key objective is to analyze the informativeness of \( p_c \), for the fundamental value \( v \) of the asset.

There are three kinds of traders in the market: for-profit traders who can learn the asset value at cost, liquidity traders who has to trade a given quantity due to liquidity need, and index funds who want to adjust their holdings at the close price to match their benchmark. Traders arrive at the beginning of period 0. There are \( N \) for-profit traders, many liquidity traders and many index funds. Liquidity traders are risk-averse, while the rest are risk-neutral. Each trader can potentially trade one unit of the asset using a market order.

For-profit traders can acquire, at a cost, perfect information about \( v \), and thus become informed traders. The information-acquisition costs are distributed across for-profit traders, with a differentiable cumulative distribution function \( F : [0, \infty) \to [0, 1] \). After observing \( v \), informed traders submit buy orders to either venue if \( v = +\sigma \), or submit sell orders if \( v = -\sigma \). For-profit traders
who do not acquire the information do not trade. Let \( N_I \) be the number of informed traders. \( L^+ \) liquidity buyers and \( L^- \) liquidity sellers arrive at the market with unit demand. Both \( L^+ \) and \( L^- \) are random variables with mean \( 0.5\mu_L \) variance \( 0.5\sigma_L^2 \). Index funds arrive at the market with a net trading demand \( I \), which is normally distributed with mean 0 variance \( \sigma_I^2 \).

One risk-neutral bank provides guaranteed close service to the traders. The bank is able to identify index funds and knows for sure their orders convey no information about the asset’s value, but knows nothing about the identity of the other traders, thus cannot tell informed traders from liquidity traders. Therefore, the bank observes the orders from the index funds \( I_b \), and the aggregate net orders from the other traders \( u \). Then bank matches buy and sell orders, then submits a market order including imbalance orders and a position on its own account of \( X(I, u) \) to the close auction.

Liquidity traders and index funds incur a convenience cost if they choose to trade via the bank. Specifically, for each trader \( i \), the convenience cost per unit of asset per capita is:

\[
c_i = \gamma_i \sigma
\]

where the multiplicative constants \( \gamma_i \) represent the types of the traders and funds and have a twice-differentiable cumulative distribution function \( G : [0, \Gamma) \rightarrow [0, 1] \), for some \( \Gamma \in (1, \infty] \). The convenience cost represents the cost of building connections with the bank.

Figure 4 illustrates the sequence of actions in the three-period model. At period 0, traders arrived at the markets, informed traders learn the value of \( v \), market maker of regular session announce bid and ask prices, the bank announce the fee \( \mu_b \), then traders choose venues to trade and orders at the regular session are executed immediately. At period 1, the bank first observes its order flow and submits orders accordingly to the close auction, then the market maker in the close auction observe orders, announce close price \( p_c \) and executes the orders. Then at period 2, the value \( v \) is revealed and paid.

![Figure 4: Time line of the three-period model.](image)

Finally, random variables \( v, L, I \) and the costs of information-acquisition are all independent, and their probability distributions are common knowledge.
5.2 Equilibrium

An equilibrium consists of the quoting strategy of the exchange market makers, the fee charged by
the bank, the information acquisition decision of the for-profit traders, and the trading strategy of
the informed traders, liquidity traders, index funds and the bank. In equilibrium, the competitive
market makers break even in expectation both in the regular session and in the close auction. All
traders and the bank maximize their expected net profits. We solve for the equilibrium in two
steps. First, we derive the equilibrium strategies of all market participants given the bank’s fee
along with all the exogeneous variables. Second, we derive the profit of the bank charging any given
fee \( \mu_b \), and find the optimal fee to charge.

Let \( \alpha_r \) and \( \alpha_c \) be candidates for the equilibrium fractions of liquidity traders who send orders to
the regular session and the close auction. The remaining fraction \( \alpha_b = 1 - \alpha_r - \alpha_c \) send orders
to the bank. Let \( \beta \) be the fraction of index funds who send orders to the close auction, and the
remaining \( 1 - \beta \) send orders to the bank. Let \( \theta_r \) and \( \theta_c \) be the fraction of informed traders who
send orders to the regular session and the close auction. The remaining fraction \( \theta_b = 1 - \theta_r - \theta_c \)
send orders to the bank.

We firstly derive equilibrium bid and ask in the regular session, given equilibrium participation
fractions \( (\theta_r, \theta_c, \beta, \alpha_r, \alpha_c) \) and \( N_I \). Because of symmetry and the fact that the unconditional mean
of \( v \) is zero, the midpoint of the market maker’s bid and ask is zero. Therefore, the exchange ask
is some \( S > 0 \), and the exchange bid is \( -S \). Given the participation fractions \( (\theta_r, \theta_c, \beta, \alpha_r, \alpha_c) \), the
number of informed traders in the regular session is \( \theta_r N_I \), the expected total orders from liquidity
traders is \( \alpha_r \mathbb{E}(|L^+| + |L^-|) = \alpha_r \mu_L \). The market maker breaks even on average over all the orders,
so the spread satisfies

\[
0 = -\theta_r N_I (\sigma - S) + \alpha_r \mu_L S
\]

which implies that

\[
S = \frac{\theta_r N_I}{\theta_r N_I + \alpha_r \mu_L} \sigma
\]

Next, we derive the equilibrium number \( N_I \) of informed traders. In equilibrium, a marginal informed
trader can get no more profit in close auction or via bank than in regular session, since otherwise
someone trading in the regular session would deviate to another venue. Given the value of the
information \( \sigma \) and the exchange spread \( S \), the net profit of a marginal informed trader is \( \sigma - S - \mu_r \).
An for-profit trader with information acquisition cost \( \sigma - S - \mu_r \) will be indifferent between being
informed and not. So, the number of informed traders in equilibrium is \( \text{int}[F(\sigma - S - \mu_r) N] \). So
we have

\[ N_I = \text{int} \left[ F(\sigma - S - \mu_r)N \right] = \text{int} \left[ F(\frac{\alpha_r \mu_L \sigma}{\theta_r N_I + \alpha_r \mu_L} - \mu_r)N \right] \]  

(9)

Then we derive equilibrium conditions in the close auction. The close auction executes the orders at the close price \( p^c \). Given the participation fractions \((\theta_r, \theta_c, \beta, \alpha_r, \alpha_c)\), the number of informed traders in the close auction is \( \pm(1 - \theta_r)N_I \), the total number of orders from liquidity traders is \( (1 - \alpha_r)N = (1 - \alpha_r)(L^+ - L^-) \), and the total number of orders from the index funds is \( \beta I \). The market maker observes the total order flow \( y \), and set the close price to be the conditional expected value of \( v \).

\[ p^c = \mathbb{E}(v|y) = [\mathbb{P}(v = +\sigma|y) - \mathbb{P}(v = -\sigma|y)]\sigma \]  

(10)

where

\[
\begin{align*}
y &= (1 - \alpha_r)L + I + (1 - \theta_r)N_I + X(I, u) \text{ if } v = +\sigma \\
y &= (1 - \alpha_r)L + I - (1 - \theta_r)N_I + X(I, u) \text{ if } v = -\sigma \\
\end{align*}
\]

(11) \hspace{1cm} (12)

We look for a Nash equilibrium in trading strategies. A justification for the method is given in Section 5 of Kyle 1989.

**Proposition 5.1.** A linear Nash equilibrium of the model described above is given by

\[ p^c(y) = \lambda y + \mu_c \text{sign}(y) \]  

\[ X(I, u) = A + BI + Cu \]  

(13) \hspace{1cm} (14)

where \( y = (1 - \alpha_r)L + I \pm (1 - \theta_r)N_I + X(I, u) \) is the net orders that go to the market maker in close auction.

Here the main difference between our model and traditional models in the literature is the \( \mu_c \) term. We emphasis the role of transaction fees in our model because it is the main incentive for people to use bank’s close service rather than directly in close auction. When transaction fees are included in the model, the market maker knows that traders would underinvest due to the fee and thus adjust accordingly in the price. Notice that the sign of this adjustment term depends on the sign of total net order \( y \). For simplicity, let’s first assume that the force of informed orders are strong enough so that the probability of \( yX < 0 \) is small enough to be neglected when making decisions, WLOG, assume \( y > 0, X > 0 \). We will go back to the more general case with expectations on the sign later.

The proof of the proposition can be found in the appendix. In equilibrium, we have
\[ A = -\frac{\mu_c}{\lambda} \]  \hspace{1cm} (15)

\[ B = -\frac{1}{2} \]  \hspace{1cm} (16)

\[ C = \frac{\hat{\mu}}{\sigma_u \lambda} \left( \sigma + \lambda[(1 - \alpha_r)(1 - \theta_r - \theta_c) - (1 - \theta_r)]N_I \right) - \frac{1}{2}(1 - \alpha_r) \]  \hspace{1cm} (17)

where \( \hat{\mu} = (1 - \theta_r - \theta_c)N_I/\sigma_u \), for the bank’s strategy \( X(I, u) = A + BI + Cu \). And

\[ \lambda = \frac{2\hat{\eta}\sigma}{\sigma_y} \]  \hspace{1cm} (18)

where

\[ \hat{\eta} = [(C + 1)(1 - \theta_r) - C\theta_c]N_I \]  \hspace{1cm} (19)

\[ \sigma_y^2 = (1 + B)^2\sigma_I^2 + [(C + 1)(1 - \alpha_r) - C\alpha_c]^2\sigma_L^2 \]  \hspace{1cm} (20)

for the market maker’s strategy.

Next we derive equilibrium venue choices of the traders. Since \( \mathbb{E}p^c = \mathbb{E}(\mathbb{E}[v|y]) = 0 \), the expected utility of a risk-averse liquidity buyer with unit demand in the regular session, in the close auction and via the bank are

\[ T_r = -S - \mu_r \]

\[ T_c = -\mu_c - \alpha \text{Var}(p^c) \]

\[ T_b = -\mu_b - c - \alpha \text{Var}(p^c) \]  \hspace{1cm} (21)

where \( \text{Var}(p^c) = \lambda^2(\sigma_I^2 + [(C + 1)(1 - \alpha_r) - C\alpha_c]^2\sigma_L^2 + [(C + 1)(1 - \theta_r) - C\theta_c]^2N_I^2) + \lambda E[|y|]\mu_c + \mu_c^2 \).

In order that the liquidity trader is indifferent between the regular session and the close auction, we have

\[ -S - \mu_r = -\mu_c - \alpha \text{Var}(p^c) \]  \hspace{1cm} (22)

The liquidity trader who is indifferent between the close auction and the bank has \( c = \mu_c - \mu_b \), so

\[ \frac{\alpha_c}{(1 - \alpha_r)} = 1 - G(\mu_c - \mu_b) \]  \hspace{1cm} (23)
The expected profits of an index fund in the close auction and via the bank are

\[
R_c = -\mu^c \\
R_b = -\mu^b - c
\]  

The index fund who is indifferent between trading in the close auction and the bank has \( c = \mu_c - \mu_b \), so the fraction of index fund choosing the bank is

\[
1 - \beta = G(\mu_c - \mu_b)
\]  

The expected profits of an informed buyer in the regular session, in the close auction and via the bank are

\[
W_r = \sigma - S - \mu_r \\
W_c = \sigma - p^c(y) - \mu_c \\
W_b = \sigma - p^c(y) - \mu_b
\]  

In equilibrium, an informed buyer chosen to go to the close auction does not want to deviate and choose to trade via the bank, and vice versa.

\[
\mathbb{E}[-p^c(y)] - \mu_c \geq \mathbb{E}[-p^c(y + C)] - \mu_b \]  

\[
\mathbb{E}[-p^c(y - C)] - \mu_c \leq \mathbb{E}[-p^c(y)] - \mu_b
\]  

\[
\Leftrightarrow \lambda C = \mu_c - \mu_b
\]  

An informed buyer chosen to go to the regular session does not want to deviate and choose to trade in the close auction, and vice versa.

\[
-S - \mu_r \geq \mathbb{E}[-p^c(y + 1)] - \mu_c
\]

\[
-S - \mu_r \leq \mathbb{E}[-p^c(y)] - \mu_c
\]

\[
\Leftrightarrow \lambda[(C + 1)(1 - \theta_r) - C\theta_c]N_I + \lambda + \mu_c - S \geq \mu_r - \mu_c
\]

\[
\geq \lambda[(C + 1)(1 - \theta_r) - C\theta_c]N_I + \mu_c - S
\]

Now we can characterize the first step of the equilibrium when \( \mu_b \) is given. Given exogenous parameters: fees \( \mu_r, \mu_c, \mu_b \), trade demand \( \mu_L, \sigma_L, \sigma_I, N \), fundamental value \( \sigma \), information acquisition cost CDF \( F(\cdot) \), bank convenience cost CDF \( G(\cdot) \), risk aversion coefficient of liquidity trader \( \alpha \). Equilibrium quantities \( (S, N_I, \lambda, A, B, C, \alpha_r, \alpha_c, \beta, \theta_r, \theta_c) \) can be solved by Equations (8) bid-ask
spread, (9) the number of informed traders $N_I$, (18) market maker’s decision rule, (15) (16) (17) bank’s trading strategy, (22) (23) (25) indifference conditions of liquidity traders and index funds, (31) and (35) informed traders NE condition.

The bank then calculates the expected profit from both trading and fees for any given $\mu_b$, and pins down the optimal $\mu_b$.

$$\max_{\mu_b} E[(v-p)X - \mu_c|X||I, u|\mu_b] + \mu_b E[(1 - \beta)|I] + \alpha_b(L^+ + L^-) + \theta_b N_I |\mu_b]$$  \hspace{1cm} (36)

The numerical solutions and comparative statics of the model will be given soon in future work.

5.3 Price informativeness

In equilibrium, the price impact is

$$\lambda = \frac{2\hat{\eta}\sigma}{\sigma_y}$$  \hspace{1cm} (37)

where $\hat{\eta} = [(C + 1)(1 - \theta_r) - C\theta_c]N_I$.

When $v = +\sigma$, we know the close price is

$$p_c = \frac{2\hat{\eta}^2 N_I}{\sigma_y} + \frac{2\hat{\eta}}{\sigma^2} \left[I + \frac{1}{2} (C + 1)(1 - \alpha_r) - C\alpha_c)L\right]\sigma$$  \hspace{1cm} (38)

which is normally distributed around the mean $\frac{2\hat{\eta}^2 N_I}{\sigma_y}$. Similarly, when $v = -\sigma$, we know the close price is normally distributed around the mean $-\frac{2\hat{\eta}^2 N_I}{\sigma_y}$. Here for simplicity we ignored the price adjustment term $\mu_c \text{sign}(y)$ since it doesn’t affect our analysis of changes in price informativeness with or without a bank.

We measure the informativeness of the close price by its closeness to the fundamental value $v$, in the sense of mean squared error

$$MSE(p_c - v|v = +\sigma) = \sigma^2 \left[\left(\frac{2\hat{\eta}N_I}{\sigma_y} - 1\right)^2 + \left(\frac{\hat{\eta}}{\sigma^2}\right)^2 + \left(\frac{2\hat{\eta}}{\sigma_y}((C + 1)(1 - \alpha_r) - C\alpha_c))\right]^2\sigma_L^2\right]$$  \hspace{1cm} (39)
Change of price informativeness after adding the bank venue is

$$\Delta MSE \approx \sigma^2 \left[ \left( \frac{4\hat{\eta}I}{\sigma_y} - 2 \right) \Delta \frac{2\hat{\eta}I}{\sigma_y} + \left( \frac{2\hat{\eta}}{\sigma_y} \Delta \frac{\hat{\eta}}{\sigma_y} - 3 \left( \frac{\hat{\eta}}{\sigma_y} \right)^2 \sigma_y^2 \right)^2 + 8 \left( \frac{\hat{\eta}}{\sigma_y} \right)^2 \left( 1 - \alpha_r \right) \left( -\Delta \alpha_r + C \Delta \alpha_b \right) \sigma_y^2 \right]$$

where

$$\Delta \hat{\eta} \approx (-\Delta \theta_r + C \Delta \theta_b)NI$$  \hspace{1cm} (41)

$$\Delta \sigma_y \approx -\frac{3}{4} \sigma_L^2 + 2(1 - \alpha_r)(-\Delta \alpha_r + C \Delta \alpha_b)$$  \hspace{1cm} (42)

$$\Delta \frac{\hat{\eta}}{\sigma_y} \approx \frac{-\Delta \theta_r + C \Delta \theta_b}{\sigma_y} \sigma_y - \hat{\eta} \left( \frac{-3}{4} \sigma_L^2 + 2(1 - \alpha_r)(-\Delta \alpha_r + C \Delta \alpha_b) \right)$$  \hspace{1cm} (43)

The bank has three effects on close price informativeness: amplifying or absorbing informed orders, absorbing index fund orders and amplifying or absorbing liquidity orders. When $\sigma_z$ is small enough such that $C > 0$, the bank rationally believe its order flow reveals more information than noise, so amplifies both informed orders and liquidity orders, making the price more informative. Conversely, if $\sigma_z$ is large enough such that $C < 0$, the bank rationally believe its order flow reveals less information than noise, so absorbs informed orders and liquidity orders, still making the price more informative. Detailed analysis and conditions for these results to be true will be presented after we solve for the numerical solutions of the model.

References


Ye, Mao (2010), “Non-execution and market share of crossing networks,” *Available at SSRN 1719016*.
Appendix

A.1 Robustness Tests

To address the concern that the spike of off-exchange closing activity starting from early 2017 implies some structural changes, we estimate the effect in samples in different time periods: 2012-2018 and 2012-2015. Results remain unchanged. Currently we haven’t controlled for return volatility, but we will do so soon.

Table 6: Predictability of Close Price on Next Day’s Open Price using Extended Sample

<table>
<thead>
<tr>
<th>Last5m_Open</th>
<th>S&amp;P500 Firms 2012-18</th>
<th>All Firms 2012-18</th>
<th>S&amp;P500 Firms 2012-15</th>
<th>All Firms 2012-15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Last5m_Close</td>
<td>0.516*** (67.067)</td>
<td>0.578*** (446.366)</td>
<td>0.543*** (53.207)</td>
<td>0.592*** (337.839)</td>
</tr>
<tr>
<td>Last5m_Close × Frac_Off Ex</td>
<td>3.060*** (4.759)</td>
<td>1.995*** (11.310)</td>
<td>4.120*** (3.016)</td>
<td>2.149*** (7.715)</td>
</tr>
<tr>
<td>log(Market Cap)</td>
<td>0.000*** (18.038)</td>
<td>-0.000*** (-10.533)</td>
<td>0.001*** (21.078)</td>
<td>0.000*** (15.210)</td>
</tr>
<tr>
<td>log(Total Volume)</td>
<td>0.000*** (10.870)</td>
<td>0.000*** (66.217)</td>
<td>0.000*** (4.761)</td>
<td>0.000*** (41.672)</td>
</tr>
<tr>
<td>Frac_Close Auction</td>
<td>0.001*** (4.629)</td>
<td>0.000 (1.152)</td>
<td>0.001*** (3.919)</td>
<td>0.000 (0.272)</td>
</tr>
</tbody>
</table>

| N       | 797955 | 7330811 | 458058 | 4092908 |
| R²      | 0.343  | 0.168   | 0.411  | 0.224   |
| Stock FE | YES    | YES     | YES    | YES     |
| Date FE | YES    | YES     | YES    | YES     |

$t$ statistics in parentheses  
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table reports the estimation results of the relationship between predictability of close price on next-day open price and off-exchange MOC activity using the extended sample covering 2012-2018. For the outcome variable, we used the return calculated by next-day open price/last-5-min VWAP. We regress the outcome variables on Last5m_Close, calculated by close price/last-5-min VWAP, and its interaction with Frac_Off Ex. Frac_Off Ex is each individual stock’s fraction of daily trades that occurred off exchange yet at the close price. The rest are control variables. log(Total Volume) is log total daily trading volume for each individual stock. Frac_Close Auction is each individual stock’s fraction of daily trades that occurred in the close auction. log(Market Cap) is log market capitalization based on CRSP data. Variables were aggregate to daily frequency. Stock fixed effects and date fixed effects are added to all regressions. For robustness, outliers in each variable are trimmed at 1% tails.

Table reports robustness test results using last 15 minutes to close(open) returns, in the 2012-2018 and 2012-2015 samples. Results remain unchanged.
Table 7: Predictability of Close Price on Next Day’s Open Price using Extended Sample

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Last15m_Close</td>
<td>0.587***</td>
<td>0.603***</td>
<td>0.616***</td>
<td>0.620***</td>
</tr>
<tr>
<td></td>
<td>(92.773)</td>
<td>(536.070)</td>
<td>(75.035)</td>
<td>(412.769)</td>
</tr>
<tr>
<td>Last15m_Close × Frac_Off Ex</td>
<td>2.667***</td>
<td>2.319***</td>
<td>3.885***</td>
<td>2.803***</td>
</tr>
<tr>
<td></td>
<td>(5.576)</td>
<td>(15.729)</td>
<td>(3.989)</td>
<td>(12.314)</td>
</tr>
<tr>
<td>log(Market Cap)</td>
<td>0.000***</td>
<td>-0.000***</td>
<td>0.001***</td>
<td>0.000***</td>
</tr>
<tr>
<td></td>
<td>(18.408)</td>
<td>(-10.823)</td>
<td>(21.382)</td>
<td>(15.531)</td>
</tr>
<tr>
<td>log(Total Volume)</td>
<td>0.000***</td>
<td>0.000***</td>
<td>0.000***</td>
<td>0.000***</td>
</tr>
<tr>
<td></td>
<td>(11.051)</td>
<td>(67.656)</td>
<td>(4.961)</td>
<td>(42.088)</td>
</tr>
<tr>
<td>Frac_Close Auction</td>
<td>0.001***</td>
<td>0.000**</td>
<td>0.001***</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>(4.741)</td>
<td>(2.310)</td>
<td>(4.037)</td>
<td>(0.946)</td>
</tr>
<tr>
<td>N</td>
<td>801532</td>
<td>7401619</td>
<td>459823</td>
<td>4132382</td>
</tr>
<tr>
<td>R²</td>
<td>0.344</td>
<td>0.175</td>
<td>0.412</td>
<td>0.231</td>
</tr>
<tr>
<td>Stock FE</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Date FE</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
</tbody>
</table>

_t_ statistics in parentheses

* _p < 0.1, ** _p < 0.05, *** _p < 0.01

Table reports the estimation results of the relationship between predictability of close price on next-day open price and off-exchange MOC activity using the extended sample covering 2012-2018. For the outcome variable, we used the return calculated by next-day open price/last-15-min VWAP. We regress the outcome variables on Last15m_Close, calculated by close price/last-15-min VWAP, and its interaction with Frac_Off Ex. Frac_Off Ex is each individual stock’s fraction of daily trades that occurred off exchange yet at the close price. The rest are control variables. log (Total Volume) is log total daily trading volume for each individual stock. Frac_Close Auction is each individual stock’s fraction of daily trades that occurred in the close auction. log(Market Cap) is log market capitalization based on CRSP data. Variables were aggregate to daily frequency. Stock fixed effects and date fixed effects are added to all regressions. For robustness, outliers in each variable are trimmed at 1% tails.
A.1. Model Variable Definitions

Table 8: Variables and Parameters in the Baseline Model

<table>
<thead>
<tr>
<th>Exogenous Variables</th>
<th>Endogenous Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_r$ Transaction fee in the regular session</td>
<td>$\mu_b$ Transaction fee of the bank’s closing service</td>
</tr>
<tr>
<td>$\mu_c$ Transaction fee in the close auction</td>
<td>$S$ Transaction fee of the bank’s closing service</td>
</tr>
<tr>
<td>$\mu_L$ Mean number of liquidity traders</td>
<td>$N_I$ Number of informed traders</td>
</tr>
<tr>
<td>$\sigma^2_L$ Variance of number of liquidity traders</td>
<td>$\lambda$ Price impact in close auction</td>
</tr>
<tr>
<td>$\sigma^2_I$ Variance of total demand from index funds</td>
<td>$A, B, C$ Bank’s linear decision rule in proposition 1.1</td>
</tr>
<tr>
<td>$N$ Number of for-profit traders</td>
<td>$\alpha_r$ Fraction of liquidity traders trading in regular session</td>
</tr>
<tr>
<td>$\sigma$ Fundamental value of the asset</td>
<td>$\alpha_c$ Fraction of liquidity traders trading in close auction</td>
</tr>
<tr>
<td>$\alpha$ Risk aversion coefficient of liquidity traders</td>
<td>$\beta$ Fraction of index funds trading in close auction</td>
</tr>
<tr>
<td>$\theta_r$ Fraction of informed traders trading in regular session</td>
<td>$\theta_c$ Fraction of informed traders trading in close auction</td>
</tr>
</tbody>
</table>

B.1 Proof of Proposition 1.1.

The bank chooses a position $X(L, u)$ that maximizes his expected profits, given the uninformed orders $(1 - \beta)L$ from index funds and other unidentifed orders $u$.

$$\max_X \mathbb{E}[(v - p)X - \mu_c | X | I, u]$$

where $p = \lambda y + \mu_c = \lambda ((1 - \alpha_r)L + I \pm (1 - \theta_r)N_I + X(I, u)) + \mu_c$.

If $v = +\sigma$, then $u = u(+) = (1 - \alpha_r - \alpha_c)L + (1 - \theta_r - \theta_c)N_I$. If $v = -\sigma$, then $u = u(-\sigma) = (1 - \alpha_r - \alpha_c)L - (1 - \theta_r - \theta_c)N_I$. Assume $L = L^+ + L^-$ is normally distributed $N(\mu_L, \sigma^2_L)$, we know the following.
\[ u \sim N((1 - \theta_r - \theta_c)N_I, \sigma_u^2) \text{ if } v = +\sigma \]
\[ u \sim N(-(1 - \theta_r - \theta_c)N_I, \sigma_u^2) \text{ if } v = -\sigma \]

where \( \sigma_u^2 = (1 - \alpha_r - \alpha_c)^2 \sigma_L^2 \).

The bank updates its belief after observing \( u \),

\[
P(v = +\sigma | u) = \frac{\Phi\left(\frac{u-(1-\theta_r-\theta_c)N_I}{\sigma_u}\right) - \Phi\left(\frac{u+(1-\theta_r-\theta_c)N_I}{\sigma_u}\right)}{\Phi\left(\frac{u-(1-\theta_r-\theta_c)N_I}{\sigma_u}\right) + \Phi\left(\frac{u+(1-\theta_r-\theta_c)N_I}{\sigma_u}\right)}
\]
\[
P(v = -\sigma | u) = 1 - P(v = +\sigma | u)
\]

The updated belief can be simplified by linear approximation if we assume the number of informed traders \( N_I \) is small enough compared to the variance of liquidity trader’s trading demand \( \sigma_u^2 \).

Denote \( \hat{\mu} = (1 - \theta_r - \theta_c)N_I/\sigma_u \).

\[
P(v = +\sigma | u) = \frac{\Phi\left(\frac{u-(1-\theta_r-\theta_c)N_I}{\sigma_u}\right)}{\Phi\left(\frac{u-(1-\theta_r-\theta_c)N_I}{\sigma_u}\right) + \Phi\left(\frac{u+(1-\theta_r-\theta_c)N_I}{\sigma_u}\right)}
\]
\[
= \frac{e^{-(u/\sigma_u - \hat{\mu})^2}}{e^{-(u/\sigma_u - \hat{\mu})^2} + e^{-(u/\sigma_u + \hat{\mu})^2}} = (1 + e^{-4u\hat{\mu}/\sigma_u})^{-1}
\]
\[
= (2 - 4u\hat{\mu}/\sigma_u)^{-1} + o(u\hat{\mu}/\sigma_u) = \frac{1}{2}(1 + 2u\hat{\mu}/\sigma_u) + o(u\hat{\mu}/\sigma_u)
\]

Substituting in for \( p = \lambda y + \mu_c = \lambda((1 - \alpha_r)L + I \pm (1 - \theta_r)N_I + X(I, u)) + \mu_c \) and \( Z|v = +\sigma, u = u - (1 - \theta_r - \theta_c)N_I, Z|v = -\sigma, u = u + (1 - \theta_r - \theta_c)N_I \). Then the bank’s problem becomes

\[
\max_X \left[ \sigma - \lambda((1 - \alpha_r)(u - (1 - \theta_r - \theta_c)N_I) + I + (1 - \theta_r)N_I + X) - \mu_c - \mu_c \right] X P(v = +\sigma | u) \]
\[
+ \left[ - \sigma - \lambda((1 - \alpha_r)(u + (1 - \theta_r - \theta_c)N_I) + I - (1 - \theta_r)N_I + X) - \mu_c - \mu_c \right] X(1 - P(v = +\sigma | u))
\]

Take first order conditions, then we obtain
2\lambda X = (2\mathbb{P}(v = +\sigma|u) - 1)(\sigma + \lambda(1 - \alpha_r)(1 - \theta_r - \theta_c)N_I - \lambda(1 - \theta_r)N_I) - \lambda((1 - \alpha_r)u + I) - 2\mu_c \tag{54}

X \approx \frac{\hat{\mu}}{\sigma_u \lambda}(\sigma + \lambda[(1 - \alpha_r)(1 - \theta_r - \theta_c) - (1 - \theta_r)]N_I)u - \frac{1}{2}((1 - \alpha_r)u + I) - \frac{\mu_c}{\lambda} \tag{55}

So \( A = -\frac{\mu_c}{X}, B = -\frac{1}{2}, C = \frac{\hat{\mu}}{\sigma_u \lambda}(\sigma + \lambda[(1 - \alpha_r)(1 - \theta_r - \theta_c) - (1 - \theta_r)]N_I) - \frac{1}{2}(1 - \alpha_r). \)

Next we derive the close price \( p^c \) and show that it takes the form in the proposition. The competitive market maker sets

\[ p^c = \mathbb{E}(v|y) = \mathbb{P}(v = +\sigma|y) - \mathbb{P}(v = -\sigma|y) \sigma \tag{56} \]

where

\[ y = (1 - \alpha_r)L + I + (1 - \theta_r)N_I + X(I, u) \text{ if } v = +\sigma \tag{57} \]
\[ y = (1 - \alpha_r)L + I - (1 - \theta_r)N_I + X(I, u) \text{ if } v = -\sigma \tag{58} \]

Since \( X(I, u) = A + BI + Cu \), and that \( u = (1 - \alpha_r - \alpha_c)L + (1 - \theta_r - \theta_c)N_I \) if \( v = +\sigma \), \( u = u = (1 - \alpha_r - \alpha_c)L - (1 - \theta_r - \theta_c)N_I \) if \( v = -\sigma \), we have

\[ y = (1 + B)I + [(C + 1)(1 - \alpha_r) - Ca_c]L + [(C + 1)(1 - \theta_r) - C\theta_c]N_I + A \text{ if } v = +\sigma \tag{60} \]
\[ y = (1 + B)I + [(C + 1)(1 - \alpha_r) - Ca_c]L - [(C + 1)(1 - \theta_r) - C\theta_c]N_I + A \text{ if } v = -\sigma \tag{61} \]

\[ \mathbb{P}(v = +\sigma|y) = \frac{1}{2}(1 + 2(y - A)\hat{\eta}/\sigma_y) + o((y - A)\hat{\eta}/\sigma_y) \tag{62} \]

where

\[ \hat{\eta} = [(C + 1)(1 - \theta_r) - C\theta_c]N_I \tag{63} \]
\[ \sigma_y^2 = (1 + B)^2\sigma_I^2 + [(C + 1)(1 - \alpha_r) - Ca_c]^2\sigma_L^2 \tag{64} \]

So

\[ p^c = \mathbb{E}(v|y) = (2\mathbb{P}(v = +\sigma|y) - 1)\sigma = 2\frac{(y - A)\hat{\eta}}{\sigma_y} \tag{65} \]
We have the following equilibrium condition:

\[ \lambda = \frac{2\eta \sigma}{\sigma_y} \]  

(66)

Then \( p^c = \lambda(y - A) = \lambda y + \mu_c \). The proposition is proved.