Debt covenants and the value of commitment

Abstract

We analyze the value of shareholders’ commitment created by empirically observed debt covenants. We show that the renegotiation following covenant violation improves the ex post firm value at an ex ante cost and can lead to value losses similar to those under no commitment. Therefore, renegotiation frictions are key for covenants to increase the ex ante firm value. In a dynamic model, the main driver of the value loss is the no-commitment issue and the rigid debt policy ensuing from it. Hence, covenants that discipline the leverage improve firm value the most because they restore the flexibility of the debt policy. Instead, debt and asset sweeps, which are designed as ex post debt protections, are less efficient because they do not discipline the leverage policy. An efficient covenant has positive effects also on firm investment, and alleviates the agency conflicts between existing claim holders.

Keywords: Dynamic contracting, debt covenants

JEL classification: G32, G33, D86
Introduction

The way commitment improves contracting efficiency in corporate finance is an important question which has not yet been sufficiently addressed. Exceptions are Admati, DeMarzo, Hellwig, and Pfeiderer (2018) and DeMarzo and He (2017), who emphasize the cost of shareholders’ lack of commitment and its effect on firms’ leverage policy; i.e., the leverage ratchet effect. According to the received wisdom, debt covenants can be used to increase commitment: by forcing shareholders to renegotiate with debt holders at verifiable violations states, covenants discipline the shareholders’ policy and alleviate the cost of no-commitment.¹

We look into the relationship between debt renegotiation and the (lack of) commitment and provide a different answer: if debt covenants can be renegotiated ex post, then they may not be per se effective commitment devices. We illustrate this point with a simple example, in which shareholders can continuously issue new debt but a fixed leverage covenant is added to the debt contract to curb the leverage ratchet effect. We show that in equilibrium with renegotiation, shareholders and debt holders jointly decide to renegotiate the covenant and issue more debt constantly to maximize ex post firm value. Hence, the covenant proves ineffective, because the leverage ratchet effect is still the equilibrium outcome.

More specifically, there are two main economic forces that render covenants ineffective in a setting with dynamic leverage policy. The first is that new debt is issued to prospective debt holders, who do not participate in the renegotiation between the existing debt holders and shareholder and take the outcome as given. While the ex post renegotiation solves the narrowly defined agency conflicts between the existing claim holders, it does not make their interests aligned with the ones of the new claim holders. The latter is important to explain the divergence between the ex ante and ex post incen-

¹Debt covenants are viewed as ways of “allocating control rights to debt holders” by involving the debt holder to renegotiate at covenant violations, so that they can influence firm policies. The role of financial contracts in allocation of control rights is discussed as early as Aghion and Bolton (1992). Some empirical researches postulate debt holders use debt covenants to exert control and affect firm policies (e.g., Roberts and Sufi (2009)). Some emphasize the role of covenants as ways to include more information into a debt contract and to induce more monitoring (e.g., Christensen, Nikolaev, and Wittenberg-Moerman (2016)). Also, covenants are interpreted as ways of “improving contractual completeness” by using verifiable violation points as contractible conditions (see Matvos (2013)). All these views hinge upon the role of covenants as ways to increase ex post commitment.
tive which drives the leverage ratchet effect: although ex ante the firm wants to restrict its future debt issuance for high debt price, the high price of debt motivates excessive debt issuance ex post.

The second is that the control right given by the covenant is priced. When renegotiating a covenant, the shareholder has to transfer some value to the debt holders for their agreement. But the fair market price of debt reflects the future transfer, which benefits shareholders at debt issuance. As the increase of debt price and the present value of the future transfer offset each other, the inclusion of the covenant has neutral effect on the shareholders’ ex ante debt issuance policy.\(^2\) Overall, the divergence between ex post efficiency and ex ante optimality, which is the crux of the non-commitment problem in contracting, is not solved by debt covenants (or more generally, by ex post renegotiation with debt holders) per se.

This new perspective provides a different rationale for the covenants used in practice, and exactly for the way they are, as opposed to the way they should be if compared to the theoretical covenants. Our main point is: for covenants to be effective commitment devices, the presence of frictions that limit ex post renegotiation of the debt contract is essential. Those frictions are not in the letter of the contract, because as such they could be eliminated by simply re-writing it. Rather, they are features of the debt arrangement that determine the likelihood and the success of the renegotiation of the contract. Example of these frictions are debt holding structure and access to insurance against credit risk. As for the first, a common view (e.g., Rajan (1992)) is that a disperse debt holding increases coordination costs and reduces the likelihood of a successful renegotiation. As for the second, Bolton and Oehmke (2011) show that access to credit insurance makes renegotiation harder. Clearly, these circumstances transcend the debt contract and cannot be renegotiated.

While renegotiation frictions make commitment possible, debt covenants determine the set of policies that firms are committing to. Therefore, given the same friction, different covenants can be differently effective. We do not pursue a concept of first-best contract in our dynamic model. Assuming perfect commitment, a first-best contract that

\(^2\)To illustrate, suppose a protective covenant prevents a value transfer from debt to equity in some future states, and the present value of the transfer is $1. While the debt holder is willing to pay $1 more for such covenant to be included in the debt contract, the shareholders is indifferent to the inclusion: the debt price increase compensates the loss of future chance to transfer value from debt holders.
specifies state-by-state firm policies is theoretically possible, but such complete contract is not likely in reality. Rather, neither renegotiation nor commitment are perfect and therefore we focus on the realistic case in which ex post renegotiation is possible, but not certain.

Under (partial) commitment, we evaluate a covenant by its effect on ex ante firm value, which is a trade-off between ex post cost and disciplinary effect on firm policy. On the one hand, commitment is costly ex post, because the firm could be prevented from adopting policies that maximize the ex post firm value. On the other, commitment leads to direct or indirect discipline which may increase ex ante firm value. Covenants are designed to directly discipline the shareholders’ decisions. For example, committing to leverage restrictions means that high leverage states are less likely for a firm. The indirect discipline results from inefficient resolutions due to the inability to renegotiate at covenant violations. Because the inefficient resolutions reduce both equity and debt value at covenant violations, they motivate shareholders to adjust their policy ex ante to minimize the occurrence of these costly states. By restricting different policies, different debt covenants achieve a different trade-off of ex post cost versus the ex ante benefit of discipline.

We evaluate different types of covenants (financial/accounting covenants, asset sweeps, debt sweeps) that are commonly observed in reality, as documented by the empirical literature, in a neoclassical dynamic corporate finance model, in which shareholders are not able to commit to ex post debt adjustments and investments. In this standard model, the covenant that improves firm value the most is a financial covenant that restricts the ratio of debt over earnings. The reason is that this covenant imposes an effective discipline on the leverage policy of the firm, which helps to restore flexibility of the debt dynamics.

The intuition of why such financial covenant works is directly related to the commitment problem. With full commitment, the ex ante optimal debt issuance is pro-cyclical: in line with standard trade-off theory, more debt is issued after a positive productivity shock to capture the tax benefits, whereas debt is reduced after negative shocks to reduce...

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3In this sense, the cost of commitment is the difference of ex post unconstrained maximum firm value and the ex post value under covenant restrictions.

4Among others Smith and Warner (1979), Billett, King, and Mauer (2007), Chava and Roberts (2008), Roberts and Sufi (2009), and Nini, Smith, and Sufi (2009).
expected bankruptcy costs. However, in the no-commitment equilibrium where debt is issued without any covenants, firms never reduce outstanding debt, and debt increase is infrequent and in large amounts. The lack of debt reduction is consistent with the leverage ratchet effect in DeMarzo and He (2017), which could be explained by shareholders’ lack of incentive.\(^5\) The infrequent debt issuance, on the other hand, makes debt dynamics *rigid* and unable to respond to productivity shocks. With rigid debt dynamics, firms either have zero leverage, or very high leverage which makes bankruptcy likely. In either case, firms cannot capture the interest tax shield. The addition of the financial covenant (that can be committed to) in the debt contract has a direct disciplinary effect on debt dynamics, and the commitment restores debt flexibility: the debt issuance is now linked to productivity shocks. As a consequence, the covenant can be instrumental in achieving a target leverage ratio.

We compare the financial covenant to two other popular classes of covenants, debt sweeps and asset sweeps, which restrict dilution of existing debt claim through debt issuance and asset sales, respectively.\(^6\) We find that, for the same level of commitment, the financial covenant improves ex ante firm value more than what debt and asset sweeps do. This is due to a fundamental difference of how these covenants work. The sweep covenants are designed as protections of debt value in some ex post states. Our result implies that in a dynamic model the ex ante benefit of such ex post protection is at best moderate. Intuitively, similar to the case when covenant could be renegotiated and debt holder’s control at renegotiation is priced, covenant protection does not change the shareholders’ leverage policy in equilibrium and does not reduce the rigidity of the debt policy. The lesson is that covenants generate more ex ante value through their discipline on firm policy, rather than through ex post protection on debt holders value.

The superior efficiency of the financial covenant has implications on potential solutions of agency conflicts among existing claim holders. With the financial covenant, ex post agency issues still exist (particularly, those caused by the debt dilution motive of shareholders) in some states, but the expected agency costs fall. The reason is that the

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5While the lack of incentive to reduce debt in our model is consistent, the infrequent and impulsive debt issuance is different from that is predicted by the continuous-time model in DeMarzo and He (2017). In our discrete-time setup, it is impossible for the firms to split a debt issuance into many small quantities and issue them one by one, trying to maximize the issuance price.

6A debt sweep covenant usually requires shareholders to use the proceeds of new debt issuance to pay back the existing debt. An asset sweep covenant requires shareholders to use the proceeds of asset sales to pay back the existing debt.
financial distress states, in which the debt-equity conflicts due to shareholders’ limited liability manifest themselves, are less likely if the firm commit to a healthy leverage policy. In a way, the standard trade-off theory would work in a dynamic model with various agency conflicts, but the required debt adjustments are made costly by the no-commitment problem. If we introduce sufficient commitment on leverage policy, the firm value can be improved and the debt dynamics could be similar to one that is predicted by the trade-off theory.

Our setup allows us to find important implications of how covenants affect investment policy. The first order effect of covenant on investment is through improvement of the firm value. The investment policy is mainly a consequence of the marginal firm value of capital stock. The traditional argument of debt overhang, whereby investment is determined by the marginal equity value of capital, is no longer true in a dynamic model with endogenous leverage. As more asset can be used to back more debt issuance, the shareholder actually capture both the equity and debt value of investment. Therefore, if a covenant improves firm value, such effect is transferred to investment.

A secondary effect is that there are states in which the covenant is (or close to be) stringent. In such states, a covenant restriction can distort investment, which may not maximize the unconstrained ex post firm value. While under the debt and asset sweeps these restrictions can be severe, with a covenant that disciplines the leverage policy, these distortions are less likely. Therefore, the investment is most efficient in terms of responding to productivity shocks under the financial covenant.

The rest of the paper continues as follows. In Section 1 we shows that covenants may not per se be commitment devices. We do it in a convenient continuous-time setup that enables us to use closed-form solutions to illustrate the main economic message. In Section 2 we introduce a more flexible model to discuss the efficiency of several real-life covenants. In Section 3 we illustrate the main results of our analysis. Section 4 concludes.
1 A simple model

We illustrate the main point of the paper regarding covenants with a simple model in continuous time, which allows closed-form solutions.

1.1 Model Setup

We add to Abel and Eberly (1997) model of corporate investment a dynamic capital structure. A firm has after tax cash flow \( x_t k_t (1 - \tau) dt \) over \([t, t+dt]\), where \( k_t \) is the capital stock of the firm at \( t \), \( \tau \) is the corporate tax rate, and \( x_t \) is capital productivity, which follows a geometric Brownian motion with drift rate \( \mu \) and diffusion rate \( \sigma \). The law of motion of capital stock is \( dk_t = I \gamma dt \), where \( I_t \) is the scale of investment and \( \gamma \in (0, 1) \) ensures a finite positive optimal investment. We assume that the price of the investment good is proportional to the productivity of capital, and so the cost of investment is \( C(x_t, I_t) = x_t I_t \).\(^7\) To value the corporate securities we assume risk-neutrality of all the agents and a constant risk-free rate, \( r \).

The unlevered firm value is found by solving the HJB equation

\[
rv^u = kx(1-\tau) - C(x, I) + V^u_k I \gamma + V^u_x \mu x + \frac{1}{2} V^u_{xx} \sigma^2,
\]

and is the sum of the values of asset in place and of future investment options,\(^8\)

\[
V^u(x_t, k_t) = A^u(x_t, k_t) + \Gamma^u(x_t).
\tag{1}
\]

As it is standard in the literature, \( A^u(x_t, k_t) = q^u(x_t) k_t \), where

\[
q^u(x) = \frac{1 - \tau}{r - \mu} x
\]

is the marginal value of capital for an unlevered firm.

\(^7\)This specification of the technology ensures that the value of the investment option is linear in \( x_t \), which makes our analysis tractable. In the same spirit, capital depreciation is assumed to be zero. A positive depreciation rates would make the effective discount rate more complicated and would add technicalities that are unnecessary for our main point.

\(^8\)All derivations are in Appendix A.
At $t$, the optimal investment is found by solving the program

$$\max_{I_t} q^u(x_t)I_t^\gamma - x_tI_t,$$

that is

$$I^*_t = I^* = \left( \frac{1 - \tau}{r - \mu} \right)^{\frac{1}{1 - \gamma}}.$$

The present value of future growth options is

$$\Gamma^u(x_t) = \mathbb{E}_t \int_t^\infty e^{-r(s-t)} [q^u(x_s) (I^*_s)^\gamma - x_sI^*_s] \, ds = \left( \frac{1 - \tau}{r - \mu} \right)^{\frac{1}{1 - \gamma}} \gamma^{\frac{\gamma}{1 - \gamma}} (1 - \gamma) \frac{x_t}{r - \mu},$$

which is independent of $k$.

In this setup, we introduce debt financing using a contract with a covenant. In particular, we discuss the case of a constant book leverage covenant, which requires that the outstanding debt is proportional to the capital stock at all times.

In what follows, we first show that, if the shareholders commit to it, the covenant partially restores the investment incentive by alleviating the debt overhang issue. This provides a justification of the existence of such covenant. Next, we show that the covenant is not renegotiation-proof. Namely, even with a constant book leverage covenant, the shareholders have an incentive to increase the leverage and will propose a renegotiation of the covenant. We will then derive an equilibrium under the assumption the shareholders do not commit to the covenant. In that case, even if the renegotiation is ex post efficient, we will show that only a second-best investment policy is achievable, which further motivates shareholders to payout cash dividends and liquidate the firm’s asset.

### 1.2 Constant book leverage covenant

We denote $b_t$ the face value of outstanding debt and $f_t = b_t/k_t$ the book leverage at $t$, with initial value $f_0$. The shareholders promise to pay a constant coupon rate $c$ on the debt, and the flow of total interest payment over $[t, t+dt]$ is $cf_t k_t dt$. The constant book leverage covenant in the debt contract requires that the book leverage of the firm, $f_t$, stays at the target leverage $f^c$. Shareholders’ commitment implies that $f_t = f^c$ for all $t$. As the firm is investing in new capital stock over time, the nominal debt increment
is \( db = f^c dk = f^c I^c dt \), which is assumed to be issued to new debt holders. The raised capital is based on the market price of the issued debt, \( \Delta(x, I) dt = f^c I^c p^f(x; f^c) dt \), where \( p^f(x; f^c) \) is the market price per unit of face value of debt.\(^9\) Because of the covenant, the existing debt holders have no reasons to oppose the debt issuance. Indeed, since \( f^c \) units of new debt issued are always backed by one unit of new capital stock, the new debt issuance does not dilute their claim.

We make three assumptions regarding the bankruptcy process, which is triggered by default on coupon service. First, the tax shield is lost, which is a bankruptcy cost. Second, the debt holders and the shareholders negotiate to split the unlevered value of asset in place \( q^u(x_d) \), where \( x_d \) is the default threshold. Denoting \( \alpha \in (0, 1] \) the debt holders’ bargaining power, and \( k_T \) the capital stock at default date \( T \), the recovery value for debt holders is \( \alpha q^u(x_d) k_T \) and the shareholders get the remaining \( (1 - \alpha) q^u(x_d) k_T \). The split of firm value at bankruptcy could be viewed as an outcome of strategic debt service, e.g., Mella-Barral and Perraudin (1997). Third, to preserve tractability, we assume that at default the shareholders do not lose the value of their growth option \( \Gamma^u(x_d) \), which is assumed to be put in an unlevered firm.\(^10\) This assumption can be motivated by the fact that the investment option is related to the shareholders’ inalienable human capital. Technically, this assumption is tantamount to assuming that the debt is only collateralized by the existing asset, so these two parts of firm value can be derived separately in the analysis.

Under these assumptions, the value of levered equity is

\[
V^l(x_t, k_t; f^c) = A^l(x_t, k_t; f^c) + \Gamma^l(x_t, k_t; f^c),
\]

in the continuation region, \((x_d, \infty)\), in which the firm is solvent. The default threshold \( x_d \) is determined to maximize the total equity value \( V^l \), i.e.,

\[
\frac{\partial V^l(x, k; f^c)}{\partial x} \bigg|_{x=x_d} = \left. \frac{\partial [(1 - \alpha) q^u(x) k_T + \Gamma^u(x)]}{\partial x} \right|_{x=x_d}.
\]

Notably, \( x_d = x_d(f^c, k) \); i.e., the default threshold is a function of \( f^c \) and \( k \).

\(^9\)As it will be clear later on, the price of debt is a function of three arguments, \( x, f_t \), and the covenant, \( f^c \). Because we are discussing the case with commitment (i.e., \( f_t = f^c \)), it is sufficient to denote the price of debt as \( p^f(x; f^c) \).

\(^10\)Overall, the shareholders get \( (1 - \alpha) \) of the capital stock, \( k_T \), plus the investment option, which can be used to grow this initial capital. Note that \( \Gamma^u(x_d) \) is independent of \( k_T \), as shown in equation (4).
In (5), the value of asset in place is 

\[ A^l(x_t, k_t; f^c) = q^l(x_t; f^c)k_t, \]

where

\[ q^l(x; f^c) = q^u(x) - \alpha q^u(x_d) \left( \frac{x}{x_d} \right)^{\beta} - (1 - \tau) f^c \frac{C}{r} \left[ 1 - \left( \frac{x}{x_d} \right)^{\beta} \right]. \quad (7) \]

In this expression, \( \beta \) is the negative root of \( \frac{1}{2} \sigma^2 \beta (\beta - 1) + \mu \beta - r = 0 \). On the right-hand side of (7), the second term is the present value of bankruptcy deadweight costs and the third term is the effective burden of debt. The value of debt, given \( x_d \), is

\[ p^l(x; f^c) = \frac{c}{r} \left[ 1 - \left( \frac{x}{x_d} \right)^{\beta} \right] + \alpha q^u(x_d) \left( \frac{x}{x_d} \right)^{\beta}, \quad (8) \]

where the second term on the right is the recovery value for a unit of debt. In (5), the present value of future growth options \( \Gamma^l \) is the solution of

\[ r \Gamma^l = \max_l q^l \Gamma^c + f^c p^l \Gamma^c - C(x, I) + \Gamma^l x \mu x + \frac{1}{2} \Gamma^l x^2 \sigma^2 x^2, \quad (9) \]

in the continuation region \((x_d, \infty)\), where the first term on the right-hand side is the value of newly installed capital, the second term are the proceeds of the debt issuance that is backed by the new installed capital, and the third term is the investment cost. At \( x = x_d \), \( \Gamma^l(x_d) = \Gamma^u(x_d) \). Solving for optimal investment, we have

\[ \Gamma^l(x_t; f^c) = \gamma^{\frac{1}{1-\gamma}} \left( 1 - \gamma \right) \mathbb{E}_t \left[ \int_t^T e^{-r(s-t)} x_s^{\frac{-\gamma}{1-\gamma}} \left( q^l + f^c p^l \right)^{\frac{1}{1-\gamma}} ds + e^{-r(T-t)} \Gamma^u(x_d) \right], \quad (10) \]

where \( T \) is the default stopping time.

Under the maintained assumption that the equity holders commit to the covenant, the optimal leverage \( f^c \) is chosen at \( t = 0 \) to maximize the total firm value:

\[ \hat{f}^c = \arg \max_{f^c} V^l(x_0, k_0; f^c) + f^c p^l(x_0; f^c) k_0, \]

which shows that \( \hat{f}^c \) depends on the initial profitability \( x_0 \) and initial capital \( k_0 \). If \( k_0 \) is large, the leverage decision is close to the one that maximizes the value of asset in place plus the value of debt: \( (q^l + f^c p^l) k_0 \). At the opposite extreme, if the firm has no asset
in place, $k_0 = 0$, the leverage decision maximizes only the value of the growth options. In general, a non-trivial $\hat{f}^c$ exists such that

$$V^l(x_0, k_0; \hat{f}^c) + \hat{f}^c p^l(x_0; \hat{f}^c) k_0 > V^u(x_0, k_0),$$

(11)

based on a standard trade-off argument (e.g. Leland 1994).

To conclude, if the shareholders commit to the constant leverage covenant, under which the choice of an optimal initial leverage is possible, the value of levered equity is increased by the value of the tax shield. Underinvestment due to debt overhang is still present in the model, because $q^l < q^u$. This can be seen by considering that the two terms that are subtracted from $q^u$, on the right-hand side of equation (7), are positive. At the same time, the cash flow from financing, $f^c p^l$, compensates the shareholders to the point they actually benefit from investment. This is because in (9) the marginal benefit of each unit of additional capital is $q^l + f^c p^l$, with $f^c p^l > 0$.

However, the constant leverage covenant only adds value to the firm if the shareholders commit to it. Indeed, if renegotiation is allowed, the covenant will not be effective, because the shareholders have an incentive to deviate from the constant leverage rule implied by the covenant. This is clear by comparing the marginal benefit and cost of issuing an incremental unit of debt. On the one hand, by issuing one dollar of face value of debt the cost to the shareholders is the present value of after tax coupon payments, $c(1 - \tau)$, until default, which is

$$(1 - \tau) \frac{c}{r} \left[ 1 - \left( \frac{x}{x_d} \right)^\beta \right].$$

On the other hand, the benefit given by the market price of one unit of debt is $p^l(x; f^c)$, as defined in (8). The difference between benefit and cost is $(1 - (x/x_d)^\beta) \tau c/r + (\alpha q^u(x_d)/f^c) (x/x_d)^\beta \geq 0$, which is strictly positive in the continuation region, $(x_d, \infty)$. Indeed, the shareholders benefit from issuing more debt because of the tax shield. No-

\[11\text{In this simple case we only consider static leverage, in the sense of Leland (1994), and } \hat{f}^c \text{ is optimal at } (x_0, k_0), \text{ when constrained to static leverage policies. There could be other debt arrangements (e.g., a dynamic capital structure setting in the sense of Goldstein, Ju, and Leland (2001), or the flexible leverage with instantaneously maturing debt, in the sense of Tserlukevich (2008)) that achieve a higher firm value. The purpose of the simple example is to show how a fixed leverage covenant may increase firm value, as opposed to derive the best debt contract that maximizes the ex ante firm value.}\]
tably, although a debt increase has a positive impact on the default threshold, this has no bearings for equity holders owing to the envelope theorem.

Overall, the shareholders have an incentive to deviate from the constant book leverage covenant. If one considers that covenants are generally designed to realign the incentives of debt and equity holders by triggering renegotiation at some contractible events, it is clear that the covenant is not renegotiation-proof, because of the incentive described above.

1.3 Renegotiation equilibrium

To derive an equilibrium with covenant renegotiation, we restrict ourselves to a specific form of renegotiation. While the bankruptcy resolution is the same as in the previous case, when the shareholders intend to violate a given fixed book leverage covenant they can renegotiate this decision with the debt holders.

To illustrate, suppose at time \( t \) the firm is solvent, the covenant is not violated, \( f_t^c = f_t \), and the shareholders want to deviate from it. The proposed change of book leverage is \( df_t \), or in terms of the total face value of debt

\[
db_t = k_t df_t + f_t^c dk_t = k_t df_t + f_t^c I^\gamma dt,
\]

where \( f_t^c I^\gamma \) is the issuance of new debt due to increased capital stock, pursuant to the covenant, and \( k_t df_t \) is the deviation from the covenant. Hence, the shareholders renegotiate to agree with the debt holders on the new debt policy of the firm, \( f_{t+dt} = f_t + df_t \). After \( df_t \) is agreed upon, the covenant is rewritten and \( f_{t+dt}^c = f_{t+dt} = f_t^c + df_t \), so that the covenant is consistent with the new leverage.\(^{12}\) We conjecture (and later will verify) that \( df_t \geq 0 \). The new debt is issued in the debt market at price \( p(x_t; f_{t+dt}^c, f_{t+dt}) \) on a pari passu basis.

We focus on an equilibrium with smooth leverage policy, \( df_t = g_t dt \), as in DeMarzo and He (2017), and the renegotiation is ex post efficient. Ex post efficiency is defined

\(^{12}\)For tractability, we do not allow \( f_{t+dt}^c \) to differ from \( f_{t+dt} \) after renegotiation. Hence, along an equilibrium path, \( f_t \) is the state variable for both the actual book leverage and the covenant. This feature enables us to focus on a Markov perfect equilibrium, which is sufficiently characterized by the state variable \( x_t, k_t, \) and \( f_t \).
by the requirement that $g_t$ maximizes the value to the firm’s current claim holders (per unit of capital stock):

$$
\max_{g_t} \quad p(x_t; f^c_{t+dt}, f_{t+dt})g_t + [p(x_t; f^c_t, f_{t+dt}) - p(x_t; f^c_t, f_t)] f_t \\
+ [V(x_t, k_t; f_{t+dt}) - V(x_t, k_t; f_t)] / k_t,
$$

where $V$ is the value of equity. In the objective function above, the first term is the cash flow from new debt issuance, the second term is the impact of such issuance on existing debt, and the third is the impact of the issuance on equity per unit of capital stock. Notably, the price of debt issued after renegotiation, $p(x_t; f^c_{t+dt}, f_{t+dt})$, is different from the value of debt for the existing debt holders at the renegotiation, $p(x_t; f^c_t, f_{t+dt})$. This is because the existing debt holders take part in the renegotiation (and receive part of the surplus), while the new debt holders, who pay $p(x_t; f^c_{t+dt}, f_{t+dt})$, are not. This difference will become clear in the following discussion on the mechanism to implement the objective.\(^{13}\)

In Appendix A we show that a necessary condition for ex post efficiency in the renegotiation is the first order condition

$$
\frac{\partial (pf + V/k)}{\partial f} = p + pf + \frac{V_f}{k} = 0,
$$

where $p_f$ is a short form for $\partial p/\partial f$. In (13), the marginal change of ex post firm value (per unit of capital) comprises three parts: proceeds of new debt issuance, the impact on the market value of current debt, and the impact on the equity value.\(^{14}\) Intuitively, with ex post efficient renegotiation, the leverage policy is determined so that the sum of the three parts equals zero.

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\(^{13}\)Because we require $f^c_t = f_t$ for all $t$ on an equilibrium path, the value $p(x_t; f^c_t, f_{t+dt})$ might seem “off-equilibrium”, when $f^c_t \neq f_{t+dt}$. While we recognize this is an abuse of our notation, still it is convenient to denote this way the value of debt at the renegotiation, when the proposed leverage, $f_{t+dt}$, is different from the existing covenant, $f^c_t$, and the holder of this claim has the right to call for a renegotiation.

\(^{14}\)In the non-commitment equilibrium by DeMarzo and He (2017), since the value of the current debt is ignored by the shareholders, the marginal benefit to shareholders when deciding the new debt is $p + V_f/k$. Differently from theirs, in our case, because the debt policy is restricted by a covenant and the shareholders must renegotiate to deviate from the prescribed policy, all three parts are relevant.
We can derive the equilibrium debt policy required by (13) under a specific bargaining game. In this game, the shareholders determine the leverage policy \( f_{t+dt} \) and take the proceeds of the new debt issuance, \( p(x_t; f_{c,t+dt}, f_{t+dt})g_t \). At the same time, they compensate the current debt holders for their loss due to the increased leverage with the payment

\[
\Theta dt = -[p(x_t; f_{c,t+dt}, f_{t+dt}) - p(x_t; f_{c,t}, f_{t})]b_t = -g_tp_f dt,
\]

which is distributed evenly across all debt holders (notably, \( p_f < 0 \)). Upon receiving this compensation, the current debt holders agree to rewrite the covenant to \( f_{c,t+dt} \). Therefore, \( p(x_t; f_{c,t+dt}, f_{t+dt}) \) is different from \( p(x_t; f_{c,t+dt}, f_{t+dt}) \) because the former contains the cash compensation \( \Theta dt/b_t = -g_tp_f dt \). This difference implies that

\[
\frac{\partial p}{\partial f_{c,t}} = \frac{p(x_t; f_{c,t+dt}, f_{t+dt}) - p(x_t; f_{c,t}, f_{t+dt})}{g_t dt} = \frac{g_tp_f dt}{g_t dt} = p_f.
\]

The equity value under this equilibrium is the solution of the HJB equation

\[
rV(x, k; f) = \max_g (1 - \tau)xk - C(x, I) - (1 - \tau)cb + (gk + f\Gamma) p + gp_f b
\]

\[
+ gV_f + V_k\Gamma + V_x\mu x + \frac{1}{2}V_{xx}\sigma^2 x^2,
\]

in the continuation region \((x_b, \infty)\), with default threshold \( x_b \). On the right-hand side of (14), the first line is the cash flows to equity (after-tax earnings, minus investment cost, minus effective debt service, plus proceedings of new debt issuance net of the compensation, \( \Theta \), paid to the debt holders to renegotiate the covenant), and the second line is the change in value due to its dependence of the state variables. The first order condition for optimal \( g \) is \( pk + p_f b + V_f = 0 \), which coincides with the condition of ex post optimal renegotiation in equation (13).

As before, the equity value is given by the value of asset in place plus the value of growth options,

\[
V(x_t, k_t; f_t) = q(x_t; f_t)k_t + \Gamma(x_t),
\]

where \( q \) is the value of a unit of capital under the renegotiation equilibrium, and \( \Gamma \) is the value of the growth option which, as we show in Appendix A, is not a function of \( f_t \) and \( k_t \).
In equilibrium, the solution of $q$ is

$$q(x_t; f_t) = q^u(x_t) - \alpha q^u(x_b) \left( \frac{x_t}{x_b} \right)^\beta - (1 - \tau) \frac{c}{r} f_t \left[ 1 - \left( \frac{x_t}{x_b} \right)^\beta \right], \quad (16)$$

with

$$x_b = -\frac{\beta}{\alpha(1 - \beta)} (r - \mu) \frac{c}{r} f_t.$$ 

From (13) and the boundary condition

$$p(x_b; f_T) = \frac{\alpha q^u(x_T)}{f_T}, \quad (17)$$

where $f_T$ is the leverage at default date, the market price of debt is

$$p(x_t; f_t) = (1 - \tau) \frac{c}{r} \left[ 1 - \left( \frac{x_t}{x_b} \right)^\beta \right] + \frac{\alpha q^u(x_b)}{f_t} \left( \frac{x_t}{x_b} \right)^\beta.$$ 

Comparing (18) to (8), ignoring the difference between two default boundaries $x_d$ and $x_b$ and that $f_t$ is changing, the price the debt under renegotiation equilibrium could be viewed as the price of debt with a committed fixed leverage $f_t$ minus the expected tax shield.

In Appendix A, we show that the optimal debt issuance in excess of what is required by the covenant is

$$g_t^* = -\frac{\tau c}{p_f}. \quad (19)$$

Because $p_f < 0$, we have $g_t^* > 0$ along the equilibrium path, so that in equilibrium the covenant is always violated. The optimal debt issuance is determined by trading off the tax benefit of the new debt issuance with the decrease of newly issued debt price.\(^\text{15}\) Our model implies that the leverage ratchet effect is not specific for equity. Even when the objective is to maximize the joint value of existing debt and equity, tax benefit is still enticing and we observe increase of debt over time.

\(^{15}\)The optimal debt issuance is the same as in DeMarzo and He (2017).
To see that the price of debt is rational (i.e., the present value of all future cash flows), note that the HJB equation for $p$ is

$$rp = c - gp_f + g \frac{\partial p}{\partial f^c} + g \frac{\partial p}{\partial f} + p_x \mu x + \frac{1}{2} p_{xx} \sigma^2 x^2. \quad (20)$$

The third term on the right hand side implies that when the covenant is renegotiated and $f^c$ increases, the protection of the debt in terms of future compensation becomes weaker. The fourth term implies that as the leverage increases, the probability of default increases and the value of the debt decreases. Although the current debt holders are compensated (the second term) of the loss of value due to the leverage increase in the current renegotiation, the debt loses its value from covenant protection over time as leverage goes up. To understand, compare two debt prices at different dates, $p(x_s; f^c_s, f_s)$ and $p(x_t; f^c_t, f_t)$ with $s < t$. Even if $x_s$ is equal to $x_t$, the price at $s$ is higher than that at $t$ for two reasons: $f_s < f_t$ and $f^c_s < f^c_t$. From the first condition, the default probability is lower. From the second condition, the time-$s$ debt is a claim on compensations to be received on covenant renegotiation from $s$ to the default date $T$, whereas the time-$t$ debt is a claim only on compensations on the shorter period from $t$ to $T$.

Using (19), equation (20) simplifies to

$$rp = c - \tau c + p_x \mu x + \frac{1}{2} p_{xx} \sigma^2 x^2$$

and it is easily verified that $p$ in (18) is a solution to this HJB equation with the boundary condition in (17). This suggests another way to interpret (20): the current compensation, $-gp_f$, offsets the loss of present value of future compensations, $g \frac{\partial p}{\partial f^c}$, so that the pricing equation above could be viewed as if the debt received no compensation for the increased leverage, along an equilibrium path. Hence, the effect of increasing leverage over time is to make the debt price lower compared to the one under commitment.

The loss of debt value due to increasing leverage also offsets completely the tax benefit of the existing debt. To see this, per unit of debt, the rate of debt price loss is $-g^*p_f = \tau c$, and the rate of the cash flow from the tax benefit is also $\tau c$. Due to the

\(^{16}\)Note that $p_f = \frac{\partial p}{\partial f^c}$ in equilibrium.
absence of tax benefit, if we define \( q + fp \) as the present value of the cash flows from asset in place,\(^{17}\), we have

\[
q + fp = q^u,
\]

which means the value of asset in place with dynamic leverage \( f \) maximizing the ex post firm value is equal to the value of asset in place of an unlevered firm.

The equality of the value of asset in place also implies the equality of the growth option. Using (21), the value of future growth options is

\[
\Gamma(x_t) = \mathbb{E}_t \left[ \int_t^T e^{-r(s-t)} \max_{I_s} \left[ \left( q(x_s, f_s) + f_s p(x_s, f_s) \right) I_s^\gamma - x_s I_s \right] \, ds + e^{-r(T-t)} \Gamma^u(x_T) \right].
\]

Therefore, the optimal investment \( I_s^* \) is the same as the one for unlevered case, in (3), it is time-invariant, \( I_s^* = I^* \), and the value of future growth options, \( \Gamma(x_t) \), is the same as in (4).

We conclude that total firm value in the renegotiation equilibrium and for the unlevered case are the same:

\[
V + pb = (q + fp) k + \Gamma = q^u k + \Gamma^u = V^u.
\]

This shows that total firm value in the renegotiation equilibrium is independent of current leverage. Hence, based on (11), we state our main result:

\[
V(x_0, k_0; f_0) + p(x_0; f_0) f_0 k_0 < V^l(x_0, k_0; \hat{f}^c) + p(x_0; \hat{f}^c) \hat{f}^c k_0,
\]

for any \( f_0 \). That is, the value of the firm is lower with renegotiation (regardless of \( f_0 \)) than under commitment at the optimal leverage.

The renegotiation equilibrium features ever-increasing leverage (the leverage ratchet effect) and the inability of shareholders to capture the tax benefit. While these results are in common with the no-commitment equilibrium in DeMarzo and He (2017), our mechanism is different, and it allows to understand the effect of covenants in financial

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\(^{17}\)This definition is motivated by the fact that both \( p \) and \( q \) are supported by the cash flows generated by the asset in place.
contracting. In our model, the fixed book leverage covenant can be renegotiated at all times and the book leverage is ever increasing (until default). Moreover, the covenant does not improve the value of the firm above the unlevered firm.

What covenant does in the model is to ensure the pledge-ability of the firm’s cash flow. In other words, with renegotiation in the bankruptcy resolution, and without such covenant no debt can be supported by the cash flows of firms. To see this, if the debt holders recover \( \alpha q u k_T \) at default, at any point before bankruptcy without the covenant the shareholders would issue a sufficient amount of debt to capture such value through the proceeds of the issuance. Combining this value to their recovery, \((1-\alpha)q u k_T + \Gamma u\), the shareholders can effectively appropriate the whole value of the unlevered asset around default. As a result, no cash flows of the firm can be pledged to debt holders, and the debt capacity of the firm becomes zero.

A debt capacity is ensured by what happens to the cash flows at default: with the covenant, if more debt is issued near default, the debt holders receive a compensation that exactly offsets their loss of recovery value. Hence, given the covenant, shareholders will have no incentive to issue debt just to expropriate the debt holders of \( \alpha q u k_T \). Then, positive debt values can be supported in the renegotiation equilibrium. Therefore, the existence of the covenants makes the cash flow of the firm pledge-able to debt in a setting with renegotiation.

1.4 Summary

This example shows that a debt covenant, if shareholders can commit to it, can improve firm value because it allows to capture the tax benefit and to alleviate agency costs through a compounding effect: a leverage covenant actually increases the incentive to invest. However, the incentive of the shareholders is to violate the covenant to capture more tax shield, and the covenant is designed to favor renegotiation with the debt holders. Even in the case of perfect renegotiation, assuming ex post firm value is maximized, the ex ante firm value is lower than in the commitment case. This result is in line with the general notion that an ex post efficient policy may not be ex ante optimal.

Our results show the vulnerability of covenants to renegotiation. Even with a covenant designed to avoid excessive leverage, renegotiation implies that the covenant
will be almost surely removed on an equilibrium path. Contrary to the empirical literature, which argues covenants are effective because they trigger renegotiation, we offer a counterexample, in which renegotiation is what renders covenants ineffective.

Therefore, with renegotiation covenants themselves cannot establish ex ante discipline. For covenants to be useful contracting devices to improve ex ante firm value, some renegotiation frictions are necessary. In what follows we will assume that some exogenous frictions make renegotiation *imperfect*. With imperfect renegotiation, covenants may be useful, but finding the channels of their impact and measuring their effects require a numerical model to be addressed.

Another implication of our discussion is that, if debt covenants are introduced for a purpose, in equilibrium they must be effective in ways other than by protecting debt holders’ value. In our renegotiation equilibrium, the debt value is fully protected ex post by the covenant, because existing debt holders have all the bargaining power and get full compensation for their value loss due to the proposed deviations from the prescribed policy. Still, the value of the firm is not improved relative to the unlevered case. In fact, in renegotiation the debt price increases because of the compensation the debt holders receive due to covenant deviations, and such increase exactly offsets the cost (i.e., the compensation) to the shareholders.

2 A general model to analyze debt covenants

The continuous-time setup in the previous section conveniently allows for closed-form solutions, but it imposes a number of restrictions when considering real-life covenants. In this section we introduce a more flexible setup to analyze debt contracting, in particular how the effectiveness of covenants depends on their structure. This flexibility will come at the cost of resorting to numerical solutions.

While the setup used in this part is borrowed from the structural corporate finance literature, to keep the intuition in line with the one of the simple model in Section 1 we exclude several of the frictions typically used in that literature to give realism to the model, like macro-economic risk, stochastic discount factor, capital adjustment costs, and equity issuance costs, which are less important in our analysis.
We model firm’s investment and financing decisions in a neoclassic framework with infinite-horizon and discrete-time. We denote productivity of capital stock by $x$, and assume that it is an autoregressive process with autocorrelation $\rho$ and conditional standard deviation $\sigma$. The operating cash flow before taxes is $\pi(x, k) = e^{x} k^{\gamma} - \varphi$, where $k > 0$ is the stock of capital with returns to scale parameter $\gamma \in (0, 1)$, and $\varphi > 0$ is a fixed cost that includes all expenses. The capital stock depreciates at a rate $\delta \in (0, 1)$.

The company’s debt consists of callable risky consol bonds of equal priority. The debt contract specifies the face value $b \geq 0$ and a fixed coupon rate equal to the risk free rate $r$. The debt contract may or may not be associated with debt covenants and we will explore the possibility that covenants impose further restrictions on new debt issuance. For simplicity, we assume the interest tax shield is the only incentive to using debt financing. Corporate taxes are levied on corporate earnings, $\pi - rb - \delta k$. We denote $\tau$ the marginal net tax rate.

At any date, given the current state $(x, a)$, where $a = (k, b)$, the firm may decide to change the capital stock for next period, $k'$, where the sign of investment, $k' - (1 - \delta)k$, is unrestricted. Subject to limitations imposed by covenants, the firm may also decide at any date to change its debt from $b$ to $b'$, under the same debt contract. Additional debt is issued at market value, and old and new debt have equal seniority. We assume that the debt level can be reduced by repurchasing a portion of the debt at its par value, rather than its market value.\(^{18}\)

The overall firm’s dynamic is described as follows: given $(x, a)$, new levels of capital, $k'$, and debt, $b'$, are simultaneously chosen. While the debt contract is incomplete in that a state-contingent face value and interest payment are not possible, debt covenants can be contracted on the state $(x, a)$ and impose restrictions on the policy $a' = (k', b').\(^{19}\)

\(^{18}\)Mao and Tserlukevich (2014) show that in the absence of frictions, lenders will sell their debt back to the firm only at par value. Even with frictions, debt will be repurchased at a premium to market value, although the size of the premium depends on the nature of the frictions. Our result does not materially change, as long as the firm’s debt is repurchased at a premium over the before-change market price. Mao and Tserlukevich (2014) also detail institutional considerations, including securities and tax regulations, that make it more likely that debt is repurchased at or close to par value.

\(^{19}\)The assumption that the underlying state $(x, a)$ is contractible implies that the net worth, $w = (1 - \tau) (\pi(x, k) - rb - \delta k) + k - b$, is contractible. Empirical studies show that covenants based on net worth, interest coverage ratio, debt-to-cash-flow ratio, and other equivalent accounting covenants are common. See for instance Dichev and Skinner (2002). Hence, our setup has sufficient generality to discuss real-life covenants.
As we will show later when we examine specific types of covenant in detail, the effects on cash flows and policies of a renegotiation triggered by covenant violation is summarized by a state- and policy-dependent cost function to equity, $\Theta^e$ and payment function to debt, $\Theta^d$. By specifying two contingent functions $\Theta^e$ and $\Theta^d$, not only we can model the effect of covenant restrictions on firm’s policies, but also capture the effect of relative bargaining power between the shareholders and debt holders at renegotiations upon covenant violation. Particularly, it will turn out that the case with no covenants (unprotected debt), corresponds to $\Theta^e = 0$ and $\Theta^d = 0$.

We derive a Markov perfect equilibrium of the dynamic game between equity holders and debt holders under the assumption of perfect and symmetric information for all parties. In what follows, we define the equilibrium by a state-contingent default, investment and financing policy of the firm, and the value of debt and equity supported by those policies.

If in $(x, a)$ the firm is solvent and makes decision $a'$, the cash flow to equity is

$$e(x, a, a') = (1 - \tau) (\pi - rb - \delta k) - (k' - k)$$

$$+ \chi_{\{b' \leq b\}} (b' - b) + \chi_{\{b' > b\}} p(x, a') (b' - b) - \Theta^e(x, a, a'), \quad (23)$$

where $\chi_\mathcal{E}$ is the indicator function of event $\mathcal{E}$ and $p(x, a')$ is the ex-coupon market price per unit of face value of debt in state $x$ given policy $a'$. The first line of (23) is the after-tax operating profit minus the investment cost; the second line is the cash flow from altering the debt level (the first term is for a decrease in debt, and the second term is for a debt increase), net of the cost function of debt covenants. If the optimal residual cash flow for shareholders is negative, then funds are raised by issuing new equity.

The levered equity value is defined as the fixed point of

$$V^l(x, a) = \max \left\{ (1 - \alpha) V^u(x, a), \max_{a'} e(x, a, a') + \beta \mathbb{E}_x \left[ V^l(x', a') \right] \right\}, \quad (24)$$

where, $V^u(x, a)$ is the current value of unlevered firm, $\alpha$ is the debt holders’ bargaining power, and the expectation $\mathbb{E}_x[\cdot]$ is conditional on the current state $x$, and $\beta$ is a risk-free

---

20 The functional form of $\Theta^e$ and $\Theta^d$ will be derived in Section 3.
21 We assume zero equity issuance cost because this aspect is inessential to our analysis of debt covenants. Therefore, from the shareholders’ perspective, internal and external equity are equivalent, and therefore, without affecting the firm’s other policies, we do not model a savings policy.
discount factor. If $V^l \leq (1 - \alpha)V^u$, the shareholders (strategically) default on servicing debt and split the unlevered firm value with the debt holders, based on their bargaining power. Otherwise, the optimal policy is $a^* = (k^*, b^*)$. The indicator $\omega(x, a)$ is one if the firm defaults and zero otherwise.

To determine the (ex-coupon) market value of debt at $t$ conditional on the firm being solvent and to decision $(k', b')$, we denote

$$d(x', a'\prime, a''\prime) = (1 - \omega(x', a')) \left\{ r + \chi_{\{\nu^r \leq \nu\}} \left[ p(x', a'\prime) \frac{b''}{b'} + 1 - \frac{b''}{b'} \right] 
+ \chi_{\{\nu^r > \nu\}} p(x', a'\prime) + \Theta d(x', a', a'\prime) \right\} + \omega(x', a') \alpha \frac{V^u(x', a')}{b'} \right]. \quad (25)$$

the value to debt holders at $t + 1$ in state $(x', a')$, in which the firm will make decisions $a''$ conditional on being solvent. To interpret equation (25), at $t + 1$, if the firm is solvent the debt value is equal to the coupon plus the payment received due to covenant requirements. This depends on the ensuing policy: if the debt level is reduced or remains unaltered, it is the new debt price of the residual debt plus the book value of the reduced debt $(1 - b''/b')$; if the debt is increased, it is the price, $p(x', a'')$, at the policy $a''$. In default, the value to debt equals the fraction of unlevered firm value received by the debt holders in proportion of their stake in the firm. Hence, the ex-coupon market value of debt is

$$p(x, a') = \beta \mathbb{E}_x [d(x', a', a'\prime)]. \quad (26)$$

Notably, the price of debt at $t$ depends on debt holders’ equilibrium beliefs regarding default, $\omega(x', a')$, and optimal policy, $a''$, at $t + 1$.

To close the model we need to define the unlevered firm value, which is the fixed point of

$$V^u(x, a) = \max \left\{ 0, \max_{k'} \left\{ (1 - \tau)(\pi - \delta k) - (k' - k) + \beta \mathbb{E}_x [V^u(x', a')] \right\} \right\}. \quad (27)$$

To summarize, we have set up a baseline model to examine the effect of debt covenants and renegotiations when these covenants are violated. A natural benchmark is the case with a unprotected debt (i.e., $\Theta^e = \Theta^d = 0$), because we are interested in how and how much debt covenants improve the value of the firm.
3 Results

In this section, we use the model described in Section 2 to explore the effects of covenants on investment and financing policies and the value of the firm.

The following analysis is based on a numerical approximation of the model described in Section 2. The base case parameters for our analysis, as shown in Table 1, are similar to those used in comparable dynamic models. We take \( \rho = 0.8 \) and \( \sigma = 0.15 \), which is not far from the one used by Hennessy and Whited (2005). We use risk-free discounting at \( \beta = 1/1.05 \) (i.e., \( r = 5\% \)). The return-to-scale parameter \( \gamma = 0.4 \) is set to the average of values used by Hennessy and Whited (2005) and Zhang (2005). The fixed production cost of \( \varphi = 0.8 \) is useful to set the average leverage (however, no consistent fixed cost value is used in the literature, largely due to differences in model features). We use an annual depreciation rate of \( \delta = 0.12 \) as in Kuehn and Schmid (2014). We use a higher marginal corporate tax rate, \( \tau \), than typically used in the literature with the sole purpose of better highlighting the effect of capital structure on firm value. The debt holders' bargaining power at default, \( \alpha \), is set to 100%.

First, we will focus on a restricted version of the model, in which we remove the investment option to confirm the results of Section 1 in this different setup. Next, we show how agency conflicts affects financing and investment dynamics under the case of debt with no covenants. Finally, we show how inclusion of different types of debt covenants that we find in the empirical literature affect these dynamics.

3.1 Constant capital stock

If we restrict \( k = 1 \) at all dates (and therefore investment always equals depreciation), we remove the effect of decreasing returns to scale, and therefore the implications can be compared to the model in Section 1 of a firm with constant returns to scale. To ensure consistency of the contracting features between two models, we create two cases in the numerical model: a case with fixed book leverage, i.e., \( b \) is a constant (and optimally

\footnote{The solution to equation (24), with constraint (26), which is a system of two non-linear equations with unknowns \( V^l \) and \( p \), is found numerically by a discrete state-space method based on value function iteration done simultaneously on both equations.}
chosen to maximize the ex ante firm value) as in Section 1.2; a case with a renegotiable leverage covenant. For the latter case, following Section 1.3, we set\(^{23}\)

\[
\Theta^e = \chi_{\{v>b\}} b \left[ p(x, b) - p(x, b') \right] \quad \text{and} \quad \Theta^d = 0.
\]

While the shareholders offer a compensation to the existing debt holders for the value loss due to a debt increase from \(b\) to \(b'\), the actual compensation received by the debt holders is zero because the transfer from debt to equity is offset by the debt value loss due to a weakened covenant. Hence, the debt is actually priced in the market as if there was no compensation.

Comparing the numerical results in the numerical model with the continuous-time model under the same contract and renegotiation is not only to verify the leverage ratchet effect and the loss of firm value due to no-commitment, but also serves the purpose of understanding the difference between two types of models.

Figure 1 shows the value of the firm at a given point in time \(t\), against the exogenous shock, \(x\). We report the firm value and the change of debt for the two cases mentioned above as well as the case with no debt. Because the ex ante level of debt that maximizes the value of the firm under commitment is \(\hat{b} = 2.3\), to make the comparison fair we report the value of the firm under renegotiation at the same current debt.

The figure confirms the fundamental result presented in Section 1.3: the value of the firm under commitment is higher than under no-commitment. Therefore, even if ex post renegotiation is aimed at maximizing the value of the firm, ex ante this results in a lower value than in the case the constant leverage covenant is maintained at all dates. This result is consistent with the one in Section 1.3: from Table 2, we see that in the whole simulated sample the change of debt is always non-negative, i.e., the debt outstanding for the firm can only be increased.

There are two differences with the model in Section 1.3, though. The firm value under renegotiation is higher than an unlevered firm and the increase of debt outstanding is not very frequent, while the continuous-time model predicts that the two firm values should be equal and the firm should constantly issue new debt. While there is the obvious difference that in the numerical model the firm makes decisions at discrete times, and

\(^{23}\)We change our notation slightly, by removing \(k = 1\) from the arguments of the price of debt. See Appendix B for detailed derivation of \(\Theta^e\) and \(\Theta^d\).
thus the trade-off at each time point might be different relative to the trade-off on marginal values in the continuous-time setting, the main reason lies in the different probabilistic assumptions of the tho models.

To illustrate, suppose in the discrete-time model the productivity shocks are purely transitory (that is, the shock are i.i.d.), and the model is essentially an infinite sequence of single-period models. Facing a negative shock and a possible liquidity default (when the operating cash flow is low relative to the coupon), the shareholders may cover the shortfall by injecting equity capital if the shock leaves a positive continuation value of equity. This gives the shareholders an incentive to “stay in the game” and service the debt. Also, it makes excessive debt issuance ex ante more costly, alleviating the leverage ratchet effect.

A random walk assumption for profitability represents the opposite side of the spectrum. In this case, the shocks have a permanent impact on future states. Therefore, a negative shock has a much negative impact on the continuation value of equity, reducing the shareholders’ incentive to service the debt and keep the firm ongoing. Hence, the random walk assumption, used in the continuous-time model, makes the no-commitment problem more severe, everything else equal.

The AR(1) assumption for the productivity shocks in our numerical model (which is also the common assumption for other models) is in between these two extremes. On one hand, the productivity reverts to its long term mean in the stationary distribution, which provides higher continuation value of equity and makes shareholders more interested to serve the current debt. On the other hand, a negative shock could be persistent which dampens the shareholder’s incentive. Because our theory is motivated by debt renegotiation in a dynamic setup, we choose \( \rho = 0.8 \), larger than value used in other contributions (e.g., Hennessy and Whited (2005) use \( \rho = 0.62 \)), to highlight the impact of the no-commitment issue.\(^{24}\)

Under the AR(1) assumption a debt increase is not as frequent as that in a random-walk model. To make this decision, the firm trades off short-term benefits with long-term costs. A debt increase is optimal only when the productivity shocks are large enough, because the tax shield is large and the market price of debt high. In other states, the

\(^{24}\)On the other hand, setting \( \rho \) higher than 0.8 would lead to extremely high bankruptcy rate in our simulated sample. Though we do not aim to match the moments, extreme samples do not provide useful inferences on how debt contracting functions in a real-world setting, so we do not set \( \rho \) above 0.8.
long-term cost of increasing debt can be significant, and a typical firm would stay put at
the same amount of debt for a long period of time, generating the debt rigidity that we
observe in our simulated economies, even absent debt transaction costs. The leverage
ratchet effect, in the sense that debt debt is never reduced, is still pervasive in the
numerical model. The lack of incentive to reduce debt in the future actually increases
the cost of issuing more debt for the long run (simply because increased debt cannot be
reduced) and makes the debt even more rigid.

To conclude, the numerical solutions confirms that ex post debt renegotiation does
not lead to ex ante firm value maximization. Although renegotiation is allowed and the
debt policy can be changed at every period, still the leverage ratchet effect determines
the debt policy making it even more rigid. The main implication of Section 1.3 and of this
section is to show that the leverage ratchet effect is not caused by the narrowly-defined
agency conflicts (i.e., those between the existing claim holders), rather by conflicts be-
 tween existing and new claim holders. Although the ex post renegotiation eliminates the
conflicts between the equity holders and existing debt holders, the cause of the leverage
ratchet effect is the lack of commitment by both equity and existing debt versus new
debt holders.

These findings suggest that to study the effects of debt covenants, instead of the
traditional focus on narrowly-defined agency, renegotiation, and how the debt value is
protected, one should focus on the effect of covenants as commitment devices that limit
ex post renegotiation. In reality covenant renegotiation is limited by a number of frictions
(which we discuss later) and firm can (at least, partially) commit to (or not to) certain
behaviors by including the covenants in the debt contract. In the following sections,
we show that covenants improve firm value through contracting by enable flexible debt
policies when the firm can commit to them.

3.2 Unprotected debt

As a benchmark for the cases with covenants, we first consider a firm with unprotected
debt, that is, a perpetual and defaultable debt contract defined the face value, $b$, the
coupon rate, $r$, and no covenants. Table 3 show some sample moments of common
metrics for the simulated economy under this case.
The impact of the agency conflicts under unprotected debt financing is apparent. The average and median firm values are 7.58 and 7.17. These are much bigger than the values in Table 2 because the unconditional average of the asset is here 3.44 instead of 1. However, as we will show later, the firm value is lower than the ones under the cases with covenants. There are two main forces in the model leading to a lower firm value: the inability of capturing the tax benefit through capital structure decisions, and inefficient investment decisions. Of these two, the former is first order since investment is driven by marginal value of capital, similarly to what is seen Section 1. Table 3 shows that the debt policy: the median book debt (market leverage) is 0.37 (0.07) which indicates at least half of the firm operates under a very low leverage, but the 75th percentile is 6.09 (0.62) which indicates excessive debt for at least a quarter of firms.

Such distribution of debt is the result of an impulsive debt issuance policy. From the 5th percentile to median, the change of debt is zero, half of the firms are not issuing or reducing debt. Only at the right side of the distribution we observe some extreme debt issuances. Figure 2 shows the debt policy of the firm at an average level of asset, for the 25th, 50th, and 75th percentile of the distribution of the debt. For low debt outstanding, it is optimal to issue debt only for a positive productivity shock, whereas there are no debt changes for negative shocks. Firms with median to high debt outstanding, however, issue debt regardless of the shock. While debt issuance at positive productivity shocks is justified by the high market price of debt, at negative shocks debt issuance is solely driven by the cashing out incentive of the shareholders, who issue debt before bankruptcy and pay out the proceeds to themselves. In this way shareholders capture the recovery value of debt at default.25

Due to the discrete nature of the model, shareholders cannot predict bankruptcy perfectly, so cashing out is triggered by a sufficiently high probability of next period default. The cashing out incentive is sensitive to the current level of debt outstanding. Compared to the upper panel of Figure 2, where it is absent, the firm in the middle panel displays cashing out at negative productivity shocks. Given the small difference in current debt between the top and the medium panel, we conclude that debt issuance in a dynamic setup is self-amplifying: as the firm takes the initial steps to lever up, the

25Another way of cashing out is to liquidate the firm’s capital stock and pay out. As we assume the debt holders claim the unlevered firm value at default, issuing debt is usually a more profitable option for the shareholders.
probability of larger debt issuance increases significantly. This mechanism explains the polarized unconditional leverage we observe in the simulated economy.

Given the above described debt dynamics, the firm will not be able to properly capture the tax shield. One the one hand, if the debt is low, the tax benefit is low. On the other, if the debt is high, with high credit risk the value of debt is mostly due to the recovery value, which is independent of the tax shield, because it is proportional to the unlevered firm value.

While we confirm, under unprotected debt, the leverage ratchet effect, our mechanism is different from the one discussed by DeMarzo and He (2017). In our model, for negative productivity shocks not only the debt is never reduced, but a firm with an already high debt issues more debt making bankruptcy more likely. In a way, the rigid debt policy amplifies the impact of negative productivity shocks on the firm value.

Traditional trade-off-theory arguments suggest that a firm should decrease leverage to negative productivity shocks to avoid excessive bankruptcy probability, but these arguments are established on a view of ex ante firm value maximization. Under no-commitment, the debt flexibility to respond to negative shocks is immediately exhausted, and one wonders if financial contracting may be employed to restore it. We investigate whether and to what extent different debt covenants can give such flexibility. We present the results in the following sections.

### 3.3 Debt covenants

While a debt contract is usually incomplete as it cannot specify state-contingent face values and coupon rates, debt covenants can be contracted on some verifiable events. Following the paradigm of Aghion and Bolton (1992), covenants that are tied to these events allow creditors to gain control and restrict the firm policies even in the absence of a missed debt payment. As shown by the empirical literature (e.g., Smith and Warner (1979) and Nini, Smith, and Sufi (2009)), creditors exercise their control rights through renegotiations at triggering events. Therefore, previous research on debt covenant mostly pay attention to how covenant-induced control right transfers ex post protect debt value and alleviate agency conflicts.
However, the results in Section 3.1 suggest a different story. In a dynamic setup, although covenant-induced perfect renegotiation can eliminate the agency conflicts, we still find the firm value is lower than in the case under commitment, and the debt rigidity is not solved.\(^{26}\) While solving agency conflicts ex post, renegotiation prevents commitment and attainment of a higher ex ante firm value.\(^{27}\) Therefore, the analysis of covenant effectiveness should focus on how debt covenants can be used as commitment devices to establish ex ante discipline, rather than on how renegotiation solves agency conflicts.

With costless renegotiation, covenants cannot establish any commitment as they can be renegotiated. Hence, a necessary condition for covenants to create ex ante value is that in reality there are non-negotiable circumstances that make debt renegotiation infeasible or at least sufficiently costly. For example: if public debt is dispersedly held, mis-coordination among creditors might prevent renegotiation; if debt holders buy credit default swaps to insure their claim, they are not interested in renegotiation. These circumstances cannot be changed ex post and make covenants (at least partially) committable. Since these are factors outside debt contracts per se, we model them as an exogenous renegotiation frictions.

Given the same level of renegotiation cost, not all covenant are equally effective. Our purpose is not to design the optimal debt covenants in our setup.\(^{28}\) Instead, we want to evaluate the common covenants used in practice and investigate their mechanism. It is understood that these real-life covenants are usually contracted on some simply verifiable accounting information and impose simple restrictions on firm policies. Hence, one naturally anticipates that they are incomplete, because they do not lead to fine-tuned firm policies in every possible state. However, the main point of our analysis (and in fact of this paper) is that, although the covenants are non-optimized, may generate ex

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\(^{26}\) As another piece of evidence, comparing the firm under full-time covenant protection in Section 3.1 with the firm under an unprotected debt contract in Section 3.2, the rigid debt dynamic that impedes the use of tax benefit is not solved by covenant protection and renegotiation, as shown by the polarized debt distribution in both cases.

\(^{27}\) This notion of renegotiation has been one of the key insights of the contract theory, but it has not been addressed enough in the literature of debt covenants where most attention has been paid to the ex post value of renegotiation.

\(^{28}\) Usually an optimal contract is model specific. In our numerical setup, as long as the capital \(k\), debt level \(b\) and the cash flow which could be used to infer productivity \(x\) are contractible, assuming full commitment, writing an optimal complete contract on each state is possible. But such practice of finding the optimal contract has little real implications since full commitment is unlikely in the real world and firms are usually much more complicated than our purposefully simplified model.
post inefficient policies, and sometimes might even exacerbate the agency conflicts, still they have an *ex ante positive effect* by restoring commitment and changing the dynamics policy of the firm.

Our setup provides a useful testing environment for investigating each specific debt covenants. As Section 3.2 implies that the low firm value is linked to rigid debt dynamic, we focus on three different type of covenants that target to restrict the debt level contingent on certain corporate events: i) a debt sweep covenant; ii) an asset sweep covenant; and iii) a financial accounting covenant that restricts the ratio of debt over EBITDA. These three types of covenants are most prevalent empirically, as presented in the relevant literature: see Bradley and Roberts (2015), Billett, King, and Mauer (2007), Chava and Roberts (2008), Roberts and Sufi (2009), among others. For each covenant, we specify the cost for equity and debt holders, $\Theta^e$ and $\Theta^d$ respectively, to represent the effect of renegotiation frictions.

### 3.3.1 Debt Sweep Covenant

A debt sweep covenant requires that if shareholders decide to increase the debt, the proceeds from new debt issuance must go towards paying back existing debt. For simplicity, and for consistency with similar restrictions imposed in the capital structure literature (e.g., Fischer, Heinkel, and Zechner (1989)), we assume that the covenant requires that the outstanding debt has to be fully retired if the firm issues some new debt. As a result, when the covenant is strict, in order to increase the debt level, the firm must buy back all debt (at par) and reissue new debt. The motivation of this covenant, as generally discussed in the literature, is to prevent the shareholders from diluting current debt holders’ claim by increasing leverage.

Shareholders have incentive to renegotiate the covenant as in general they have no incentive to reduce debt as shown in Section 3.2. To make the covenant committable, a
renegotiation friction is necessary. Specifically, we assume when the debt sweep covenant is triggered (as the firm increases outstanding debt), the cost functions are

$$\Theta^e(x, a, a') = \Theta^d(x, a, a') = \theta \chi_{\{\nu > 0\}} b [1 - p(x, a')]$$.

(27)

The interpretation is straightforward: with probability $\theta$, some renegotiation frictions make shareholders fully commit to the covenant. $\theta$ is a key parameter we use to gauge the degree of commitment (or frictions of renegotiation).

In Table 4 we report statistics of key metric of the simulated economy under the debt sweep covenant, for $\theta = 0.5$. The covenant protection gives an average firm value, 8.19, higher than the one under unprotected debt. Shareholders’ incentive of issuing more debt at negative productivity shocks is offset by the cost in (27), regardless of the current debt, as shown in Figure 3. As a result, the firm’s cash flow will be able to support more debt. For example, the 25th percentile of book debt (market leverage) is 3.34 (0.41), higher than those under unprotected debt. At the same time, the credit spread is lower. In Figure 4, credit spread is zero for positive productivity shocks, under moderate leverages. In these states, the firm is able to capture some tax benefit with partial commitment.

The mechanism that induces commitment in our specification of the cost functions is quite intuitive: $\Theta^e$ is proportional to the difference between the face value and the market value of debt, and it is higher the more negative the shock or the higher the outstanding debt. As the cost is countercyclical, the renegotiation friction is sufficient to subdued the self-amplifying behavior of debt issuances we observed under unprotected debt. Because of this, Table 5 shows that even a moderate cost (such as at $\theta = 0.1$) is effective, as the firm value does not improve much when $\theta$ goes from 0.5 to 1.$^{31}$

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$^{29}$The derivation is in Appendix B.1. Notably, the renegotiation following the violation of a debt sweep covenant results in a transfer of an amount from the equity to the debt holders, as $\Theta^e = \Theta^d$. This is not generally the case for other covenants, as it may be $\Theta^e \neq \Theta^d$.

$^{30}$An alternative interpretation of (27) is that renegotiation results in waiving the covenant with probability $(1 - \theta)$. Under full commitment, shareholders pay the existing debt at par, and thus their cost is $[1 - p(x, a')]$, which is also the compensation to the debt holders. Notice that, when $b' > b$ and the shareholders choose to retire the existing debt at face value without violating the covenant, the existing debt is risk free, and $\Theta^e = \Theta^d = 0$.

$^{31}$Comparing among the different values of $\theta$ in the table indicates that the key factor contributing to firm value is the policy dynamics, measured by standard deviation, as opposed to the magnitude of the cost function.
Although the debt sweep covenant curbs the shareholders’ incentive to cash out, by design it does not induces debt reductions. This generates rigidity of the debt policy, which reduces the firm’s ability to take full advantage of the tax shield and to further improve the ex ante value, a major drawback with this covenant. In Table 4, only the 95th percentile of change of debt over asset is strictly positive (and equal to 5.41), while the leverage ratchet effect is not removed, which makes the issuance of new debt even more infrequent. Overall, a firm will keep the same debt for longer, with the capital structure not responding to productivity shocks.

Furthermore, as this covenant imposes additional cost on debt issuances, it might reduce the incentive to issue debt in states in which it would otherwise be efficient (especially positive productivity shocks), which further strengthens debt rigidity.\textsuperscript{32} The sensitivity analysis on parameter $\theta$ is shown in Table 5. The higher $\theta$ the higher the unconditional average of the ex ante firm value, because the more costly for the shareholders is to increase debt ex post, which leads to lower expected bankruptcy costs. The average firm value under $\theta = 1$ is 8.44, which is the highest possible achieved by the covenant. The firm value increase comes from the increase of debt value, at a cost of more rigid debt dynamics (the 95th change of debt over asset is 0, and the 99th is 12.45).

\subsection{Asset Sweep Covenant}

An asset sweep covenant requires that if shareholders voluntarily disinvest, the proceeds from the sale must be used to reduce the outstanding debt level. Bradley and Roberts (2015) document that asset sweeps are very common in private debt contracts, particularly in the latter years of their sample (e.g., 93.8\% of the loans in their 2001 sample had asset sweeps). In reality, by forcing proceeds from assets to be used to pay down debt, the asset sweep covenant prevents the shareholders from liquidating assets in order to make large payouts, which may be particularly enticing to shareholders as the firm approaches default.

In our model shareholders have an incentive to issue debt near default, because the debt is collateralized on the unlevered value of the firm. However, in these states a firm with large capital stock is better off liquidating capital, due to diminishing returns to

\textsuperscript{32}The debt sweep covenant will also amplify the debt overhang effect on investment when it is binding, which also contributes to its weakness. We discuss the effect in Section 3.3.4.
In this case, if this firm is at the same time highly levered, using the proceeds of asset sales to pay down debt may be a good option, as for firm value maximization the leverage should be reduced facing a negative shock.

As before, we assume the commitment to the covenant is not perfect, and a cost function is imposed if shareholders deviate from it:\(^{34}\)

\[
\Theta^e(x, a, a') = \eta_a \chi_{\{I < 0\}} \max \{0, b' - \max \{0, b + I\}\}, \quad \text{and} \quad \Theta^d = 0, \tag{28}
\]

where \(I = k' - (1 - \delta)k\). \(\Theta^e\) can be interpreted as a probabilistic assessment of the event the shareholders will waive the covenant at the end of the period.\(^{35}\) Again, \(\eta_a\) gauges the degree of commitment: the higher it is, the more unlikely are the shareholders to waive the covenant restriction in renegotiation. Because \(\Theta^d = 0\), the debt holders do not gain from enforcing the covenant, while the value of the covenant is reflected in \(\Theta^e\) restricting the shareholders’ action.

As shown in Table 6 for the case \(\eta_a = 0.35\), the asset sweep covenant does not improve the firm value relative to the unprotected debt case. The average firm value, the average asset level, and the average Q-ratio are all lower than in that case. While the effect of the asset sweep covenant also depends on the cost parameter, \(\eta_a\), in Table 7 we show that changing \(\eta_a\) from 0.05 to 0.35 does not significantly change the characteristics of the simulated sample.

The reason for the relative inability of the asset sweep covenant to increase the value of the firm is the debt policy induced by this covenant. In Table 6, we find that the debt is never reduced, which is also confirmed by the debt decision of typical firms in Figure 5. The implication is that the incentive of not reducing the debt is very strong, and given the covenant restrictions, shareholders either forgo asset sale opportunities even when they would be efficient, or they pay the cost to renegotiate with the debt holders.

Therefore, although the covenant is designed to tie asset sales to debt reductions in poor states of the firm (which is meant to be efficient), the actual (and unintended) effect

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\(^{33}\)Investment is perfectly reversible in our model, which makes liquidating capital less costly and more profitable.

\(^{34}\)The derivation is in Appendix B.2.

\(^{35}\)As documented by Chen and Wei (1993) and Nini, Smith, and Sufi (2009), creditors will often waive this right, in case demanding immediate payment.
is the distortion of asset sales decisions due to such tie. From Figure 6, the distortion is apparent, as for a moderate negative shock, the firm delays asset sale decisions to keep positive investment (inferred from the convexity of the investment policy around $x = 0$). Firms with high amount of outstanding debt, when hit by a negative productivity shock, choose an inefficient solution with no asset sales and cashing out by issuing excessive debt, instead of liquidating capital stock and reduce the debt, as shown in the bottom panels of Figures 5 and 6.

It is worth noticing that the cost parameter $\eta_a = 0.35$ is sufficiently high so in the simulation the covenant violation frequency is low at 4.94%. Even with this high commitment level, the asset sweep covenant does not provide incentive for shareholders to reduce debt ex post and does not alleviate the rigidity of debt policy (as shown by the distribution of the change of debt over asset in Table 6). Additionally, it has a negative impact on the asset sale policy of the firm.

As an alternative interpretation of these results, by forcing to use the proceeds of asset sales to pay the debt holders, the asset sweep covenant creates a debt overhang problem for asset sales.\textsuperscript{36} The asset sweep covenant provide an example of how in a covenant that links two or more policies together, distortions might transfer through one policy to the other.

### 3.3.3 Financial Covenant

Finally, we study a covenant that combines both balance sheet and income statement metrics, namely a maximum Debt/EBITDA ratio.\textsuperscript{37} Chava and Roberts (2008) document that this is the most prevalent covenant in their sample of private loans, even more common than minimum net worth and interest coverage covenants. We find it to be a good proxy for alternative covenants (e.g., on net worth) that impose state-contingent restrictions on financial policies.

\textsuperscript{36}The classic debt overhang issue is that the shareholders’ incentive to invest is reduced, because the benefit of such investment would accrue to the existing debt holders. Here, the covenant demands the benefits of asset sales accrue first to the debt holders.

\textsuperscript{37}Bradley and Roberts (2015) and Billet, King, and Mauer (2007) report that covenants based on financial statement metrics are common both in private and public debt contracts. Some covenants are based solely on balance sheet values (e.g., a minimum net worth) or income statement items (e.g., minimum interest coverage ratios).
More specifically, we assume that the maximum Debt/EBITDA ratio written in the financial covenant is $\psi$. In the case when the selected $b'$ exceeds $\psi \pi(s, k')$, that is, the debt-on-EBITDA ratio is expected to be violated, technical default occurs. Committing to the covenant means that shareholders must then select new asset, $k'$, and debt, $b'$, subject to the restriction that the Debt/EBITDA ratio does not exceed $\psi$ if next period’s productivity shock, $x'$, is the same as the current period’s, that is, $b'/\pi(x, k') \leq \psi$. This rule proxies for a requirement on how the firm’s policies should adjust to the covenant’s restrictions.

As the asset sweep covenant, the financial covenant is also a restriction on the firm’s policy that ties the size of EBITDA with debt level (although it does not specify a monetary penalty if the equity holders deviate). To model renegotiation, we take the similar approach as the asset sweep covenant, and assume the covenant restriction is applied in a probabilistic fashion:

$$\Theta^e(x, a, a') = \eta_f \chi\{b' > \psi \pi(x, k')\} \left[b' - \psi \pi(x, k')\right] \quad \text{and} \quad \Theta^d = 0,$$

(29)

where $\eta_f$ gauges the debt holders’ bargaining power. In our baseline case, we set $\psi$ at 5 and $\eta_f$ at 0.35.

As shown in Table 8, the financial covenant significantly improves the firm value, relative to the other covenants. The simulated sample features flexible debt dynamics. The distributions of change of debt over asset are more dispersed and we see active debt reduction. Therefore, the distribution of debt is also less dispersed, with average book debt at 4.55, which is the highest among all the cases. Despite the high leverage, the credit spread is the lowest among all the cases, indicating low bankruptcy probability and high debt valuation.

The debt decisions of typical firm, in Figure 7, displays positive issuance in high states and active debt reductions in low states, except when the debt overhang problem is severe, which happens if the current debt is high. From these debt policies, a target leverage emerges from the simulated economy, and firms reduce (increase) leverage from this target responding to negative (positive) productivity shocks. Due to the response of leverage to productivity shocks, the impact of productivity shocks on credit spread is efficiently managed. Figure 8 shows a positive relation between credit spread and

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38The derivation of these functions is in Appendix B.2.
productivity shock, owing to the effect of leverage policy on the default probability, except when the current debt is high (bottom panel).

The combination of higher leverage and low credit spread resulting from this covenant is worthy of further discussion. Given a positive shock, a firm can issue more debt at a lower cost if it can credibly commit to reduce the debt in case of future negative shocks. If future debt reductions cannot be committed to, as it was the case with unprotected debt and the other two covenants, increasing debt increases bankruptcy probability, thus increasing the cost of debt. Even worse, in those cases, the current debt issuance increases the incentive to cash out. Because of the ensuing impulsive debt issuance behavior, in equilibrium, the tax benefits cannot be captured (debt holders anticipate the cashing out incentive and the pricing of debt does not include the tax benefit). Differently from these cases, the financial covenant creates a credible commitment to future debt reductions, and thus it enables the firm to issue debt to capture tax benefits. Because it avoids the leverage ratchet effect, the debt policy is \textit{flexible}; i.e., it responds to productivity shocks in an efficient way.

In our numerical model, ex ante optimal capital structure can be derived according to the classic trade-off theory.\textsuperscript{39} The achievement of optimal leverage requires firms to ex post reduce debt at negative productivity shocks and increase debt at positive shocks. But the shareholders cannot commit to such plan, under unprotected debt contract or with the two covenants considered previously, because their ex post incentive is to increase debt. This is why a covenant that restricts a maximum Debt/EBITDA ratio forces the shareholders to reduce debt at negative shocks. The result is that the debt policy approximates the one that maximizes the ex ante firm value.

Although impact of agency costs on firm value is not the main focus of the paper, the results have strong implication for agency costs in a dynamic model. Compared to unprotected debt and the two sweep covenants, under a financial covenant shareholders’s cashing out is minimized (as seen from the high likelihood and extent of debt reduction at negative shocks in Table 8).\textsuperscript{40} The leverage is a pre-condition for cashing out. Only when the firm faces a significant default probability, the interests of shareholders and

\textsuperscript{39}The trade-off theory still applies even if we assume zero bankruptcy cost as debt holders recover unlevered firm value at default. This is because as leverage increases, the tax benefit when the firm is solvent increases, which is traded off with the increased probability losing the tax benefit at default.

\textsuperscript{40}As we will show later, the impact of debt overhang on investment is also minimized.
debt holders diverge sufficiently and the debt policy will be distorted by the agency conflicts. The financial covenant induces a flexible debt policy that enables to minimize the default probability. Therefore, the ability of committing to an ex ante efficient leverage policy is the key to alleviate possible agency conflicts.

The above results are obtained under the condition $\eta_f = 0.35$. To interpret the parameter, for each dollar of face value of debt above of what is prescribed by the covenant, the shareholders lose 35 cents. Such high cost of deviation ensures low covenant violation frequency (and the flexible debt policy), relegating deviations to some extreme cases. In the bottom panel of Figure 7 a firm with high debt outstanding and a very negative productivity shock will not reduce the debt (as it would have been efficient). However, it does not issue debt either as the incentive to do so is curbed by the high cost of deviation from the covenant restriction.

Because $\eta_f$ is a key parameter, Table 9 shows a sensitivity analysis. For low values of this parameter, there is more debt issuance at negative shocks. If the cashing out behavior is predominant, the debt policy becomes rigid. For instance, if we set $\eta_f = 0.05$, there is no debt reduction, and the simulated sample becomes very similar to one under unprotected debt, with low firm value. Table 9 shows that, although committing to the financial covenant enhances debt flexibility and firm value, overall the ex post incentive to deviate from the covenant is strong and it requires sufficient renegotiation friction to build effective commitment.

### 3.3.4 Investment Dynamics

The notion of debt overhang by Myers (1977) is that shareholders invest to maximize equity value, and therefore current debt negatively affects the investment incentive. This notion does not apply to a model of dynamic debt issuances. Indeed, in a dynamic model shareholders make investment not only to increase future dividends, but also to increase cash flows and capital stock, which will collateralize future debt issuances to take advantage of the tax shield. The important implication is that, if a higher debt allows the firm to capture more tax benefits, in equilibrium a firm may invest more now to be able to back more debt issuances in the future.\(^{41}\)

\(^{41}\)Note that for a clean illustration of investment policy, we shut down frictions involved with investment by assuming zero investment cost and perfect reversibility.
This remarkable result is visible in our simulations, in which the only difference among the cases is the covenant. In Table 8, the average (median) asset under the financial covenant is 4.13 (4.21), which is significantly higher compared to 3.57 (3.37) under the debt sweep covenant in Table 4, 3.41 (3.37) under the asset sweep covenant in Table 6, and 3.44 (3.36) with unprotected debt in Table 3. Overall, under the financial covenant, a firm grows larger (i.e., more asset) than under the other covenants, and this is solely due to its more efficient debt policy.

The efficiency of investment policy can also be judged by ex post investment functions for typical firms under different covenants. In general, following the same logic as above, ex post investment should respond to the marginal firm value, which is determined by the productivity shocks and firm value loss due to non-commitment problems and ex post agency conflicts. However, the link between investment and firm value breaks down when: (1) in a financial distress state investment is determined with high bankruptcy probability; (2) the covenant restrictions are binding for investment decisions.

For example, in Figure 9 with unprotected debt, when the agency conflicts become severe (i.e., high current debt), there is a kink of the sensitivity of investment to productivity shocks, at about $x = 0.2$. The interpretation is that with shocks below 0.2, the sensitivity of investment to the shock, and therefore the dis-investment incentive, increase. This is the impact of financial distress on the investment policy. For shocks lower than $-0.3$, the shareholders wish to liquidate the whole firm. Also under the debt sweep covenant, Figure 10, the sensitivity of investment to productivity shocks is higher for negative shocks if the debt is high, although the firm appears more resilient to negative shocks than under unprotected debt, as the critical shock for liquidation is below $-0.5$.

As observed before, the asset sweep covenant inefficiently restricts the firm from liquidation of capital. In Figure 11, for a high level of debt, the firm chooses not to liquidate capital at any negative shocks, even in cases in which asset is high and decreasing returns to scale would make liquidation an efficient choice. This is due to the fundamental debt overhang issue created by this covenant, as we noted at the end of Section 3.3.2.

Under the financial covenant, the debt distribution in the simulated sample is not polarized as in the other cases. Hence, the investment behavior is almost the same at
Comparing Figure 12 to Figure 13, we see that investment is mostly driven by the marginal firm value (approximated by average $q$), except for the case when covenant is binding, and the investment is distorted. Facing a negative shock, firm can either reduce debt or keep investing to avoid covenant violations. In Figure 12 at average shocks ($x$ around 0), we see evidence of upward distortion of investments.

The analysis of ex post investment under different covenants shows that commitment is ex post costly, because it induces inefficient investment in financial distress states. What drives the inefficiency is the conflict of interests of shareholders and debt holders in bankruptcy, and its effect is visible when default is likely. Committing to a covenant does not restore efficiency in these ex post states. Rather, the ex ante efficiency gain from covenants derives from minimizing the probability of these distress states. Not surprisingly, covenants that can be effective disciplines on leverage policies (like the financial covenant in our model) provide most efficiency gain since these states with significant agency conflicts are mostly driven by excessive leverage.

4 Conclusion

We assess the effect of debt covenants as commitment devices on firm value and investment in a context of dynamic leverage decisions. Our analysis emphasizes that covenant-induced renegotiation will make covenant ineffective to generate ex ante effects on firm value and leverage policy. Therefore, renegotiation frictions are necessary to make covenants useful commitments.

When combined with renegotiation frictions, covenants that discipline firm’s leverage policy is the most effective to improve ex ante firm value. In a dynamic setting, the no-commitment on leverage policy has a first order effect on the firm value loss, due to the resulting leverage ratchet effect and rigid debt dynamics. Covenants that better induce active debt reductions in response to negative productivity shocks restore debt flexibility and improve firm value the most. Under commitment, the debt issuance decision is pro-cyclical, which is close to what will be predicted by the standard trade-off theory.

When the firm can actively adjust its debt in both directions with the financial covenant, the firm value is not sensitive to current debt, because the firm can quickly adjust the debt to the optimal level. The same argument applies to the expected tax benefit.
Also, with flexible debt policy, other debt-equity conflicts are less of concern due to the minimization of incurrence of financial distress states in which these conflicts are significant. The firm value improvement could be transferred to investment decisions.

The important link between renegotiation frictions and effect of covenant on ex ante firm value and leverage policy could be a future research direction.
References


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A Simple model: derivation of the main equations

A.1 Base case - unlevered firm

The value of the unlevered firm is

\[ V_u(x_t, k_t) = \frac{1 - \tau}{r - \mu} x_t k_t + \left( \frac{1}{r - \mu} \right)^{\frac{1}{\gamma}} \gamma^{\frac{1}{1 - \gamma}} \left( 1 - \gamma \right) \frac{x_t}{r - \mu}, \]

or \( V_u(x, k) = A^u(x, k) + \Gamma^u(x) \). The value of the asset in place, \( A^u \) satisfies the differential equation

\[ rA^u = kx(1 - \tau) + A^u_x \mu x + \frac{1}{2} A^u_{xx} \sigma^2, \]

or, given the linearity in capital stock, \( A = q^u k \); equation

\[ rq^u = (1 - \tau)x + q^u_x \mu x + \frac{1}{2} q^u_{xx} \sigma^2. \]

The solution of this equation is (2). \( \Gamma^u \) solves the HJB equation

\[ r\Gamma^u = \max_{I} A^u_k I^\gamma - C(x, I) + \Gamma^u_x \mu x + \frac{1}{2} \Gamma^u_{xx} \sigma^2. \]

From this equation, optimal investment is derived from the first-order conditions as in (3) and the value of growth options is

\[ \Gamma^u(x_t) = \mathbb{E}_t \int_t^{\infty} e^{-r(s-t)} \max_{I_s} [A^u_k I_s^\gamma - C(x_s, I_s)] \, ds \]

\[ = \mathbb{E}_t \int_t^{\infty} e^{-r(s-t)} [A^u_k (I^*)^\gamma - C(x_s, I^*)] \, ds \]

\[ = \mathbb{E}_t \int_t^{\infty} e^{-r(s-t)} \left[ x_s^{1 - \frac{\tau}{r - \mu}} \left( \gamma \frac{1 - \tau}{r - \mu} \right)^{\frac{1}{1 - \gamma}} - x_s \left( \gamma \frac{1 - \tau}{r - \mu} \right)^{\frac{1}{1 - \gamma}} \right] \, ds \]

\[ = \left( \frac{1 - \tau}{r - \mu} \right)^{\frac{1}{1 - \gamma}} \gamma^{\frac{\gamma}{1 - \gamma}} (1 - \gamma) \mathbb{E}_t \int_t^{\infty} e^{-r(s-t)} x_s \, ds \]

\[ = \left( \frac{1 - \tau}{r - \mu} \right)^{\frac{1}{1 - \gamma}} \gamma^{\frac{\gamma}{1 - \gamma}} (1 - \gamma) \frac{x_t}{r - \mu}. \]
A.2 Constant book leverage covenant

The value of equity is fund by solving the differential equation

\[ rV^l(x, k; f^c) = \max_l (1 - \tau)xk - C(x, I) - (1 - \tau)c f^c k + f^c I^\gamma p^l + V^l_k I^\gamma + V_x \mu x + \frac{1}{2} V_{xx} \sigma^2 x^2 \]

in the continuation region, \((x_d, \infty)\). We conjecture and verify that the value of equity has the following functional form

\[ V^l = q^l k + \Gamma^l, \]

where \(\Gamma^l\) does not explicitly depends on \(k\), although \(k\) may indirectly affect \(\Gamma^l\) through the bankruptcy threshold \(x_d\). The marginal value of capital for equity is

\[ \frac{\partial V^l}{\partial k} = q^l + \frac{\partial (q^l k + \Gamma^l)}{\partial x_d} \frac{\partial x_d}{\partial k} = q^l, \]

because the second term in the right-hand side in the above equation is zero since \(x_d\) is optimally chosen to maximize the equity value. Therefore, we can separately value the asset in place from growth option.

The value of the asset in place is found by solving the differential equation

\[ rq^l k = k x(1 - \tau) - c (1 - \tau) f^c k + k q^l_x \mu x + \frac{1}{2} k q^l_{xx} \sigma^2 x^2 \]  \(\text{(30)}\)

in the continuation region, \((x_d, \infty)\). From (30) we have

\[ rq^l = (1 - \tau)(x - cf^c) + q^l_x \mu x + \frac{1}{2} q^l_{xx} \sigma^2 x^2 \]

with solution

\[ q^l = (1 - \tau) \left[ \left( \frac{x}{r - \mu} - \frac{c}{r} f^c \right) - \left( \frac{x_d}{r - \mu} - \frac{c}{r} f^c \right) \left( \frac{x}{x_d} \right)^\beta \right], \]

which can be written as in equation (7).
The market price of the debt is the solution of

\[ rp^l = c + p_k^l \Gamma^\gamma + p_x^l \mu x + \frac{1}{2} p_{xx}^l \sigma^2 x^2, \]

in the region \((x_d, \infty)\), with boundary condition \(p^l(x_d) = \alpha q^u(x_d)/f^c\). In the expression above, the increase of capital stock does not change the debt price (i.e., \(p^l_k = 0\)) because each dollar of face value of debt is backed by \(1/f^c\) units of asset. Therefore, the value of debt is as in equation (8).

The value of the growth options is the solution of (9) in the continuation region, plus the boundary condition \(\Gamma^l(x_d) = \Gamma^u(x_d)\). The optimal investment, \(I^*\), is the solution of

\[
\max_I q^l I^\gamma + \Delta(x, I) - C(x, I) = \max_I \left( q^l + f^c p^l \right) I^\gamma - xI,
\]

from which

\[
I^*(x) = \left[ \frac{\gamma}{x} \left( q^l + f^c p^l \right) \right]^{1-\gamma},
\]

and the present value of future growth options is

\[
\Gamma^l(x_t) = \mathbb{E}_t \int_t^T e^{-r(s-t)} \max_{I_s} \left[ q^l I_s^\gamma + \Delta(x_s, I_s) - C(x_s, I_s) \right] ds + e^{-r(T-t)} \Gamma^u(x_d)
\]

\[
= (1-\gamma) \mathbb{E}_t \int_t^T e^{-r(s-t)} \left[ q^l(x_s; f) + f^c p^l(x_s) \right] I^*(x_s)^\gamma ds + e^{-r(T-t)} \Gamma^u(x_d),
\]

which gives (10).

### A.3 Renegotiation equilibrium

For \(dt \to 0\) in the smooth equilibrium where \(df_t = g_t dt\), the objective function of renegotiation in (12) is equal to

\[
\max_{g_t} p(x_t; f^c_t, f_t) g_t + \frac{\partial p}{\partial f} g_t f_t + \frac{1}{k_t} \frac{\partial V}{\partial f} g_t.
\]
The first order condition with respect to \( g_t \) gives (13). On the other hand, taking a deviation \( \Delta f \) from the debt policy \( f_t \) implied by (13), we consider the problem

\[
\max_{\Delta f} \Delta f \cdot p(x_t; f^c_t + \Delta f, f_t + \Delta f) + [p(x_t; f^c_t, f_t + \Delta f) - p(x_t; f^c_t, f_t)] f_t \\
+ [V(x_t, k_t; f_t + \Delta f) - V(x_t, k_t; f_t)] / k_t.
\]

If the equilibrium smooth policy \( f_t \) is indeed optimal, the objective function above is solved by zero \( \Delta f \). Then the objective function can be written as

\[
\int_0^{\Delta f} p(x_t; f^c_t + \Delta f, f_t + \Delta f) d\delta + \int_0^{\Delta f} p_f(x_t; f^c_t + \delta, f_t + \delta) f_t d\delta \\
+ \int_0^{\Delta f} \frac{V_f(x_t, k_t; f_t + \delta)}{k_t} d\delta \\
\leq \int_0^{\Delta f} p(x_t; f^c_t + \delta, f_t + \delta) d\delta + \int_0^{\Delta f} p_f(x_t; f^c_t + \delta, f_t + \delta) f_t d\delta \\
+ \int_0^{\Delta f} \frac{V_f(x_t, k_t; f_t + \delta)}{k_t} d\delta \\
= \int_0^{\Delta} \left[ p(x_t; f^c_t + \delta, f_t + \delta) + p_f(x_t; f^c_t + \delta, f_t + \delta) f_t + \frac{V_f(x_t, k_t; f_t + \delta)}{k_t} \right] d\delta \\
= 0
\]

The inequality holds when \( \partial p / \partial f^c \leq 0 \) and \( p_f \leq 0 \), which we will verify later. Therefore, in the smooth equilibrium, \( f_t \) defined by (13) is the optimal leverage policy.

To solve equation (14), we first show that the equity value \( V \) has the functional form

\[
V(x, k; f) = A(x, k; f) + \Gamma(x) = q(x; f)k + \Gamma(x).
\]

\( ^{43} \)On the equilibrium we have \( f^c = f \). Under the equilibrium smooth policy \( f \), when a change of leverage \( \Delta f \) is proposed, the change to value to the existing debt holders reflects the change of equilibrium debt price from \( f \) to \( f + \Delta f \), that is,

\[
[p(x_t; f^c_t, f_t + \Delta f) - p(x_t; f^c_t, f_t)] f_t = \int_0^{\Delta f} p_f(x_t; f^c_t + \delta, f_t + \delta) f_t d\delta.
\]
Due to constant returns to scale, the value of asset in place for equity, $A$, as well as the total value of debt, $pb$, are linear in $k$. Therefore, we can write $A = qk$ and $pb = pfk$. Condition (13) on an equilibrium path becomes

$$ p + pf f + \frac{V_f}{k} = p + pf f + q_f + \frac{1}{k} \frac{\partial \Gamma}{\partial f} = 0, $$

which must hold for all $k$. We conjecture that $\Gamma$ is independent of $k$. This implies that $\partial \Gamma / \partial f$ is a constant with respect to $k$. Hence, for the above equation to hold identically for all $k$, we conjecture that $\partial \Gamma / \partial f = 0$ along an equilibrium path. These conjectures will be confirmed later on. From this we have $V_f = q_f k$, and thus (13) would be re-written as

$$ p + pf f + \frac{V_f}{k} = p + pf f + q_f = 0. \quad (31) $$

Finally, given the above conjecture

$$ V_k = \frac{\partial (qk)}{\partial k} = q. $$

Using these conditions in (14) we have

$$ r V = \max_{G,I} (1 - \tau) x k - C(x, I) - (1 - \tau) cb + (gk + f \Gamma) p + gkp_f f 
+ gq_f k + q \Gamma + V_x \mu x + \frac{1}{2} V_{xx} \sigma^2 x^2, $$

from which we have

$$ rqk + r \Gamma = \max_{g,I} k \left[ (1 - \tau)(x - cf) + gp + gp_f f + gq_f + q_x \mu x + \frac{1}{2} q_{xx} \sigma^2 x^2 \right] 
+ (q + fp) \Gamma - C(x, I) + \Gamma_x \mu x + \frac{1}{2} \Gamma_{xx} \sigma^2 x^2 
= k \left[ \max_g (1 - \tau)(x - cf) + gp + gp_f f + gq_f + q_x \mu x + \frac{1}{2} q_{xx} \sigma^2 x^2 \right] 
+ \max_I (q + fp) \Gamma - C(x, I) + \Gamma_x \mu x + \frac{1}{2} \Gamma_{xx} \sigma^2 x^2, $$

47
because given current $k$, maximization with respect to $g$ is independent of maximization with respect to $I$. The above program is equivalent to solving equations

$$rq = \max_g (1 - \tau)(x - cf) + gp + gp_f f + gq_f + q_x \mu x + \frac{1}{2} q_{xx} \sigma^2 x^2,$$  \hspace{1cm} (32)

and

$$r \Gamma = \max_I (q + fp) \Gamma - C(x, I) + \Gamma_x \mu x + \frac{1}{2} \Gamma_{xx} \sigma^2 x^2,$$  \hspace{1cm} (33)

simultaneously. Using the first-order condition for optimal $g$ in (32), we get

$$rq = (1 - \tau)(x - cf) + q_x \mu x + \frac{1}{2} q_{xx} \sigma^2 x^2.$$  \hspace{1cm} (34)

The solution in equation (16) and the bankruptcy boundary $x_b$ can be found assuming $g = 0$ along an equilibrium path. With this simplification, the solution for $q$ is straightforward.

The price of debt in (18) is found solving (31). The solution is

$$p(x; f) = (1 - \tau) \frac{c}{r} \left[ 1 - \frac{1}{1 - \beta} \left( \frac{x}{x_b} \right)^\beta \right].$$

Given $x_b$, the price of debt can be rewritten as

$$p(x; f) = (1 - \tau) \frac{c}{r} \left[ 1 - \frac{1}{1 - \beta} \left( \frac{x}{x_b} \right)^\beta \right]$$

$$= (1 - \tau) \frac{c}{r} \left[ 1 - \left( \frac{x}{x_b} \right)^\beta \right] + \frac{-\beta}{1 - \beta} (1 - \tau) \frac{c}{r} \left( \frac{x}{x_b} \right)^\beta$$

$$= (1 - \tau) \frac{c}{r} \left[ 1 - \left( \frac{x}{x_b} \right)^\beta \right] + \frac{\alpha (1 - \tau)}{f (r - \mu)} x_b \left( \frac{x}{x_d} \right)^\beta$$

$$= (1 - \tau) \frac{c}{r} \left[ 1 - \left( \frac{x}{x_b} \right)^\beta \right] + \frac{\alpha q_u(x_b)}{f} \left( \frac{x}{x_b} \right)^\beta$$

which proves (18). It is immediate to see that $q + fp = q^u$. Therefore, equation (33) becomes

$$r \Gamma = \max_I q^u \Gamma - C(x, I) + \Gamma_x \mu x + \frac{1}{2} \Gamma_{xx} \sigma^2 x^2,$$
and the solution for Γ is the same as Γ^u, where the latter is defined in (4). This verifies that the conjectures that \( \partial \Gamma / \partial f = 0 \) and that Γ is independent of k were correct.

To derive equation (19), we note that the pricing equation for the firm value, \( fp + q \), is

\[
r(fp + q) = (1 - \tau)x + \tau cf + gp + \frac{\partial^2 (fp + q)}{\partial f^2} g + \frac{\partial^2 (fp + q)}{\partial x} \mu x + \frac{1}{2} \frac{\partial^2 (fp + q)}{\partial x^2} \sigma^2 x^2.
\]

This equation is implied by the valuation from the existing claim holders of the asset in place: at time t, the existing debt holder and shareholders jointly claim the operating cash flow (the first term on the right hand side), the interest tax shield (second term), the cash flow from financing which is also backed by the asset in place (third term), and their claim value will be affected by the drift of x and f (the remaining terms on the right-hand side). Taking the partial derivative with respect to f on both side and using (31), we have \( g = -\tau c/p_f \), which is (19).

\section*{B Renegotiation and specification of \( \Theta^e \) and \( \Theta^d \)}

\subsection*{B.1 Debt sweep covenant}

When the restriction of the debt sweep covenant applies, the debt holders can demand the current debt \( b \) to be repaid at the face value, before new debt, \( b' \), is issued. Given the specification of the cash flow to equity in equation (23), the debt holders exercise this right in the event \( \{b' > b\} \), which we refer to as covenant violation. We model renegotiation of the restriction contingent upon violation.

In our specification, debt sweep covenant demands a cash payment from shareholders to debt holders in case of violation. Therefore, the functions \( \Theta^e \) and \( \Theta^d \) are specified as (27). The interpretation of (27) is straightforward. When the event \( \{b' > b\} \) is triggered and the debt sweep covenant applies, the shareholders must pay the full face value \( b \) to the current debt holders. As a result, the debt holders in addition to the value \( p(x, a')b \) of debt absent covenant protection, receive from the shareholders a compensation \( b - p(x, a')b \) due to the covenant. The compensation is the result of the renegotiation between the shareholders and the debt holders. We assume the cash payment is only
enforced with probability \( \theta \), which is a modeling device used to measure the bargaining power of the debt holders in renegotiation.

### B.2 Asset sweep covenant and financial covenant

The asset sweep and Debt/Ebitda covenants result in a restriction of the firm policy, \( a' = (k', b') \), differently from the debt sweeps which require a payment from equity to debt. We incorporate such covenant restrictions and renegotiations in our dynamic model in a tractable way. In what follows, without limitation of our arguments, we will focus for simplicity on the case in which the firm is solvent at the current date.

In general, the restriction imposed on policy \( a' \) by a covenant can be conceptualized as a constraint \( \phi(x, a, a') \leq 0 \), for some function \( \phi \). When the covenant is never waived or relaxed, the optimal policy and equity value result from the constrained maximization problem

\[
V^l(x, a) = \max_{a'} \left\{ e(x, a, a') + \beta \mathbb{E}_x [V^l(x', a')] - \lambda(x, a) \phi(x, a, a') \right\} ,
\]

where \( \lambda(x, a) \) is the Lagrangian multiplier of the covenant constraint in the optimized solution at state \( (x, a) \). Note that (35) is expressed in a recursive formulation and next period equity value \( V^l(x', a') \) will in turn result from next period covenant restriction, with a state-contingent Lagrangian multiplier \( \lambda(x', a') \).

The optimal policy \( a^* = (k^*, b^*) \) and \( \lambda^*(x, a) \) from (35) correspond to the bargaining outcome when the debt holders always have all the bargaining power, whereby the covenant is fully enforced, and we define

\[
C^1(x, a, a') = \lambda^*(x, a) \phi(x, a, a')
\]

the state-contingent cost of violating the covenant, because it is \( C^1 = 0 \) when \( \lambda^* = 0 \) (i.e., the covenant is not binding) and \( C^1 > 0 \) when \( \lambda^* > 0 \) (i.e., the covenant is binding).

To model a renegotiation in which shareholders retain some bargaining power, we consider the following variation on (35):

\[
V^l(x, a) = \max_{a'} \left\{ e(x, a, a') + \beta \mathbb{E}_x [V^l(x', a')] - C^\theta(x, a, a') \right\} ,
\]

\( 50 \)
with
\[ C^\theta(x, a, a') = \theta C^1(x, a, a') = \theta \lambda^* (x, a) \phi(x, a, a'), \] (37)
where \( \theta \in [0, 1] \). Relative to (36), (37) is the cost function of violating the covenant with probability \( \theta \). As before, \( \theta \) gauges the bargaining power of the debt holder: when \( \theta = 1 \), the covenant surely applies; with \( \theta < 1 \), the covenant restriction is waived with probability \( 1 - \theta \).

Particularly, \( \theta = 0 \) is the case with unprotected debt, as shareholders have no cost in choosing an optimal policy that maximizes their value. \( \theta > 0 \) means that it is costly to deviate from the covenant restriction \( \phi(x, a, a') \) for shareholders and as a result, the covenant could be used as a commitment device.

To define \( C^\theta \) in our dynamic setting, we need to determine \( \lambda^*(x, a) \) at the optimum of (35). This can be found using the first order conditions (under regularity conditions of \( e \) and \( E \) which we assume, for simplicity) at \( a^* \):
\[
\frac{\partial e(x, a, a^*)}{\partial a^*} + \beta \frac{\partial E_x [E(x', a^*)]}{\partial a^*} - \lambda^*(x, a^*) \frac{\partial \phi(x, a, a^*)}{\partial a^*} = 0.
\]
However, such an approach is cumbersome to use in our numerical optimization procedure as it forces us to solve the above equation at each \((x, a)\).

For tractability, we take a reduced-form approach by approximating \( \theta \lambda^*(x, a) \) using \( \eta \chi_{\{\phi(x, a, a') > 0\}} \), in which \( \eta > 0 \) is a constant and \( \{\phi(x, a, a') > 0\} \) is the event the constraint is violated. While this approach retains the property that when the constraint is not binding (i.e., \( \{\phi(x, a, a') < 0\} \)), the cost is zero, we assume a constant \( \eta \) in place of the state-contingent Lagrange multiplier. A higher \( \eta \), which reflects higher shadow costs and higher bargaining power of the debt holders, makes shareholders more constrained ex post.

With this approximation, to model the effect of covenants (and their renegotiations), we use the cost function
\[
\Theta^\varepsilon(x, a, a') = \eta \chi_{\{\phi(x, a, a') > 0\}} \phi(x, a, a')
\] (38)
to discipline shareholders’ policies ex post.
The ‘best’ value of $\eta$ for a given covenant is determined as the one that gives the highest unconditional average of total firm value when we use program (37). Given the limitations of this approach, we also do a sensitivity analysis to show how specified magnitudes of $\eta$ influence the results.

As for the debt payment function $\Theta^d$, note that $\lambda^*(x,a) \phi(x,a,a')$ is a shadow cost that restrains the policy $a'$, but it is not an actual cash flow. By the same token, the cost function $\Theta^e$ restrains the policy under renegotiation, which is interpreted as a dead-weight cost on the future equity value due to covenant restrictions, but it is not an actual cash payment. Therefore, in our implementation of renegotiation of asset sweep and financial covenants, there is no cash transfer between debt and equity at renegotiation, and $\Theta^d = 0$. This means that a covenant affects debt value through the firm’s policies, as opposed to through the cash flow to debt.

To conclude, specific cost function for the asset sweep covenant is defined as in (28), and for the Debt/Ebitda covenant as in (29).
Figure 1: **Constant capital stock - firm value.** We compare the value of the firm against $x$, for three alternative cases, for the model with constant $k = 1$. The first is the case with commitment, in which the shareholders stick to a constant book leverage covenant set at time zero. The optimized initial debt is $b = 2.3$. The second case, while observed at the same initial debt, $b = 2.3$, allows perfect renegotiation of the covenant, and the renegotiation is ex post efficient. The third case has no debt.
Figure 2: **Unprotected debt - debt policy.** For the case with unprotected debt, this figure plots the debt policy (in terms of change relative to the current debt) against $x$, for three possible levels of $b$ corresponding to the 25th, 50th, and 75th percentile of the distribution of face value of debt, setting $k$ at the median level of 3.36.
Figure 3: **Debt sweep covenant - debt policy.** This figure plots the debt policy (in terms of change relative to the current debt) against $x$, for three possible levels of $b$ corresponding to the 25th, 50th, and 75th percentile of the distribution of face value of debt, setting $k$ at the median level of 3.37.
Figure 4: **Debt sweep covenant - credit spread.** This figure plots the credit spread against $x$, for three possible levels of $b$ corresponding to the 25th, 50th, and 75th percentile of the distribution of face value of debt, setting $k$ at the median level of 3.37.
Figure 5: **Asset sweep covenant - debt policy.** This figure plots the debt policy (in terms of change relative to the current debt) against $x$, for three possible levels of $b$ corresponding to the 25th, 50th, and 75th percentile of the distribution of face value of debt, setting $k$ at the median level of 3.37.
Figure 6: Asset sweep covenant - investment policy. This figure plots the investment policy against $x$, for three possible levels of $b$ corresponding to the 25th, 50th, and 75th percentile of the distribution of face value of debt, setting $k$ at the median level of 3.37.
Figure 7: Financial covenant - debt policy. This figure plots the debt policy (in terms of change relative to the current debt) against $x$, for three possible levels of $b$ corresponding to the 25th, 50th, and 75th percentile of the distribution of face value of debt, setting $k$ at the median level of 4.21.
Figure 8: **Financial covenant - credit spread.** This figure plots the credit spread against \( x \), for three possible levels of \( b \) corresponding to the 25th, 50th, and 75th percentile of the distribution of face value of debt, setting \( k \) at the median level of 4.21.
Figure 9: **Unprotected debt - investment policy.** This figure plots the investment policy against $x$, for three possible levels of $k$ corresponding to the 25th, 50th, and 75th percentile of the distribution of asset, setting $b$ at the 75th percentile of 6.1 for the case.
Figure 10: Debt sweep covenant - investment policy. This figure plots the investment policy against $x$, for three possible levels of $k$ corresponding to the 25th, 50th, and 75th percentile of the distribution of asset, setting $b$ at the 75th percentile of 5 for the case.
Figure 11: **Asset sweep covenant - investment policy.** This figure plots the investment policy against $x$, for three possible levels of $k$ corresponding to the 25th, 50th, and 75th percentile of the distribution of asset, setting $b$ at the 75th percentile of 4.6 for the case.
Figure 12: Financial covenant - investment policy. This figure plots the investment policy against $x$, for three possible levels of $k$ corresponding to the 25th, 50th, and 75th percentile of the distribution of asset, setting $b$ at the 75th percentile of 6.9 for the case.
Figure 13: **Financial covenant - Q-ratio.** This figure plots the Q-ratio against $x$, for three possible levels of $k$ corresponding to the 25th, 50th, and 75th percentile of the distribution of asset, setting $b$ at the 75th percentile of 6.9 for the case.
Table 1: **Base case parameter values.** This table provides the parameters used in the optimizations and simulations of the baseline model and its variants.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>time discount factor</td>
<td>$1/1.05$</td>
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<tr>
<td>$\rho$</td>
<td>persistence of productivity shock</td>
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<tr>
<td>$\sigma$</td>
<td>conditional volatility of productivity shock</td>
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<td>$\gamma$</td>
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<td>$\alpha$</td>
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<tr>
<td>$\theta$</td>
<td>debt holders’ bargaining power for debt swift</td>
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<tr>
<td>$\eta_i$</td>
<td>cost of covenant, $i = a, f$</td>
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<tr>
<td>$\psi$</td>
<td>threshold for financial covenant violation</td>
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Table 2: **Constant capital stock.** For the case with \( k = 1 \) at all dates, this table provides unconditional sample moments based on Monte Carlo simulation for the following variables: firm value \( (V^t + d) \), cum-dividend equity value \( (V^t) \), book value of debt \( (b) \), market value of debt \( (p \cdot b) \), leverage \( (b/(b + V^t)) \), Q–ratio \( ((V^t + d)/k) \), cash flow over assets \( (\pi/k) \), credit spread \( (r/p − r) \) in basis points, default frequency (in percentage), and change in debt/assets \( ((b' − b)/k) \). The columns report unconditional mean ('m'), standard deviation ('sd'), and five quantiles. We consider three cases: one in which the equity holders commit to a constant debt covenant, one with perfect renegotiation of the debt, and the third with no debt. The results are based on the parameters in Table 1.

<table>
<thead>
<tr>
<th>Moments</th>
<th>m</th>
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<th>5th</th>
<th>25th</th>
<th>median</th>
<th>75th</th>
<th>95th</th>
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<td>commitment</td>
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<tr>
<td>Firm Value</td>
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<td>Equity</td>
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<td>2.30</td>
<td>2.30</td>
<td>2.30</td>
<td>2.30</td>
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<tr>
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<td>1.29</td>
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</tr>
<tr>
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<td>1.26</td>
<td>1.61</td>
<td>2.03</td>
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<td>3.51</td>
</tr>
<tr>
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</tr>
<tr>
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<td>0.01</td>
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<td>0.71</td>
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<td>4.67</td>
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<td>0.86</td>
<td>0.71</td>
<td>1.38</td>
<td>1.89</td>
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<td>3.60</td>
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<tr>
<td>Chg. Debt/Assets</td>
<td>0.24</td>
<td>0.87</td>
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<td>0.00</td>
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<td>Equity</td>
<td>1.67</td>
<td>0.76</td>
<td>0.49</td>
<td>1.07</td>
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<td>2.21</td>
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<td>2.63</td>
</tr>
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<td>EBITDA/Assets</td>
<td>0.23</td>
<td>0.26</td>
<td>-0.15</td>
<td>0.03</td>
<td>0.20</td>
<td>0.41</td>
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Table 3: **Unprotected debt.** For the case with unprotected debt, this table provides unconditional sample moments based on Monte Carlo simulation for the following variables: firm value ($V^f + d$), cum-dividend equity value ($V^i$), book value of debt ($b$), market value of debt ($p \cdot b$), leverage ($b/(b + V^i)$), Q–ratio ($((V^f + d)/k)$), cash flow over assets ($\pi/k$), credit spread ($r/p - r$) in basis points, default frequency (in percentage), and change in debt/.assets ($((b' - b)/k$). The columns report unconditional mean (‘m’), standard deviation (‘sd’), and five quantiles. The results are based on the parameters in Table 1.

<table>
<thead>
<tr>
<th>Moments</th>
<th>m</th>
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<th>25th</th>
<th>median</th>
<th>75th</th>
<th>95th</th>
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<td>2.89</td>
<td>3.48</td>
<td>5.58</td>
<td>7.17</td>
<td>9.23</td>
<td>12.95</td>
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<td>0.10</td>
<td>2.61</td>
<td>3.36</td>
<td>4.59</td>
<td>6.00</td>
</tr>
<tr>
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<td>1.89</td>
<td>3.21</td>
<td>4.95</td>
<td>6.15</td>
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<td>9.36</td>
</tr>
<tr>
<td>Debt (book)</td>
<td>3.42</td>
<td>5.21</td>
<td>0.00</td>
<td>0.23</td>
<td>0.37</td>
<td>6.09</td>
<td>13.00</td>
</tr>
<tr>
<td>Debt (market)</td>
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<td>0.13</td>
<td>0.20</td>
<td>1.90</td>
<td>5.22</td>
</tr>
<tr>
<td>Leverage</td>
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<td>0.29</td>
<td>0.03</td>
<td>0.05</td>
<td>0.07</td>
<td>0.62</td>
<td>0.74</td>
</tr>
<tr>
<td>Q–Ratio</td>
<td>4.56</td>
<td>8.92</td>
<td>1.77</td>
<td>2.00</td>
<td>2.16</td>
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<td>-3.53</td>
<td>0.15</td>
<td>0.23</td>
<td>0.31</td>
<td>0.41</td>
</tr>
<tr>
<td>Credit Spread (bps)</td>
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<td>123</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>269</td>
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<td>Default freq (ppt)</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<td>Investment/Assets</td>
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<td>9.27</td>
<td>-0.85</td>
<td>0.00</td>
<td>0.12</td>
<td>0.30</td>
<td>27.62</td>
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<tr>
<td>Chg. Debt/Assets</td>
<td>0.99</td>
<td>8.27</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.08</td>
<td>1.99</td>
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Table 4: **Debt sweep covenant.** This table provides unconditional sample moments based on Monte Carlo simulation for the following variables: firm value \((V^t + d)\), cum-dividend equity value \((V^t)\), book value of debt \((b)\), market value of debt \((p \cdot b)\), leverage \((b/(b+V^t))\), Q–ratio \((((V^t + d)/k)\), cash flow over assets \((\pi/k)\), credit spread \((r/p - r)\) in basis points, default frequency (in percentage), and the change in debt/assets \((b'-b)/k\). The results are based on the parameters in Table 1.

<table>
<thead>
<tr>
<th>Moments</th>
<th>m</th>
<th>sd</th>
<th>5th</th>
<th>25th</th>
<th>median</th>
<th>75th</th>
<th>95th</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm Value</td>
<td>8.19</td>
<td>2.73</td>
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<td>6.38</td>
<td>7.90</td>
<td>9.81</td>
<td>13.04</td>
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<td>1.48</td>
<td>0.92</td>
<td>2.62</td>
<td>3.37</td>
<td>4.60</td>
<td>6.00</td>
</tr>
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<td>4.93</td>
<td>1.85</td>
<td>2.44</td>
<td>3.61</td>
<td>4.63</td>
<td>6.00</td>
<td>8.35</td>
</tr>
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<td>3.06</td>
<td>0.95</td>
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<td>3.34</td>
<td>5.01</td>
<td>10.59</td>
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<td>2.69</td>
<td>2.74</td>
<td>3.70</td>
<td>4.98</td>
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<tr>
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<td>0.11</td>
<td>0.32</td>
<td>0.41</td>
<td>0.49</td>
<td>0.58</td>
<td>0.69</td>
</tr>
<tr>
<td>Q–Ratio</td>
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<td>7.39</td>
<td>1.83</td>
<td>2.11</td>
<td>2.30</td>
<td>2.51</td>
<td>11.72</td>
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<tr>
<td>EBITDA/Assets</td>
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<td>1.02</td>
<td>-1.38</td>
<td>0.15</td>
<td>0.24</td>
<td>0.31</td>
<td>0.41</td>
</tr>
<tr>
<td>Credit Spread (bps)</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>91</td>
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<tr>
<td>Default freq (pct)</td>
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<td>-</td>
<td>-</td>
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</tr>
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<td>6.69</td>
<td>-0.52</td>
<td>0.00</td>
<td>0.12</td>
<td>0.26</td>
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<tr>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>5.41</td>
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<tr>
<td>Violation freq (pct)</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
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Table 5: **Debt sweep covenant: sensitivity with respect to \( \theta \).** This table provides unconditional averages based on Monte Carlo simulation for the following variables: firm value (\( V^f + d \)), cum-dividend equity value (\( V^l \)), book value of debt (\( b \)), market value of debt (\( p \cdot b \)), leverage (\( b/(b + V^l) \)), Q-ratio (\( (V^f + d)/k \)), cash flow over assets (\( \pi/k \)), credit spread (\( r/p - r \)) in basis points, default frequency (in percentage), and change in debt/assets (\( (b' - b)/k \)). For different values of the covenant parameter, \( \theta \), we report unconditional averages, except for the change of debt over asset, in which case we report the standard deviation. The results are based on the parameters in Table 1.

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<td>3.57</td>
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<td>5.16</td>
<td>4.93</td>
<td>5.47</td>
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<td>3.13</td>
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<td>0.46</td>
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<td>0.39</td>
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<td>Q-Ratio</td>
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<td>5.39</td>
<td>3.90</td>
<td>2.82</td>
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<td>Ebitda/Asset</td>
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<td>-0.12</td>
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<td>0.01</td>
<td>0.17</td>
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<td>56.81</td>
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<td>5.66</td>
<td>4.17</td>
<td>1.82</td>
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<td>1.54</td>
<td>0.41</td>
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<td>4.78</td>
<td>7.10</td>
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Table 6: **Asset sweep covenant.** This table provides unconditional sample moments based on Monte Carlo simulation for the following variables: firm value \((V^t + d)\), cum-dividend equity value \((V^t)\), book value of debt \((b)\), market value of debt \((p \cdot b)\), leverage \((b/(b + V^t))\), Q–ratio \(((V^t + d)/k)\), cash flow over assets \((\pi/k)\), credit spread \((r/p – r)\) in basis points, default frequency (in percentage), and the change in debt/assets \(((b' – b)/k)\). The columns report unconditional mean (‘m’), standard deviation (‘sd’), and five quantiles. The results are based on the parameters in Table 1.

<table>
<thead>
<tr>
<th>Moments</th>
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<th>5th</th>
<th>25th</th>
<th>median</th>
<th>75th</th>
<th>95th</th>
</tr>
</thead>
<tbody>
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<td>5.57</td>
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<td>2.19</td>
<td>1.93</td>
<td>4.07</td>
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</tr>
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<td>1.71</td>
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<td>0.87</td>
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<td>1.66</td>
<td>1.94</td>
<td>2.14</td>
<td>2.34</td>
<td>5.88</td>
</tr>
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<td>0.15</td>
<td>0.24</td>
<td>0.31</td>
<td>0.44</td>
</tr>
<tr>
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<td>269</td>
<td>315</td>
<td>675</td>
<td>1474</td>
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<td>-</td>
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<td>-</td>
<td>-</td>
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</tr>
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<td>0.00</td>
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</tr>
<tr>
<td>Violation freq (pct)</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Table 7: **Asset sweep covenant: sensitivity with respect to** $\eta_a$. This table provides unconditional averages based on Monte Carlo simulation for the following variables: firm value $(V^t + d)$, cum-dividend equity value $(V^t)$, book value of debt $(b)$, market value of debt $(p \cdot b)$, leverage $(b/(b + V^t))$, Q–ratio ($(V^t + d)/k$), cash flow over assets $(\pi/k)$, credit spread $(r/p - r)$ in basis points, default frequency (in percentage), and change in debt/assets $((b' - b)/k)$. For different values of the covenant parameter, $\eta_a$, we report unconditional averages, except for the change of debt over asset, in which case we report the standard deviation. The results are based on the parameters in Table 1.

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<th>0.35</th>
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<td>7.41</td>
<td>7.33</td>
<td>7.28</td>
</tr>
<tr>
<td>Asset</td>
<td>3.38</td>
<td>3.43</td>
<td>3.42</td>
<td>3.41</td>
</tr>
<tr>
<td>Equity</td>
<td>6.14</td>
<td>6.21</td>
<td>6.02</td>
<td>5.56</td>
</tr>
<tr>
<td>Debt (book)</td>
<td>3.23</td>
<td>2.86</td>
<td>3.06</td>
<td>3.85</td>
</tr>
<tr>
<td>Debt (market)</td>
<td>1.18</td>
<td>1.12</td>
<td>1.22</td>
<td>1.60</td>
</tr>
<tr>
<td>Leverage</td>
<td>0.29</td>
<td>0.26</td>
<td>0.29</td>
<td>0.34</td>
</tr>
<tr>
<td>Q-Ratio</td>
<td>4.19</td>
<td>3.49</td>
<td>3.36</td>
<td>3.43</td>
</tr>
<tr>
<td>Ebitda/Asset</td>
<td>-0.03</td>
<td>0.05</td>
<td>0.07</td>
<td>0.05</td>
</tr>
<tr>
<td>Credit spread (bps)</td>
<td>27.26</td>
<td>17.04</td>
<td>15.58</td>
<td>538.34</td>
</tr>
<tr>
<td>Default freq (pct)</td>
<td>6.44</td>
<td>4.92</td>
<td>4.86</td>
<td>4.81</td>
</tr>
<tr>
<td>Violation freq (pct)</td>
<td>7.31</td>
<td>5.14</td>
<td>4.87</td>
<td>4.94</td>
</tr>
<tr>
<td>Investment/Asset</td>
<td>2.23</td>
<td>1.52</td>
<td>1.39</td>
<td>1.45</td>
</tr>
<tr>
<td>Ch. Debt/Asset (sd)</td>
<td>4.72</td>
<td>2.77</td>
<td>3.63</td>
<td>3.79</td>
</tr>
</tbody>
</table>
Table 8: Financial covenant. This table provides unconditional sample moments based on Monte Carlo simulation for the following variables: firm value \((V^t + d)\), cum-dividend equity value \((V^t)\), book value of debt \((b)\), market value of debt \((p \cdot b)\), leverage \((b/(b + V^t))\), Q–ratio \(((V^t + d)/k)\), cash flow over assets \((\pi/k)\), credit spread \((r/p - r)\) in basis points, default frequency (in percentage), and the change in debt/assets \(((b' - b)/k)\). The columns report unconditional mean (‘m’), standard deviation (‘sd’), and five quantiles. The results are based on the parameters in Table 1.

<table>
<thead>
<tr>
<th>Moments</th>
<th>m</th>
<th>sd</th>
<th>5th</th>
<th>25th</th>
<th>median</th>
<th>75th</th>
<th>95th</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm Value</td>
<td>9.24</td>
<td>2.50</td>
<td>5.49</td>
<td>7.43</td>
<td>9.19</td>
<td>10.82</td>
<td>13.40</td>
</tr>
<tr>
<td>Asset</td>
<td>4.13</td>
<td>1.24</td>
<td>2.59</td>
<td>3.09</td>
<td>4.21</td>
<td>4.64</td>
<td>6.00</td>
</tr>
<tr>
<td>Equity</td>
<td>4.58</td>
<td>1.27</td>
<td>2.72</td>
<td>3.80</td>
<td>4.53</td>
<td>5.12</td>
<td>6.56</td>
</tr>
<tr>
<td>Debt (book)</td>
<td>4.55</td>
<td>2.28</td>
<td>0.72</td>
<td>2.44</td>
<td>4.91</td>
<td>6.88</td>
<td>7.24</td>
</tr>
<tr>
<td>Debt (market)</td>
<td>4.41</td>
<td>2.14</td>
<td>0.72</td>
<td>2.43</td>
<td>4.83</td>
<td>6.49</td>
<td>6.91</td>
</tr>
<tr>
<td>Leverage</td>
<td>0.48</td>
<td>0.14</td>
<td>0.20</td>
<td>0.40</td>
<td>0.51</td>
<td>0.60</td>
<td>0.67</td>
</tr>
<tr>
<td>Q–Ratio</td>
<td>2.29</td>
<td>0.48</td>
<td>1.88</td>
<td>2.09</td>
<td>2.24</td>
<td>2.44</td>
<td>2.72</td>
</tr>
<tr>
<td>EBITDA/Assets</td>
<td>0.23</td>
<td>0.11</td>
<td>0.06</td>
<td>0.16</td>
<td>0.23</td>
<td>0.30</td>
<td>0.40</td>
</tr>
<tr>
<td>Credit Spread (bps)</td>
<td>12</td>
<td>42</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>13</td>
<td>28</td>
</tr>
<tr>
<td>Default freq (pct)</td>
<td>0.27</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Investment/Assets</td>
<td>0.16</td>
<td>0.40</td>
<td>-0.18</td>
<td>0.00</td>
<td>0.12</td>
<td>0.26</td>
<td>0.50</td>
</tr>
<tr>
<td>Chg. Debt/Assets</td>
<td>0.06</td>
<td>0.56</td>
<td>-0.60</td>
<td>-0.14</td>
<td>0.00</td>
<td>0.21</td>
<td>0.80</td>
</tr>
<tr>
<td>Violation freq (pct)</td>
<td>0.27</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Table 9: **Financial covenant: sensitivity with respect to** $\eta_f$. This table provides unconditional averages based on Monte Carlo simulation for the following variables: firm value ($V^l + d$), cum-dividend equity value ($V^l$), book value of debt ($b$), market value of debt ($p \cdot b$), leverage ($b/(b + V^l)$), Q–ratio (($V^l + d)/k$), cash flow over assets ($\pi/k$), credit spread ($r/p - r$) in basis points, default frequency (in percentage), and change in debt/assets ($((b' - b)/k$). For different values of the covenant parameter, $\eta_f$, we report unconditional averages, except for the change of debt over asset, in which case we report the standard deviation. The results are based on the parameters in Table 1.

<table>
<thead>
<tr>
<th>$\eta_f$</th>
<th>0.05</th>
<th>0.15</th>
<th>0.25</th>
<th>0.35</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm value</td>
<td>7.65</td>
<td>8.39</td>
<td>8.94</td>
<td>9.24</td>
</tr>
<tr>
<td>Asset</td>
<td>3.70</td>
<td>3.87</td>
<td>4.03</td>
<td>4.13</td>
</tr>
<tr>
<td>Equity</td>
<td>6.58</td>
<td>5.87</td>
<td>4.98</td>
<td>4.58</td>
</tr>
<tr>
<td>Debt (book)</td>
<td>1.76</td>
<td>2.52</td>
<td>3.89</td>
<td>4.55</td>
</tr>
<tr>
<td>Debt (market)</td>
<td>1.01</td>
<td>2.39</td>
<td>3.74</td>
<td>4.41</td>
</tr>
<tr>
<td>Leverage</td>
<td>0.21</td>
<td>0.31</td>
<td>0.43</td>
<td>0.48</td>
</tr>
<tr>
<td>Q-Ratio</td>
<td>2.52</td>
<td>2.20</td>
<td>2.27</td>
<td>2.29</td>
</tr>
<tr>
<td>Ebitda/Asset</td>
<td>0.19</td>
<td>0.24</td>
<td>0.23</td>
<td>0.23</td>
</tr>
<tr>
<td>Credit spread (bps)</td>
<td>-6.27</td>
<td>26.47</td>
<td>-0.24</td>
<td>12.48</td>
</tr>
<tr>
<td>Default freq (pct)</td>
<td>3.10</td>
<td>0.20</td>
<td>0.38</td>
<td>0.27</td>
</tr>
<tr>
<td>Violation freq (pct)</td>
<td>9.65</td>
<td>0.20</td>
<td>0.38</td>
<td>0.27</td>
</tr>
<tr>
<td>Investment/Asset</td>
<td>0.60</td>
<td>0.15</td>
<td>0.16</td>
<td>0.16</td>
</tr>
<tr>
<td>Ch. Debt/Asset (sd)</td>
<td>0.48</td>
<td>0.25</td>
<td>0.51</td>
<td>0.56</td>
</tr>
</tbody>
</table>