Raising external financing with hash-linked timestamps/Blockchain

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Abstract

Hash-linked timestamping is the key feature behind blockchain technology. It makes it possible to design financing contracts that are based on reliable and up-to-date records of transactions. For this reason, it is considered to enhance trust. This paper develops a theoretical model that uses dynamic contract theory to derive optimal financing contracts in a blockchain environment. I show that a dynamically adjusting profit sharing rule is optimal and I highlight properties that determine the splitting rule. In contrast to the view that blockchain enhances traditional contracts like debt and equity by bringing efficiency gains, I emphasise that blockchain allows borrowers to learn from data and take effort decisions more frequently, which make debt and equity contracts costlier.

Keywords: blockchain, smart contracts, FinTech, contract design, Bayesian learning, profit-sharing, equity, debt, dynamic moral hazard.

JEL codes: D82, D86, G23, G32, G35
1 Introduction

An important criterion for assessing the relevance of new technologies related to Finance (sometimes called "FinTech") is whether these technologies make access to external financing easier for firms and individuals who lack internal resources and/or find it difficult to pledge the future cash flows in return for the funds needed for an investment. Blockchain technology is seen as one of these new promising technologies. Quoting for example Casey and Vigna in MIT Technology Review May/June 2018, this technology "is all about creating one priceless asset: trust". The "lack of trust" is well known in Finance and Economics to be the prime reason why external financing can only be accessed by those with enough own assets (see e.g., Tirole 2006), why scarce and arguably expensive venture capital funding has been the best source of innovation financing (see e.g., Lerner, Leamon, Hardymon 2012), and why there is a fundamental need for money and liquid assets to start with (see e.g., Kiyotaki and Moore 2002).

The key distinguishing feature of blockchain is hash-linked timestamping, the technical details of which will be further discussed in Section 2. In essence, blockchain is a secure ledger/database which maintains a reliable, shareable and time-stamped record of transactions, ownership and rights, enabling contracting parties to have a common and verifiable history of events. Blockchain further helps enforcement as it can verifiably record and facilitate the execution of actions that are predetermined by contracting agents (via a pre-agreed, recorded and shared computer code) and it is an enabler for the development of “smart contracts”, i.e. contracts which can automatically self-adjust and execute pre-determined actions based on incoming data. All these features explain why blockchain technology could indeed enhance trust in the context of external financing by overcoming some traditional financing frictions that exist due to lack of cash flow pledgeability and commitment. Does this technology simply lead to cheaper access to external financing regardless

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1 There are real world examples of this: notable examples of platforms that incorporate smart contract functionality are Ethereum and the Hyperledger project. Ethereum is based on a permissionless and decentralized verification mechanism in the spirit of the blockchain technology behind bitcoin. Hyperledger is an open-source Linux based community that advocates permissioned (members-only) verification mechanisms and is supported by technology firms such as IBM and Cisco, as well as financial services providers such as JP Morgan, SWIFT and ABN Amro among others. While there are trade-offs in terms of costs and cryptographic security, both approaches enable similar core benefits

2 Number of these benefits have also been pointed out by academics and practitioners (see e.g., Catalini and Gans 2017; Yermack 2015, Harvey 2016, PwC 2016, Deloitte 2016, Dhar and Bose 2016).
of the type of financing contract, e.g., debt, equity, convertibles, or is it something different? Can it lead to outcomes closer to those that emerge in frictionless setting famously emphasized by Modigliani and Miller (1985, 1963)?

While blockchain can enable credible pledging of future cash flows and commitment to contractual terms, it cannot eliminate at least one crucial element of contracting frictions - the necessity of some human effort in generating cash flows and associated moral hazard. In my model the blockchain technology does not exist in isolation from data analysis capabilities that enable firms to learn about their future prospects from incoming cash flow relevant data. This in turn affects effort incentives of the entrepreneur and is important, because more accurate information does not necessarily make contracting easier (see e.g., Hirshleifer 1971 in the context of insurance, and Kaplan 2006 and Dang, Gorton, Holmström and Ordoñez 2017 in the context of banking).

I develop a model that incorporates dynamic moral hazard an learning in an environment where blockchain enables an entrepreneur to credibly pledge the future cash-flows and to pre-commit to any, possibly "smart" financing contract. The goal is to understand whether and under which conditions, the defining features of blockchain technology, such as time-stamps and related smart contract functionality, are beneficial for access to financing. I further explore how increasing frequency of learning and decision making affects access to the financing under different financing contracts, including traditional debt and equity contracts.

When transactions are recorded in a time-stamped chronological order, it is technologically feasible to write novel types of financing contracts that depend on not just whether, but also when, a transaction happened.\(^3\) That is, payoffs could be different depending on the sequence at which cash flows arrive, even if the total cash flows generated are the same. Furthermore, if the execution of a contract is done by a pre-agreed computer code, it is no more costly to execute a contract that is a complex function of cash flows than it is to execute a simple debt or equity contract. However, the capability of the contract to be "smart", i.e., to have contracting terms that dynamically adjust

\(^3\)This timing factor differentiates blockchain records from audited accounting records, which aim to ensure that the aggregate total in- and outflows are correctly recorded over a relatively long period of time. This makes contracting on project specific cash flows and their arrival times not feasible in traditional environments. This is further complicated when firms strategically time their earnings reporting and information disclosure (see e.g., Bartov, 1993 and Healy and Palepu, 2000).
based on incoming data, is valuable only if these contracts make access to financing easier compared to traditional assets.

I consider a setting where an entrepreneur (the borrower) can develop a project that will enable her/him to sell new products or services (called "widgets" in this paper) to potentially interested customers. The project requires a fixed investment and the borrower may need to or prefer to use external financing to cover this cost. I assume that raising financing is product (project) specific rather than firm specific, i.e., the entrepreneur pledges cash flows related to developing a specific product (e.g., as in the case of many forms of crowdfunding). This guarantees that incoming cash flows, effectively the sales records, is something feasible to record on blockchain and to contract on. Realistically, the entrepreneur cannot substantially influence the preferences of his target consumers and takes the distribution of potential sales opportunities as given: these are determined by the fundamental characteristics of the target market and the widget.

One of the main contributions of my paper is to explore how the core properties of potential sales distribution determine the desirable financing contracts in the described environment. For this reason, I do not impose parametric assumptions or the most usual restrictions on the joint distribution of potential sales opportunities, and I further allow the cost of effort to vary over time. I show that it is crucial to distinguish an environment where potential sales are stochastically affiliated (log-supermodular)\(^4\) from an environment where they are not. Stochastic affiliation implies that successful past sales lead the entrepreneur to Bayesian update his beliefs about future sales upward, which I call a "success raises prospects" environment. For instance, it is a realistic assumption in the case of a borrower who develops an innovative technological gadget and is unsure about whether the market will like the product or not. Sales success is in this context a proof of concept, and higher early sales can even generate further positive effects via network effects. In contrast, a "success lowers prospects" environment is one where sales are log-submodular, and past sales success leads the entrepreneur to Bayesian update his beliefs about future sales downward. It is a realistic assumption in an environment where the market or the scope for the opportunity may become

\(^4\)Stochastic affiliation implies monotone likelihood ratio property (see e.g., Milgrom, 1981 and Milgrom and Weber, 1982), which in turn often plays a central role in principal agent models.
exhausted. For instance, the firm may organize an event or concert for a well defined target group of potential fans that are difficult to find, offer a design item that target consumers value more if it is considered to be rare and owned by fewer other consumers, enter in a market where incumbents accommodate entry only as long as the entrant’s market share remains small, have an innovative business idea that is likely to be copied if proven successful. While the economics and finance literature most often focuses on environments with independent cash flow processes, and somewhat less frequently on affiliated ones, a "success lowers prospects" environment is equally plausible.

I show that in all these environment the optimal contract can be represented in a relatively simple form where each successful sale is split between the lender and the borrower according to a dynamically adjusting splitting rule on incoming cash flows/sales revenues that depend on the history sales up to that point. In fact, the splitting rule is determined by variables that are predictable and can be easily and dynamically calculated by an econometric algorithm embedded in the code that implements a "smart contract": such as the expected value of next sales conditional on the history of past sales up to that point, and such as the expected value of sales at some point in the future if no successful sale happens in between (i.e., a "worst case scenario analysis").

The optimal splitting rule adjust qualitatively differently in "success raises prospects" and "success lowers prospects" environments. I show that in the "success lowers prospects" environment it is always beneficial to use "smart" contracts and the optimal contract in this environment achieves the first best because all positive NPV projects can be financed without the need of the entrepreneur’s own funds. This is because a contract based on timestamped records can perfectly offset any distortions due to learning. The optimal contract in "success lowers prospects" environment resembles a merit based scheme whereby past success should give the entrepreneur a right to a higher share of next sale, and vice versa, to offset the fact that past success indicates greater difficulties to achieve further success.

In contrast, "success raises prospects" could also be called a "failure lowers prospects" environment, which necessitates the optimal contract to maintain the entrepreneur’s incentives after unlucky outcomes, by offering her/him a higher share of future cash flows following unlucky outcomes. Knowing this, the entrepreneur has a manipulation possibility whereby she/he strategically
witholds the effort anticipating better contractual terms, similar to what is called "information rents" in some contract theory papers discussed below. As a result, the entrepreneur can raise external financing to cover the highest positive NPV investment cost only if she/he can provide some internal funds. However, these funds are noticeably smaller than those needed under debt or equity. I show that the contracts that maximize availability of external financing would again benefit from being "smart" unless two restrictive conditions hold: effort cost is constant over time and the potential sales distribution satisfies the property of "exchangeability", meaning that the timing of expected future sales does not matter regardless of any past sales history (something that would be violated whenever there is expected cyclicity in sales prospects, for example).

This paper shows that under blockchain technology the economic outcomes are indeed closer to those in the frictionless market of Modigliani and Miller (1958, 1963) in the sense that the expected profit of a borrower who obtains external financing is as high as it would be in a frictionless market, and using own funds is not cheaper than external financing for most borrowers. Furthermore, borrowers can finance all profitable investment projects with minimal own funds, and even zero own funds in some informational environments. However, in contrast to Modigliani and Miller (1958, 1963) all financing contracts are not equivalent. I show that the reason why debt is suboptimal in this environment is that debt gives poor incentives to continue making efforts following unlucky outcomes (similar to debt overhang): for example, if a firm owes $1000, and has failed to sell in past despite best effort, it will rationally stop trying to sell if the best it can do is to sell goods worth $1000. Anticipating no effort in some states will make the investors more reluctant to lend. I also show that equity is suboptimal whenever learning from past sales reveals information about future sales prospects. With simple equity, the borrower would need to have high enough own share of revenues to maintain effort incentives in the "worst case scenario", while he would prefer to accept a contract that gives him a lower share in the "best case scenario" in return for the benefit of accessing external financing more easily ex-ante. The only special case where simple equity is optimal is when there is constant effort and no learning, i.e., when potential sales opportunities are independently and identically distributed (i.i.d.) over time.

While this paper shows that time-stamped records and smart contracts bring value in most
environments, it could be tempting to argue that pledgeability of cash flows and better enforcement of contracts are already good enough to fulfill the FinTech goal to make access to external financing easier also via traditional contracts. I emphasize that this argument would miss one important force that has to do with the increasing frequency of learning and decision making, possibly enhanced by the availability of blockchain records. I show that the aforementioned frictions associated with debt and equity become more severe when the frequency of learning and decision making increases. For this reason I argue that the "smart contracts" facilitated by blockchain technology should not just be seen as a way to make the execution of financing contracts cheaper, but could be seen as a counterforce to mitigate the negative forces generated by more frequent learning.

The early literature on credit rationing and financial contracting (see e.g., Tirole, 2016 and Allen and Winton, 1995, for reviews) has often found debt or debt-like contracts to be the "second best" optimal contracts in settings with realistic contracting frictions in traditional environments. This reinforces the idea that new technologies considered in this paper have the potential to fundamentally change the dominant financing frictions and consequently lead to different ways to raise external funds. One of the prominent theoretical arguments for the optimality of debt stems from the costly state verification literature (see e.g., Townsend, 1979, Diamond, 1984, Gale and Hellwig, 1985, Mookherjee and Png, 1989.), which highlights that debt contracts enable efficiency of monitoring and verification costs. As argued above, the key promise of blockchain technology is to eliminate (or greatly reduce) these costs by guaranteeing easily verifiable and commonly shared records of transactions. Debt also features as optimal in static models of moral hazard where the aggregate cash flows are assumed to be verifiable such as in Innes (1990), and in settings building on this. The crucial difference with my paper is that Innes (1990) does not consider the effect of frequent learning and effort decisions and assumes that the borrower makes his effort choice once and for all, just after signing the contract. As argued above, entrepreneurs are likely to learn from incoming data and can adjust their effort frequently after signing the contracts, which is the prime reason that makes debt contracts increasingly costly in my setting. Indeed, Chiesa (1992) extends Innes’s setting to allows the borrower to observe information before making an one-off effort decision and shows that debt combined with warrants does better than simple debt for a similar reason as
in my paper: debt contracts give bad incentives to make an effort after bad news. Other important arguments for debt emphasize robustness of these contracts to renegotiations (see e.g., Hermalin and Katz, 1991, and Dewatripont et. al., 2003) and enforcement (see e.g., Hart and Moore, 1998) - both of these frictions could arguably be eliminated by smart contracts, or at least be of second order importance for an individual small borrower that uses blockchain-based smart contracts (see further in Section 2).

This paper further relates to the literature on dynamic moral hazard, and the associated literature on principal-agent problems. My paper emphasizes that to fully benefit from blockchain technology, the borrowers and lenders would need to write contracts that depend not just on sales that happened, but also on when they happened. In this context it is important to clarify the connection of my paper with Holmstrøm and Milgrom (1987), often cited for justifying the optimality of linear contracts. However, the more general insight in their paper is that in many principal agent settings, the optimal dynamic scheme should only depend on the number of times a particular outcome occurs, and not on the order in which these outcomes occur, i.e., a sales history \( \{0, 1, 1\} \) should give the same payoff as a sales history \( \{1, 1, 0\} \). If that would be the case in my setting, time-stamping would not matter. The crucial difference is that the potential sales distribution is realistically not under the full control of the entrepreneur: she/he cannot dictate the preferences of her/his target consumers regardless of her/his effort, and can only decide whether to explore the opportunity or not. This is the main reason why distributional properties of sales process matter.

Furthermore, the results of Holmstrøm and Milgrom (1987) are derived in a setting where incentive compatibility constraints bind. When "success lowers prospects", I show that the borrower’s effort is easy to incentivise, because incentive compatibility constraints do not bind, and the binding constraint is the participation constraint. In such a case, it benefits the borrower and the lender if the contract fully adjusts to new information and sequence always matters.

More recent advances in dynamic moral hazard and financing contracts also highlight the benefits of more nuanced and history dependent financing contracts, but tend to also assume greater control of agents over the cash flow process compared to the setting considered in this paper and more restrictive cash flow processes. DeMarzo and Fishman (2007) and Biais et. al. (2007) find
that financing contracts that combine equity, debt, and credit line or well managed cash reserves are optimal in a dynamic setting where cash flow process is not verifiable and i.i.d. across time. DeMarzo and Sannikov (2016) consider a setting where cash flow process is governed by a Brownian motion, which is a parametric example of a "success raises prospects" environment. As in my paper there are "information rents" and equity contract is optimal under some circumstances. Similar information rents also feature in He et. al (2010) and Prat and Jovanovic (2014) that also use Brownian motion. These settings generally do not explore the "success lowers prospects" environment and the question whether contracting on timestamped records and timing of cash flows matters. It seems plausible that the combination of standard assets that constitutes the optimal contract in these papers could be also expressed as a dynamically adjusting profit sharing contract. However, these assets cannot replicate the self-adjusting profit sharing contracts in all informational environments considered in this paper.

Interestingly, the finding that the optimal contract could take a form of time-varying share contract features in Bergemann and Hege (1998). Their paper considers dynamic moral hazard in a different context, where the moral hazard is not associated with an effort to generate sales, but with the possibility to divert investor’s funds instead of investing these in the project development before the known sales outcome will be generated. These result are complementary to those in my paper. An important difference is that despite the fact that a product development process resembles a "success raises prospects" environment, a pre-sales product development requires continued investments which make the optimal contract in their paper to be more similar to the one applicable to "success lowers prospects" environment.

2 Technical background of Blockchain technology and related FinTech initiatives

Blockchain technology relies on a hash function. Hash functions map any data, be it numerical data, string of words, or a code to a value of fixed size. By construction, it is easy to verify a hash given original data and very difficult vice versa. The common feature behind technologies, nowadays referred as the Blockchain, is a system of recoding data in hash-linked and time-stamped units: the more time passes the more difficult it becomes to alter the data and this is what guarantees the integrity of the ledger. It is widely considered that the original idea stems from research by Haber and Scarpetta (1991, 1997). There have been noticeable further advances of real world relevance. For example, papers by Buldas, Laud, Lipmaa, and Villlemson (1998), Buldas and Saarepera (2004) provide a formal proof of the idea of time-stamping, and Buldas and Saarepera are further associated with a number of patents assigned to Guardtime\(^5\) - company providing blockchain based digital security services for DARPA, and Ericsson, and Estonian government, among others. It is well known that a pseudonymous individual or group named Satoishi Nakamoto (2008) built on this idea to propose an outside-the-system money, called Bitcoin. The idea of timestamped hashlinking features prominently in the context of Hyperledger - a consortium of well-known technology platform companies, financial services and Business Software companies\(^6\). The core feature of blockchain being a data structure that provides timestamped verifiable records of transactions, as emphasized in my introduction, features prominently in all these initiatives.

Blockchain technology is sometimes mistaken to be the same thing as Bitcoin, or for the idea of "proof-of-work" associated with Bitcoin and Ethereum, which enables not just shareable common records, but also a fully decentralized and permissioness verification system based on individuals’

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\(^6\)Including technology firms (Cisco, Fujitsu, Hitachi, IBM, Intel, NEC, NTT DATA, Red Hat, VMware), financial services firms (ABN AMRO, ANZ Bank, BNY Mellon, CLS Group, CME Group, the Depository Trust & Clearing Corporation (DTCC), Deutsche Börse Group, J.P. Morgan, State Street, SWIFT, Wells Fargo), Business Software companies like SAP, Systems integrators and others such as: (Accenture, Calastone, Wipro, Credits, Guardtime, IntellectEU, Nxt Foundation, Symbiont).
willingness and incentives to offer their computer power for the cause. There are also alternative proposals such as proof-of-stake, proof-of-importance, and proof-of-elapsed time. As shown above, there also exist prominent blockchains (or hash-linked and timestamped ledgers) that rely on permissioned verification mechanisms. Differences between these verification methods are interesting (e.g., permissioned systems could be better in maintaining privacy), but ultimately the particular verification mechanism does not matter for the questions I explore in this paper. It may be worthwhile to further point out that disruptions that affect the functioning of a particular blockchain verification system would not delete the past records, which are likely to be stored in multiple locations in the network and still constitute a common history of past records.

The term "smart contracts" was highlighted by Szabo (1994, 1997) as a tool that enables the automatic execution of contracts, lowers enforcement costs and minimize the need for intermediaries. Werback and Cornell (2017) emphasise that from law perspective the distinct feature of "smart contracts" is that the burden of proof would be reversed - a pre-agreed contract would first be automatically executed and disputes about the fairness of the contracts may happen later. In my setting, this effectively safeguards the lender by eliminating enforcement costs and makes it easier/cheaper to contract for the borrower (see also Rius (2018) for the analyses of Ethereum based "smart contracts" from a computer science perspective).

The mechanisms developed in this paper have practical relevance in the context of the recently emerged forms of fundraising that either rely on blockchain technology directly or use platforms that incorporate blockchain technology to facilitate contracting and borrower-lender interactions. A noticeable example is a subset of initial coin offerings (ICOs), which involves issuing tokens that give the participants rights on firm’s revenues, profits or shares8 Another example is a set of

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7 For example, Blockchain-based records and smart contracts are often referred to as immutable, because the records are verifiable and a code implementing a smart contract can be recorded on the blockchain as well. Immutability is not absolute, as a transaction or a rule can be "un-done" by a new rule. Yet, unilateral "un-doing" of a transaction is likely to be very costly and rare from a perspective of small participants such as financiers and entrepreneurs in my setting, unless there is a major issue of non-legitimate behaviour. There was a well known case of a major case of users exploiting a vulnerability of DAO that lead to a modification of Ethereum blockchain, effectively restoring the pre-attack state.

8 According to www.icodata.io, the overall funds raised via ICOs stood at $6 billion in 2017 (in comparison to $90 million in 2016), exceeding the early stage venture capital investment volume (see e.g., See Oscar Williams-Grut at Business Insider, January 31, 2018), and is already around $3.9 billion in January-April 2018. According to a recent paper by Adhami, Giudici and Martinazzi (forthcoming) tokens offering profit rights are already quite common and constitute 26.1% of ICOs surveyed, and they find that offering profit (or service) tokens, increases the probability
emerging FinTech platforms that act as new type of intermediaries that facilitate specific financing contracts, and typically use blockchain technology as a part of their business model.\textsuperscript{9} These new forms of fundraising are growing fast and are often based on equity- or profit sharing contracts. While a full implementation of blockchain based financing contracts may require wider adoption of this technology, hybrid models that benefit from some aspects of this technology, while relying on centralized parties for up-to-date data, or contact enforcement in other aspects, can also deliver at least some of the benefits my model highlights.

3 The model
3.1 The general setting

There is a blockchain-based ledger\textsuperscript{10}, which guarantees that there is a reliable, verifiable and timestamped record of each transaction, such as the sale of a widget by an entrepreneur (or an individual). Financing contracts based on these records can be any "complex" functions of cash flows, which can depend on each widget sale and on when it happens. The contracts are "smart" as they can adjust and execute automatically. Contracting parties pre-commit to pre-agreed contractual terms, which is facilitated by blockchain, as the code implementing the terms of financing contracts can be recorded on the blockchain as well, and it can be made sufficiently costly and difficult to change the contractual terms ex-post.

A would-be borrower (an entrepreneur or an individual) has a project idea that requires fixed investment $I$ at date 0. The borrower is risk-neutral and has own funds $A$, which he may invest in the project or not. Assume that lenders operate in a competitive market and there is a representative risk-neutral lender. If the project is pursued, the borrower can at each date $t = 1, \ldots, T$, make

\textsuperscript{9}To name a few that have attracted attention: Funderbeam (https://www.funderbeam.com) enables equity investments in the spirit of venture capital investments combined with blockchain-based secondary market trading, Corl (https://corl.io) proposes revenue sharing contracts where some of the contractual terms, such as the term, adjust depending on realized revenues. There are also platforms that facilitate debt contracts, such as SALT (https://www.saltlending.com), which facilitates blockchain based lending and enables the use of crypto-currencies and -tokens as collateral.

\textsuperscript{10}The immediate applications involve products that need to be, or benefit from being recorded on blockchain, such as goods sold for crypto-currencies, or benefit from being recorded on blockchain due to certification. It has been further imagined that blockchain could allow the creation of a World Wide Ledger that would record all transactions and ownership data (see e.g., Harvey 2016, Shackelford and Myers 2017, Swan 2015 and Tapscott and Tapscott 2016).
a potential sale \( s_t = \{0, 1\} \). The joint distribution of potential sales, \( p(s_1, \ldots, s_T) \), is known at date 0 to both parties, and the only restriction on this joint distribution imposed throughout the analysis is that all possible sequences of potential sales occur with strictly positive probability, i.e., \( p(s_1, \ldots, s_T) > 0 \) for any sequence \( s_1, \ldots, s_T \). While the distribution of the potential sales is determined by the nature of the borrower’s project and the environment in which he operates, the borrower needs to make efforts. The effort decision at \( t = 0, \ldots, T - 1 \) is denoted with an indicator function \( \mathbf{1}_t = \{1, 0\} \), where \( \mathbf{1}_t = 1 \) if the borrower makes an effort, \( \mathbf{1}_t = 0 \) otherwise, and the effort cost is \( e_t \geq 0 \) in monetary equivalent units. If the borrower chooses \( \mathbf{1}_t = 0 \), then the sales at \( t + 1 \) are zero with probability 1. The realized sales are denoted with \( \hat{s}_t = \mathbf{1}_{t-1} s_t \). Because effort is not contractible, the contract between the borrower and the lender is a function of realized sales, \( \hat{s}_t \), rather than the potential sales, \( s_t \). The discount rate is normalized to 1, and I assume that \( e_t \) is small enough to guarantee that it is optimal for the borrower to make an effort at each date \( t \) if he would self-finance the project, i.e., it holds that \( e_t \leq \mathbb{E}[s_{t+1} | \mathbf{1}_0 s_1, \ldots, \mathbf{1}_{t-1} s_t, \mathbf{1}_0, \ldots, \mathbf{1}_{t-1}] \) for any \( t \).

Financing contract is signed at date 0, and I assume that it is the borrower who proposes the contract to the lenders. The contract specifies the borrower’s reward, a function \( w(\hat{s}_1, \ldots, \hat{s}_T) \) which depends on realized sales \( \hat{s}_t \) for \( t = 1, \ldots, T \), and the lender’s reward which is the difference between total sales and the borrower’s reward. The borrower’s preferences depend on his expected returns and his own investment in the project. I assume lexicographic preferences, such that \( w(\hat{s}_1, \ldots, \hat{s}_T) \succ (\succeq) w(\hat{s}_1', \ldots, \hat{s}_T') \) if either "\( \mathbb{E}[U] > \mathbb{E}[U'] \)" or "\( \mathbb{E}[U] = \mathbb{E}[U'] \) and \( A_0 \leq (<) A_0' \)", where \( \mathbb{E}[U] \) is the expected net present value of the project, \( A_0 = [0, A] \) is the borrower’s own investment in the project under the contract \( w(\hat{s}_1, \ldots, \hat{s}_T) \), and \( \mathbb{E}[U'] \) and \( A_0' \) are the same variables under an alternative contract \( w(\hat{s}_1', \ldots, \hat{s}_T') \). The date \( T \) value of the project is

\[
U = w(\hat{s}_1, \ldots, \hat{s}_T) - A_0 - \sum_{t=0}^{T-1} \mathbf{1}_t e_t.
\]

A feasible contract must satisfy the lender’s break even constraint

\[
I - A_0 = \mathbb{E} \left[ \sum_{t=1}^{T} \hat{s}_t - w(\hat{s}_1, \ldots, \hat{s}_T) \right]
\]

\[11\]Discrete time captures the discrete nature of blockchain records. It takes time to confirm a transaction (10 min on average in the case of Bitcoin and 2.5 min in the case of Litecoin). After considering contracts on each sale, it is straightforward to apply the setting also for the case where effort choices and sales records occur in bigger bulks, e.g., after \( n \) periods, where the firm could sell \( 0, 1, \ldots, n \) units. See further in Section 4.
and the borrower makes optimal effort decisions at each date, i.e., for every $t = 0, \ldots, T - 1$, it holds that $1_t = 1$ if, and only if

$$E[U|\hat{s}_1, \ldots, \hat{s}_t, 1_0, \ldots, 1_{t-1}, 1_t = 1] \geq E[U|\hat{s}_1, \ldots, \hat{s}_t, 1_0, \ldots, 1_{t-1}, 1_t = 0].$$

I also assume that there is limited liability, i.e., $w(\hat{s}_1, \ldots, \hat{s}_T) \geq 0$.

It is worthwhile to discuss some of the features of this setting. Lexicographic preferences cover the standard case where the borrower just maximizes his expected returns from his project. However, as the blockchain is assumed to eliminate a number of financing frictions, we may need more structure to compare contracts that give the same expected profit. It is then natural to consider that the borrower would prefer to risk losing as little of his own funds as possible, while still trying out his project idea. Optimal contract derived under lexicographic preferences is one that maximizes the availability of outside financing to borrowers with little own assets, and thus minimizes credit rationing.

The incentive compatibility constraint (3) highlights that when making an effort decision at date $t$, the borrower has additional information compared to date 0 - he knows whether he made an effort in past, and what the sales were at dates when he tried to actively sell. Because the effort is not verifiable, the contract cannot distinguish between zero sales due to bad luck or due to lack of effort, and needs to incentivise the borrower to choose to make optimal effort decisions. As $U$ in (1) depends on all efforts between date 0 and $T$, it is also possible that when making the the effort decision at $t$, the borrower expects this decision to affects his future effort decisions.

While the setting impose minimal restrictions on the joint distribution of potential sales, $p(s_1, \ldots, s_T)$, it is useful to recall the definitions of some useful properties of random variables to structure the analysis and interpret the findings later on.

**Definition 1** Given $n$ random variables jointly distributed in $\mathbb{R}^n$ according to distribution function $f$, let $x = (x_1, \ldots, x_n)$ and $y = (y_1, \ldots, y_n)$ be two points in $\mathbb{R}^n$. The random variables are stochastically affiliated if, and only if for every $x$ and $y$

$$f(x \wedge y)f(x \vee y) \geq f(x)f(y),$$

where
\[ x \land y = (\min (x_1, y_1), \ldots, \min (x_n, y_n)) \]
\[ x \lor y = (\max (x_1, y_1), \ldots, \max (x_n, y_n)) \],

i.e., the joint distribution is log-supermodular. If random variables have this property I will refer to the environment as a "success raises prospects" environment. If the inequality in (4) is reversed, then the joint distribution is log-submodular and I will refer to the environment as a "success lowers prospects" environment.

Stochastic affiliation is a strong form of positive correlation (which implies the monotone likelihood ratio property). It can be called a "success raises prospects" environment, because a successful sale at date \( t \) makes the borrower to revise upwards his beliefs about the probability of making successful sales in the subsequent periods. For example, stochastic affiliation holds in the case where the borrower does not know for sure what share of its target market is interested in buying its product and updates his beliefs about this share as new information from sales arrives. There is a convenient parametric setting that would capture this: a sales process that is conditionally independent such that for any \( t \), it holds that \( \Pr (s_t = 1|\theta) = \theta \), where \( \theta \) is the unknown probability that an individual target consumer is interested in the firm's product, while \( \theta \) is drawn from a beta distribution. This example will be used in Sections 4 and 5, which compares different contracts.

The opposite case of stochastic affiliation, the log-submodular potential sales process captures a market where the potential sales opportunities may become exhausted. For example, the firm may have very precise idea how many potential customers its product has, but does not know how quickly it will find them. In fact, random sampling without replacement is an example of log-submodular potential sales process, and will also be used as an example in Sections 4 and 5. In such an environment a firm that is lucky enough to have successful sales in early periods will consider it increasingly difficult to continue selling. This informational structure has received somewhat less attention in information economics, and it is important to consider this as it has a very different impact on the borrower’s effort incentives. Realistic markets are likely to enable firms to both learn about the preferences of their target market as well as to exhaust the target market, while one of these effects is likely to be dominant in a particular case.
Another useful property of random variables is exchangeability:

**Definition 2** Random variables $X_1, \ldots, X_n$ are exchangeable if a sequence $(X_1, \ldots, X_n)$ is equal in distribution to $(X_{i_1}, \ldots, X_{i_n})$ for any $n!$ permutation $i_1, \ldots, i_n$ of integers $1, \ldots, n$.

Exchangeability means that the order at which potential sales arrive does not affect the joint probability. For example, with three potential sales opportunities it must be the case that joint probability $p(0,1,1) = p(1,1,0) = p(1,0,1)$, etc. Intuitively, exchangeability means that from an ex-ante perspective, potential sales opportunities are equally likely to arrive at any $t$. Exchangeability would be violated if there was some cyclicality in sales prospects.

Given these concepts, we can connect the features of the optimal contract to the characteristics of the market where the borrower operates.

### 3.2 First best and outside financing capacity

As a first best benchmark, consider a social planner who lends the borrower $I$ and chooses his optimal effort. The value of the project

$$U^{FB} = \sum_{t=0}^{T-1} 1_t s_t - 1_t e_t - I. \quad (5)$$

Expected utility at date 0 is $\mathbb{E}[U^{FB}]$ and the borrower’s optimal effort decision at each date $t$ must set $1_t = 1$ if, and only if

$$\mathbb{E}[U^{FB}|1_{0}s_1, \ldots, 1_{t-1}s_t, 1_0, \ldots, 1_{t-1}, 1_t = 1] \geq \mathbb{E}[U^{FB}|1_{0}s_1, \ldots, 1_{t-1}s_t, 1_0, \ldots, 1_{t-1}, 1_t = 0]. \quad (6)$$

From the assumption that the effort cost is small enough, it follows that it is optimal for the agent to set $1_t = 1$ for all $t$. The project is worth pursuing, if, and only if

$$I \leq I^{FB} = \sum_{t=0}^{T-1} \mathbb{E}[s_t] - \sum_{t=0}^{T-1} e_t. \quad (7)$$

Consider then the contracting problem (1)-(3). Replacing the lender’s break even constraint (2) in the borrower’s utility (1), we obtain that the borrower’s expected utility from the project at date 0 is

$$\mathbb{E}[U] = \mathbb{E} \left[ \sum_{t=1}^{T} 1_t s_t - 1_t e_t \right] - I, \quad (8)$$
which is exactly the same as $\mathbb{E}[U^{FB}]$ above. This implies that the borrower would maximize his utility by offering a contract at date 0 that would give him incentives to always make an effort.

Financing contracts that induce first best indeed exist in this environment. An obvious example is the case where potential sales are identically and independently distributed and the cost of effort is constant, i.e., $\mathbb{E}[s_{t+1}|s_1, \ldots, s_t] = \mathbb{E}[s_{t+1}] = \bar{s}$ and $e_t = \bar{e}$. The assumption that effort is always optimal means that $\bar{e} \leq \bar{s}$. It is immediate that offering a simple equity $w(\hat{s}_1, \ldots, \hat{s}_T) = \alpha \cdot (\hat{s}_1 + \ldots + \hat{s}_T)$, where $\alpha = \frac{\bar{e}}{\bar{s}}$, satisfies the incentive compatibility constraints (3), as the constraint for date $t$ simplifies to

$$\alpha \cdot \mathbb{E}[s_{t+1}] - \bar{e} \geq 0 \iff \frac{\bar{e}}{\bar{s}} \cdot \mathbb{E}[s_{t+1}] - \bar{e} \geq 0 \iff \bar{e} - \bar{e} = 0 \geq 0$$

This contract covers the break-even investment under the first best, $I^{FB} = T(\bar{s} - \bar{e})$, without the need for the borrower to invest any of his own funds to the project, i.e., $A_0 = 0$. Furthermore, a borrower who has lower investment costs would still choose $A_0 = 0$ and could offer a contract $w(\hat{s}_1, \ldots, \hat{s}_T) = \alpha_y \cdot (\hat{s}_1 + \ldots + \hat{s}_T)$, where $\alpha_y = \gamma \frac{\bar{e}}{\bar{s}}$ and $\gamma > 1$. It is straightforward that this contract satisfies effort incentive compatibility constraint (3) as well, and we can derive the value of $\gamma$ for a given $I$ from the lender’s break even constraint (2) as $\gamma = \frac{(T\bar{s} - I)}{Te}$. Indeed, a simple equity contract achieves the first best in this special case.

In the general setting where the sales are statistically dependent, we can also see that the borrower’s expected utility from the project (8) does not depend on $A_0$. Because choosing to make an effort every period maximizes the value of the project, $\mathbb{E}[U]$, it follows that any contract that induces first best effort satisfies the borrower’s primary objective. As the borrower has lexicographic preferences, it implies that he would offer a contract that minimizes $A_0$. Setting $1_t = 1$ for all $t$, the lender’s break even constraint (2) becomes

$$I - A_0 = \mathbb{E} \left[ \sum_{t=1}^{T} s_t \right] - \mathbb{E}[w(s_1, \ldots, s_T)]$$

and it follows that minimizing $A_0$ is equivalent to minimizing $\mathbb{E}[w(s_1, \ldots, s_T)]$. Furthermore, using (7) the lender’s break even constraint can be written as

$$\mathbb{E}[w(s_1, \ldots, s_T)] = A_0 + \sum_{t=0}^{T-1} e_t + (I^{FB} - I) \cdot$$
All first best investments can be undertaken if the borrower covers at least his cost of effort and own investment, which may be needed for the borrower to have an incentive to choose to make an effort at all dates. In order to derive the optimal contract, it is useful to define \( \tilde{w}(s_1, \ldots, s_T) \) as a contract offered by a borrower with \( I = I^{FB} \). We can restate the contracting problem as minimizing the borrower’s reward needed for this:

\[
\min_{\{\tilde{w}(s_1, \ldots, s_T) \geq 0\}} \mathbb{E}[\tilde{w}(s_1, \ldots, s_T)]
\] (10)

under the constraints that the borrower is willing to pursue the project

\[
\mathbb{E}[\tilde{w}(s_1, \ldots, s_T)] - \sum_{t=0}^{T-1} e_t \geq 0
\] (11)

and to choose first-best effort at each date \( t = 0, \ldots, T - 1 \)

\[
\mathbb{E}[\tilde{w}(s_1, \ldots, s_t, s_{t+1}, s_{t+2}, \ldots, s_T) - \tilde{w}(s_1, \ldots, s_t, 0, s_{t+2}, \ldots, s_T)] | s_1, \ldots, s_t] \geq e_t.
\] (12)

The solution of this problem will give the contract and the minimum downpayment necessary to cover the break-even investment cost \( I^{FB} \), which is given by \( A_0 = \mathbb{E}[\tilde{w}(s_1, \ldots, s_T)] - \sum_{t=0}^{T-1} e_t \).

This difference will also give the information rent the borrower needs to obtain to satisfy incentive compatibility constraints given by (12) at each date \( t \) and for any sales history up to \( t \). The distribution of potential sales revenues determines whether positive information rents are needed or not. Naturally, a borrower who has fixed investment cost \( I < I^{FB} \) is easier to incentivise. Such borrower maximizes his preferences by choosing

\[
A_0 = \max \left[ 0, \mathbb{E}[\tilde{w}(s_1, \ldots, s_T)] - \sum_{t=0}^{T-1} e_t - (I^{FB} - I) \right].
\] (13)

When \( \mathbb{E}[\tilde{w}(s_1, \ldots, s_T)] - \sum_{t=0}^{T-1} e_t - (I^{FB} - I) < 0 \), the borrower obtains an additional surplus. Any contract to capture this surplus that does not violate (3) is optimal. For example, one way is to distribute the surplus is to make a lump sum payment at date \( T \), and another way is to scale up all payments in a similar way to the example with independent sales above. The latter minimizes the need for an escrow account where some funds need to be kept during the period 0 to \( T \).
3.3 Main results: two date example

The intuition behind the main results and the solution method is easiest to demonstrate in the case where $T = 2$. There are four possible sales outcomes $(s_1, s_2) \in \{(0, 0), (0, 1), (1, 0), (1, 1)\}$, the corresponding probabilities and borrower’s rewards are $p(s_1, s_2)$ and $\tilde{w}(s_1, s_2)$ respectively. Denote the marginal probabilities as $p_t(1) = \Pr(s_t = 1)$ and $p_t(0) = \Pr(s_t = 0)$ for $t = 1, 2$. Note that $p_1(1) = p_2(1)$ if potential sales are exchangeable random variables; these probabilities may differ otherwise. We can further expand the constraint on effort costs

$$e_0 \leq \mathbb{E}[s_1] = p_1(1); e_1 \leq \min[\mathbb{E}[s_2|s_1 = 1], \mathbb{E}[s_2|s_1 = 0]] = \min \left[ \frac{p(1, 1)}{p_1(1)}, \frac{p(0, 1)}{p_1(0)} \right].$$

(14)

We can restate the problem (10)-(12) as

$$\min \mathbb{E}[\tilde{w}(s_1, s_2)]$$

subject to the participation constraint

$$\mathbb{E}[\tilde{w}(s_1, s_2)] \geq e_0 + e_1$$

(15)

and to the three incentive compatibility constraints

$$\mathbb{E}[\tilde{w}(s_1, s_2) - \tilde{w}(0, s_2)] \geq e_0$$

(17)

$$\mathbb{E}[\tilde{w}(0, s_2) - \tilde{w}(0, 0) | s_1 = 0] \geq e_1$$

$$\mathbb{E}[\tilde{w}(1, s_2) - \tilde{w}(1, 0) | s_1 = 1] \geq e_1$$

and the non-negativity constraint $\tilde{w}(s_1, s_2) \geq 0$.

Incentive compatibility constraints highlight that the borrower must always get a positive payoff from generating the next sale. It is immediate that a contract that keeps the borrower’s payoff constant in some states cannot guarantee effort in all states and be optimal. For example, a simple debt contract that requires repayment of more than 1, can not be optimal as the borrower has the same payoff after $s_1 = 0$ regardless of whether he sells of not at date 2, which would violate the second inequality in (17).

Note that the condition $e_1 \leq \mathbb{E}[s_2|s_1 = 1]$ requires that $e_1 \leq \mathbb{E}[s_2|s_1 = 0]$, $e_1 \leq \mathbb{E}[s_2|s_1 = 1]$ and $e_1 \leq \mathbb{E}[s_2]$. However, the last constraint is redundant because the law of iterated expectations implies that $E[s_2] = p_1(1)E[s_2|1] + p_1(0)E[s_2|0]$. 

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12 Note that the condition $e_1 \leq \mathbb{E}[s_2|s_1 = 1]$ requires that $e_1 \leq \mathbb{E}[s_2|s_1 = 0]$, $e_1 \leq \mathbb{E}[s_2|s_1 = 1]$ and $e_1 \leq \mathbb{E}[s_2]$. However, the last constraint is redundant because the law of iterated expectations implies that $E[s_2] = p_1(1)E[s_2|1] + p_1(0)E[s_2|0]$. 

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The last two incentive compatibility constraints also highlight the role of learning. A successful sale at date 1 may make the borrower to update his beliefs upward, i.e., \( \Pr(s_2|s_1 = 1) > \Pr(s_2|s_1 = 0) \). In such case, it is relatively easier to incentivise the borrower to make an effort following a success at date 1, and the borrower may need to retain a bigger share of date 2 sales following a failure. The opposite holds in the case where "success lowers prospects", and it is relatively easier to incentivise the borrower to continue to make an effort following a no-sale at date 1. The proposition below confirms this intuition.

**Proposition 3** If \( I = I_{FB} \), then the optimal contract depends on the joint distribution of sales and can be summarized as

1) If "success raises prospects", i.e., \( p(1,1)p(0,0) \geq p(1,0)p(0,1) \), then the reward the borrower retains is

\[
\begin{align*}
\hat{w}(0,0) &= 0 \\
\hat{w}(0,1) &= e_1 \frac{p_1(0)}{p(0,1)} \\
\hat{w}(1,0) &= e_0 - e_1 \frac{p_2(1)}{p_1(1)} + \hat{w}(0,1) \frac{p_2(1)}{p_1(1)} \\
\hat{w}(1,1) &= \hat{w}(1,0) + e_1 \frac{p_1(1)}{p(1,1)}
\end{align*}
\]

the date 0 expected value of the borrower’s reward is

\[
\mathbb{E}[\hat{w}(s_1, s_2)] = e_0 + e_1 + e_1 \frac{p(1,1)p(0,0) - p(1,0)p(0,1)}{p(0,1)}, \tag{19}
\]

and the project can be externally financed if \( A \geq A_0 = e_1 \frac{p(1,1)p(0,0) - p(1,0)p(0,1)}{p(0,1)} \).

2) If "success lowers prospects", i.e., \( p(1,1)p(0,0) \leq p(1,0)p(0,1) \), then the reward the borrower retains is

\[
\begin{align*}
\hat{w}(0,0) &= 0 \\
\hat{w}(0,1) &= e_1 \frac{p_1(0)}{p(0,1)} \\
\hat{w}(1,0) &= e_0 \frac{p_1(0)}{p_1(1)} \\
\hat{w}(1,1) &= \hat{w}(1,0) + e_1 \frac{p_1(1)}{p(1,1)}, \tag{20}
\end{align*}
\]
the date 0 expected value of the borrower’s reward is
\[ \mathbb{E} [\tilde{w}(s_1, s_2)] = e_0 + e_1, \]
and the project can be fully externally financed, i.e., projects with \( A \geq A_0 = 0 \) are externally financed.

**Proof.** See appendix A.1. ■

The proof of this proposition uses the Duality and Complementary Theorems: given the primal minimization problem above, there is a dual maximization problem. Duality Theorem implies that if both problems are feasible, both have a solution and the minimized (maximized) value of the primal (dual) is the same. Solving the dual problem and using Complementary Theorems then implies that if "success raises prospects" the incentive compatibility constraints (17) must all be binding and the participation constraint (16) can be slack. In contrast, if "success lowers prospects" then the participation constraint (16) must be binding and incentive compatibility constraints can be slack (17). The optimal contract can also be equivalently described via expectations.

**Corollary 4** The optimal contract can be expressed as
\[ \tilde{w}(s_1, s_2) = \alpha_1 s_1 + \alpha_2 (s_1) s_2, \]
where
\[ \alpha_2 (s_1) = \frac{e_1}{\mathbb{E} [s_2 | s_1]} \]  
and
\[ \alpha_1 = \begin{cases} \frac{e_0 - e_1}{\mathbb{E} [s_1]} + \frac{\mathbb{E} [s_2]}{\mathbb{E} [s_1]} \alpha_2 (0) & \text{if "success raises prospects"} \\ \frac{e_0}{\mathbb{E} [s_1]} & \text{if "success lowers prospects"} \end{cases} \]

**Proof.** Straightforward from (18) and (20) given Bayes’ rule and the definition of conditional expectations. ■

Corollary 4 shows how the optimal contract can be interpreted as specifying the borrower’s and the lender’s claim on each sale, i.e., the borrower receives the share \( \alpha_1 \) and \( \alpha_2 (s_1) \) from each date 1 and 2 sales respectively and the lender receives the remainder \( 1 - \alpha_1 \) and \( 1 - \alpha_2 (s_1) \).\(^{13}\) This

\(^{13}\)We can see that if the cost of effort is constant, i.e., \( e_0 = e_1 = \tilde{e} \), and sales are are exchangeable random variables, then the borrower and lender can immediately split the revenue from each sale, as \( \alpha_1, \alpha_2 (s_1) \leq 1 \), (see (14)). More generally, the contract can always take advantage of an escrow account if \( \alpha_1 > 1 \) and temporarily withhold some funds until the date 2 outcome is realized as well.
corresponds to a self-adjusting "smart" contract, which takes a form of revenue/profit sharing rule, where the borrower’s data 2 share of revenues depends on the date 1 realized outcome.

Proposition 3 and Corollary 4 highlight the following. First, the properties of joint distribution are crucial for understanding how the borrower’s incentives evolve based on observing the outcome from date 1 sale. If "success raises prospects", then \( E[s_2|s_1 = 1] \geq E[s_2] \geq E[s_2|s_1 = 0] \) and the borrower who Bayesian updates his beliefs finds it harder to sell another item if he failed to make a sale at date 1, and easier to sell another item if he succeeded. To maintain effort incentives for date 2, his marginal reward from making an effort after failure must be higher than his marginal reward after success at date 1, i.e., \( \alpha_2(0) \geq \alpha_2(1) \). If "success lowers prospects" then \( E[s_2|s_1 = 1] \leq E[s_2] \leq E[s_2|s_1 = 0] \) and the opposite holds, the borrower needs to receive a higher marginal reward following a success, i.e., \( \alpha_2(1) \geq \alpha_2(0) \)

Second, there is a need for the borrower’s own contribution to undertake the project in the case where "success raises prospects" and not in the case where "success lowers prospects". When deciding whether to pay an effort cost \( e_0 \), the borrower has a manipulation possibility in the "success raises prospects" environments - he can withhold effort and save this cost while knowing that he will receive a bigger share of date 2 sale following no realized sales at date 1, which would be interpreted along the equilibrium path as "bad luck" rather than no effort. To eliminate this manipulation incentive, the borrower must receive a large enough share of date 1 sale. This increases the expected value of the minimum reward that the borrower must receive for the contract to be incentive compatible and the borrower invest some own funds to cover information rents that arise due to the aforementioned manipulation possibility, an amount given in (19). In such an environment there may still be borrowers that are credit rationed, in the sense that they have a positive NPV project, but cannot obtain funding without having enough own assets. Yet, Section 4 shows that the potential credit rationing is small compared to the case where one would be restricted to use traditional assets, in particular simple debt contracts on blockchain. This manipulation possibility is not present if "success lowers prospects", because strategically withholding effort would give the lender an even smaller share of date 2 sales revenue. This is why even a lender who has no own funds to invest in the project can obtain external financing to cover the fixed cost of any profitable
investment project. The same applies for the case with independent potential sales as in such case
\( p(1,1) p(0,0) = p(1,0) p(0,1) \).

Third, we can identify the conditions when the optimal contract could be specified as a function
on enumeration aggregates as in Holmström and Milgrom (1987). If "success raises prospects", we
can see from (18) that \( \tilde{w}(0,1) = \tilde{w}(1,0) \) requires both: sales revenues must be exchangeable
random variables, in which case \( p_1(1) = p_2(1) \), and that effort cost must be constant, \( e_0 = e_1 \).
This assumption might not hold in all realistic environments, e.g., because of seasonality in sales
opportunities or because the borrower may become more skilled in selling over time and have a
decreasing effort cost. From (18), the borrowers reward should be relatively higher in the case of
early success compared to late success, i.e., \( \tilde{w}(1,0) > \tilde{w}(0,1) \) if the cost of effort is decreasing
over time, \( e_0 > e_1 \), and/or if marginal probability of generating a sale at date 1 is lower than the
marginal probability of generating a sale at date 2, \( p_1(1) < p_2(1) \). It is intuitive that the borrower’s
reward should be higher in states where generating a sale is more difficult, either because of the
effort cost or demand. If \( p_1(1) = p_2(1) \) and \( e_0 = e_1 \), then the optimal contract can be expressed
as a function of total sales, and it is interesting to note that under "success raises prospects", such
optimal contract is increasing and concave in total sales revenue.\textsuperscript{14} Concavity in total sales again
emphasizes the intuition that in the environment where "success raises prospects", the optimal
contract needs to encourage borrowers with lower sales revenues relative to those that have been
very successful and in this sense resembles progressive taxation.

We can further see from (20) that if "success lowers prospects" \( \tilde{w}(0,1) = \tilde{w}(1,0) \), only in the
highly particular case where \( \frac{e_1}{e_0} = \frac{p_1(1)p_2(0)}{p(0,1)} = \frac{E[x_1]}{E[x_2|x_1=0]} \). Note that this does not hold even in the
case where the effort cost is constant, \( e_1 = e_2 \), and sales are exchangeable. When "success lowers
prospects", we generally cannot express the optimal contract in terms of enumeration aggregates.
The reason is that incentive compatibility constraints do not need to be binding in this environment.

Whenever we cannot express the optimal contract as a function of enumeration aggregates, we
need to have sales data that includes reliable timestamps in order to implement the optimal contract

\textsuperscript{14}Assume that \( e_0 = e_1 = \tilde{e} \) and \( p(0,1) = p(1,0) \) \( \implies p_2(1) = p_1(1) \). Using (18), we obtain \( \frac{(0,0) + w(1,1)}{2} = \frac{w(0,1)}{2} + \frac{w(1,0)}{2} \). The optimal contract is concave in total sales if \( \frac{w(0,0) + w(1,1)}{2} \leq w(0,1) = w(1,0) \), and we obtain
\( \frac{w(1,0)}{2} + \frac{p_1(1)}{2} \leq w(1,0) \Leftrightarrow \frac{p_1(1)}{p(1,1)} \leq \frac{p_1(0)}{p(0,1)} \Leftrightarrow \frac{p_1(1) + p(1,0)}{p(1,1)} \leq \frac{p_1(0) + p(0,1)}{p(0,1)} \Leftrightarrow p(0,0) p(1,1) \geq p(1,0) p(0,1) \).
derived. As such timestamps are a feature of blockchain based records, it makes this technology beneficial for financial contracting beyond eliminating (or at least greatly reducing) verification costs.

Finally, the contracts described by (18) and (20) guarantee that the borrower can cover a break even investment cost $I = I^{FB}$. If $I < I^{FB}$, we can specify the optimal contract further. Namely, (9) implies that the borrower can retain

$$\mathbb{E}[w(s_1, s_2)] = \mathbb{E}[\tilde{w}(s_1, s_2)] + (I^{FB} - I).$$

If "success lowers prospects" then all projects with $I < I^{FB}$ can be fully externally funded with $A_0 = 0$, and there are multiple contracts that satisfy this equality, including a lump-sum payment $(I^{FB} - I)$ at date $T$, and a contract $w(s_1, s_2) = \gamma \tilde{w}(s_1, s_2)$, where $\gamma = 1 + \frac{(I^{FB} - I)}{e_0 + e_1} \geq 1$.

If "success raises prospects" and $I < I^{FB}$, then the borrower’s preferences are maximized by first reducing own investment, i.e., the borrower would set $A_0 = \max \left[0, e_1 \frac{p(1,1)p(0,0) - p(1,0)p(0,1)}{p(0,1)} - (I^{FB} - I) \right]$. If this maximum is zero, then the remaining surplus could again be shared in multiple ways, including a lump sum transfer at date $T$ and a contract that sets $w(s_1, s_2) = \gamma \tilde{w}(s_1, s_2)$, where $\gamma = 1 + \frac{(I^{FB} - I) - e_1}{e_0 + e_1 + e_2} \frac{p(1,1)p(0,0) - p(1,0)p(0,1)}{p(0,1)} \geq 1$.

The borrower cannot benefit from offering a contract that gives him an incentive not to make an effort at date 0 or 1, as the investment cost that could be covered by such a deviating contract would reduce $\mathbb{E}[U]$ and the range of investment costs that can be externally financed.

3.4 Main results: T period case

The $T$-period case is a similar linear programming problem with $2^T$ possible states and variables to solve for, and $1 + 2 + \ldots + 2^{T-1} = 2^T - 1$ incentive compatibility constraints given by (12) and the participation constraint (11). For the sake of clarity, I assume in this section that potential sales are exchangeable and that potential sales are either log-supermodular or log-submodular, i.e., "success raises prospects" or "success lowers prospects" as defined in Section 3.1.\(^{15}\) Results in Section 3.3, suggest that we should expect different optimal contracts depending on whether we

\(^{15}\)If the potential sales distribution does not satisfy these assumptions, the linear programming problem can still be solved numerically, and we can expect the same intuition as in Section 3.3 to apply, e.g., if there is cyclical in sales, the borrower should receive a higher share of sales in states that are less likely ex-ante.
are in a "success raises prospects" environment or in a "success lowers prospects" one. Let us start from the former.

**Proposition 5** If the joint distribution of potential sales is such that "success raises prospects" then the optimal contract for a borrower with \( I = I^{FB} \) is

\[
\tilde{w}(s_1, ..., s_T) = \alpha_1 s_1 + \ldots + \alpha_t (s_1, ..., s_{t-1}) s_t + \ldots + \alpha_T (s_1, ..., s_{T-1}) s_T, \tag{23}
\]

where

\[
\alpha_T (s_1, ..., s_{T-1}) = \frac{e_{T-1}}{\mathbb{E}[s_T|s_1, ..., s_{T-1}]}
\]

and for any \( t = 0, ..., T - 2 \), it holds that

\[
\alpha_{t+1} (s_1, ..., s_t) = \alpha_{t+2} (s_1, ..., s_t, s_{t+1} = 0) + \frac{e_t - e_{t+1}}{\mathbb{E}[s_{t+1}|s_1, ..., s_t]}.	ag{24}
\]

In order to be externally financed, the borrower needs to invest own funds

\[
A_0 = \sum_{t=0}^{T-2} \mathbb{E}[\Phi_t s_{t+1}], \tag{25}
\]

where \( \Phi_t \) is information rent the borrower obtains at \( t = 0, ..., T - 2 \), defined as

\[
\Phi_t \equiv e_{t+1} \Delta_t (s_1, ..., s_t) + e_{t+2} \Delta_t (s_1, ..., s_t, s_{t+1} = 0) + \ldots + e_{T-1} \Delta_{T-2} (s_1, ..., s_t, s_{t+1} = 0, ..., s_{T-2} = 0), \tag{26}
\]

where

\[
\Delta_t (s_1, ..., s_t) \equiv \frac{1}{\mathbb{E}[s_{t+2}|s_1, ..., s_t, s_{t+1} = 0]} - \frac{1}{\mathbb{E}[s_{t+1}|s_1, ..., s_t]} \geq 0 \tag{27}
\]

The expected value of the borrower’s rewards at date 0 is

\[
\mathbb{E}[\tilde{w} (s_1, ..., s_T)] = \sum_{t=0}^{T-1} e_t + \sum_{t=0}^{T-2} \mathbb{E}[\Phi_t s_{t+1}].
\]

**Proof.** See Appendix A.2 ■

**Corollary 6** If \( I < I^{FB} \), then the borrower invests

\[
A_0 = \max \left[ 0, \sum_{t=0}^{T-2} \mathbb{E}[\Phi_t s_{t+1}] - (I^{FB} - I) \right].
\]
It is low enough such that $A_0 = 0$, then any remaining surplus could be split in multiple ways including a lump-sum transfer at date $T$, or setting $w(s_1, \ldots, s_T) = \gamma \tilde{w}(s_1, \ldots, s_T)$, where $\gamma = 1 + (I^{FB} - I) \frac{\sum_{t=0}^{T-2} E[\Phi_t s_{t+1}]}{\sum_{t=0}^{T} \varepsilon_t + \sum_{t=0}^{I-2} E[\Phi_t s_{t+1}]} \geq 1$.

**Proof.** Follows from Proposition 5, (9) and (13).

Proposition 5 confirms the intuition from the two date case for "success raises prospects" environment. The optimal contract for $I = I^{FB}$ can be expressed as a dynamically adjusting profit sharing contract (23) that splits each successful sale between the borrower and the lender according to the sharing rule that depends on the sales history up to that point. If the effort cost is constant, then the borrower has the highest claim on date 1 sale, and every time he makes a successful sale, his claim on the next sale decreases. Namely, the sharing rule (24) in such a case implies that $\alpha_1 = \frac{e}{E[s_T|s_1=0, \ldots, s_{T-1}=0]}$, which is inversely related to the conditional expectations of $s_T$ conditional on $T - 1$ periods of no sales. With a joint distribution that satisfies "success raises prospects", $E[s_T|s_1 = 0, \ldots, s_{T-1} = 0]$ is the lowest value that conditional expectations at date $T - 1$ could take. If there is no sale at date 0, then the borrower maintains the same claim on the next sale $\alpha_2(s_1 = 0) = \alpha_1$. If there is a sale, then the borrower’s claim on next sale decreases as $\alpha_2(s_1 = 1) = \frac{e}{E[s_T|s_1=1, s_2=0, \ldots, s_{T-1}=0]} \leq \alpha_1$. The same argument can be repeated for all periods.

The optimal contract takes this form for the following reasons. The borrower needs to have an incentive to make an effort in $T-1$ in all states, which includes the worst case scenario, where he has not sold anything up to that point, despite best effort. Because "success raises prospects", "failure lowers prospects" also, which implies that the borrower has low expectations that he will make a successful sale due to Bayesian learning, and will only make an effort if he gets a high enough share of this last sale. The reason why the claim that the borrower has on sales at date $t < T - 1$ cannot be lower (under constant effort) as long as he has not sold any units up to that point, is to avoid manipulation possibility where he strategically withholds effort to increase his claim on future sales. This is the reason why there are "information rents". At the same time a self-adjusting contract optimally manages the borrower’s incentives. Any time the borrower manages to sell a unit, his expectations of future sales increase, and he is willing to make an effort in return for a smaller share
of the next sale. As it will be further shown in Section 4, simple equity would lack flexibility and would not optimally manage the borrower's effort incentives. The borrower would always need to keep high enough share of profits, and does not benefit from the fact that he would be willing to accept a lower share following successful early sales. This in turn would make external financing less accessible.

Figure 1 illustrates the optimal contract in a "success raises prospects" environment where \( I = I^{FB} = 5, \) \( T = 12, \) and the effort cost is constant \( e_t = \tilde{e} = 1/12. \) It shows how the borrower's expectations and \( \alpha_{t+1} (s_1, \ldots, s_t) \) evolve following a random path of successful sales history \((0, 1, 0, 0, 1, 1, 1, 0, 1, 0, 1, 1)\) (dots in panel A and B). The figure is constructed under the assumption that \( \Pr (s_t = 1 | \theta) = \theta, \) while \( \theta \) itself is unknown and drawn from the beta distribution \( Be (\lambda \theta_0, \lambda (1 - \theta_0)) \), where \( \theta_0 = 0.5 \) and \( \lambda = 4, \) which is an example of the affiliated sales process discussed in Section 3.1. The prior expectations of making a sale at any period is \( \Pr (s_t = 1) = 0.5. \)

If there is no sale, the borrower revises his expectations of making a sale in the next period downwards and if there is a sale, he revises his expectations upwards (see Panel A). The optimal contract starts with the borrower having a claim of around 62% on the first successful sale, and the borrower's claim following each successful sale adjusts downwards. At early dates, information rents are high and their importance diminishes over time (Panel C). This is because information rents are cumulative as shown in Proposition 5.

Recall that in \( T = 2, \) in a "success raises prospects" environment with constant effort cost and exchangeable sales process, the optimal contract can also be expressed as a function of total sales. Figure 2 represents the same contract as Figure 1, as a function of total sales (solid line). The dashed line shows in comparison a contract that maintains all the same assumptions, except the assumption that \( \theta \) is drawn from the beta distribution \( Be (\lambda \theta_0, \lambda (1 - \theta_0)) \), where \( \theta_0 = 0.5 \) and \( \lambda = 100. \) Both cases have the same prior mean \( \theta_0 = 0.5, \) but \( \lambda = 100 \) implies noticeably lower uncertainty about parameter \( \theta. \) Low uncertainty about \( \theta \) also implies that the borrower can learn very little from sales history. We can see that the optimal contract is again concave in total sales. Furthermore, if there is little room for learning, the optimal contract converges to a simple equity

\[ Var [\theta] = \frac{(1-\theta_0)\theta_0}{1+\lambda} \] is decreasing in \( \lambda. \)
Figure 1: **Learning and optimal contract in a "success raises prospects" environment**

Panel A shows how the borrower’s expectations evolve (solid line) given a stream of realized sales (dots); Panel B shows how the borrower’s claim evolves on next date sale $\alpha_t(.)$ given the same stream of realized sales as in Panel A; Panel C splits the borrower’s claim to compensation for effort and the information rent.

contract. At the limit, there are no information rents.

If effort cost is not constant, then the optimal contract cannot be expressed as a function of total sales and depends on the timing of sales realizations. Proposition 5 further shows how the dynamics of effort cost affect the optimal contract. If the total effort is decreasing over time then the borrower claim on last sale $\alpha_T(s_1, \ldots, s_{T-1})$ is lower, but his claims on earlier sales may need to be higher (see 24).

The limiting case in a "success raises prospects" environment is one where potential sales are independent random variables. Exchangeability implies that $E[s_t] = \bar{s}$ for any $t$ and independence implies that $E[s_{t+k}|s_1, \ldots, s_t] = E[s_{t+k}]$ for any $t$ and $k > 0$. From (23) it follows that $\alpha_{t+1}(s_1, \ldots, s_t) = \alpha_{t+1} = \frac{\bar{s}}{\bar{T}}$ and the sequence at which successful sales arrive matters. At the limit, there is very little to learn from observing an additional sale, then $E[s_{t+1}|s_1, \ldots, s_t] = E[s_{t+2}|s_1, \ldots, s_t]$ is not very different from $E[s_{t+2}|s_1, \ldots, s_t, s_{t+1} = 0]$. We can see from (27) that in that case $\Delta_{t}(s_1, \ldots, s_t)$ becomes smaller and information rents diminish, at the limit $\Delta_{t}(s_1, \ldots, s_t) \rightarrow 0$ and the need for any information rents disappears. It further shows that even in the environment where sales are independent, the optimal contract benefits from dynamic adjustments which consider the dynamics of effort cost.
Figure 2: Optimal contract as function of total sales in "success raises prospects" environment with constant effort cost.

Corollary 6 shows the optimal contract in the case where $I < I_{FB}$. It is straightforward that the borrower with lower investment costs is easier to incentivise. As long as a borrower is able to obtain external financing, his expected profits are the same as under the first best. In monetary terms, a borrower’s own funds are as cheap as external financing. Firms that have low enough investment costs to chose $A_0 = 0$, have not just the first best expected profit, but are exactly as well off as under the first best.

**Proposition 7** If the joint distribution of potential sales is such that "success lowers prospects" then the optimal contract for a borrower with $I = I_{FB}$ is

$$
\tilde{w}(s_1, ..., s_T) = \alpha_1 s_1 + ... + \alpha_t (s_1, ..., s_{t-1}) s_t + ... + \alpha_T (s_1, ..., s_{T-1}) s_T,
$$

(28)

and for any $t = 0, ..., T - 1$, it holds that

$$
\alpha_{t+1} (s_1, ..., s_t) = \frac{e_t}{\mathbb{E} [s_{t+1} | s_1, ..., s_t]}
$$

(29)

The borrower can cover $I_{FB}$ while optimally $A_0 = 0$ and the expected value of the borrower’s rewards at the time of signing the contract is $\mathbb{E} [\tilde{w}(s_1, ..., s_T)] = \sum_{t=0}^{T-1} e_t$.

**Proof.** See Appendix A.2 □
Corollary 8 If $I < I^{FB}$, then this surplus could be split in multiple ways including a lump-sum transfer at date $T$, and setting $w(s_1,..,s_T) = \gamma \tilde{w}(s_1,..,s_T)$, where $\gamma = 1 + \frac{(I^{FB}-I)}{\sum_{t=0}^{T} e_t}$.

Proof. Follows from Proposition 7, (9) and (13). ■

Proposition 7 also confirms the intuition from the two date case when "success lowers prospects": all projects that are worth undertaking can be fully externally financed, and the borrower would optimally not invest any of his own funds in the project. If $I = I^{FB}$, then the optimal contract dynamically adjusts the share retained by the borrower (29) upward if $e_t$ is higher and the updated expectation $E[s_{t+1}|s_1,..,s_t]$ is lower.

Figure 3 gives an example of an optimal contract in a "success lowers prospects" environment when $I = I^{FB}$ and $T = 12$. It shows how the borrower’s expectations and $\alpha_{t+1}(s_1,..,s_t)$ evolve following the same realized path of sales as in Figure 1. However, the figure is now constructed under the assumption that the potential sales process is random sampling without replacement from a population of 30 that is known to have exactly 15 interested buyers. As before, assume that $I^{FB} = 5$ and effort cost is $e_t = 1/12$. The borrower now revises his expectations downward following a successful sale and upward following no sale. Furthermore, the optimal contract does not need to give the borrower information rents and because of that, the borrower share adjusts both upward and downward in an opposite direction compared to the borrower’s expectations. The optimal contract in this case resembles a reward scheme that gives the borrower a higher share of the next sale following a success and a lower share following a failure. When there are less learning opportunities, then the optimal contract converges to the contract with independent sales where $\alpha_{t+1} = \frac{e_t^0}{8}$. As in Section 3.3, as long as there is at least some learning, the optimal contract cannot be expressed in terms of total sales, even if effort cost is constant.

Finally, we can see from Propositions 5 and 7 that the variables that determine the optimal splitting rule are simple conditional expectations $E[s_{t+1}|s_1,..,s_t]$, and in the case "success raises prospects" also a "conservative" estimate

$E[s_{t+k}|s_1,..,s_t, s_{t+1} = 0, \ldots, s_{t+k-1} = 0]$. These variables have an intuitive interpretation and could be estimated via an standard econometric tools. A "smart contract" could either be based on
Figure 3: **Learning and optimal contract in a "success lowers prospects" environment**

Panel A shows how the borrower’s expectations evolve (solid line) given a stream of realized sales (dots); Panel B shows how the borrower’s claims on next date sales $\alpha_t(.)$ evolve given the same stream of realized sales as in Panel A.

analyzing these scenarios ex-ante or estimate these variables on running bases via an automated estimation algorithm.

4 **Smart-contract vs. standard debt and equity.**

This section compares the optimal contract derived in Section 3 with standard equity and debt. I will maintain the assumption that these standard contracts also benefit from assumed positive features of blockchain. In the existing environment where there are positive verification and enforcement costs, both debt and equity contract cannot be less expensive. So the total gains from switching from the existing environment to the assumed blockchain environment equals to the gains derived below plus any further gains from the reduction of verification and enforcement costs. As the derived contract is optimal, there is no reason for borrowers to prefer not to use blockchain and smart contracts in this setting. This section enables to assess the magnitude of potential gains compared to traditional assets.

Because standard debt and equity depend on total sales, let us define total potential sales and realized sales respectively as

$$c_T = \sum_{t=0}^{T-1} s_{t+1} \quad \text{and} \quad \tilde{c}_T = \sum_{t=0}^{T-1} \tilde{s}_{t+1} = \sum_{t=0}^{T-1} 1_t s_{t+1}$$
The standard equity contract specifies the borrower's reward as

\[ w_E(\hat{c}_T) = \alpha_E \hat{c}_T, \]

where \( \alpha \) is fixed, and the standard debt contract specifies the borrower's reward as

\[ w_D(\hat{c}_T) = \begin{cases} \hat{c}_T - d, & \text{if } \hat{c}_T \geq d \\ 0, & \text{otherwise} \end{cases} \]

I compare three cases: independent sales, the "success raises prospects" environment and the "success lowers prospects" environment. Note that independent sales are Bernoulli trials, which implies that \( c_T \) has binomial distribution \( \text{Bin}(T, \theta_0) \). As an example of "success raises prospects", let us assume that sales are conditionally independent and depend on consumer preference parameter \( \theta \), i.e., \( P(s_t = 1|\theta) = \theta \), where \( \theta = Be(\lambda \theta_0, \lambda(1 - \theta_0)) \). As an example of "success lowers prospects" consider random sampling without replacement from a population with \( N \) with \( K = \theta_0 N \) interested buyers. Consider \( \theta_0 \) such that \( \theta_0 N \) is an integer, and \( T < K = \theta_0 N \). These assumptions further imply that \( c_T \) in a "success raises prospects" environment has beta-binomial distribution with parameters \( (T, \lambda \theta_0, \lambda(1 - \theta_0)) \) and \( c_T \) in a "success lowers prospects" environment has hypergeometric distribution with parameters \( (T, N, \theta_0 N) \). The distribution parameters are set such that the prior means are the same in all three cases and \( \mathbb{E}[s_t] = \theta_0 \) for any \( t \), and \( \mathbb{E}[c_t] = T \theta_0 \). Note that all three distributions imply that random variables are exchangeable. Assume further that the cost of effort is constant, \( e_t = \bar{e} \).

Assume for now that the borrower has sufficient own funds to cover the break-even investment cost \( I^{FB} = \mathbb{E}[c_t] - T \bar{e} = T (\theta_0 - \bar{e}) \). This benchmark is helpful as we know that the borrower's utility from the project is maximized when he makes an effort at all dates, i.e., \( \hat{s}_{t+1} = s_{t+1} \) for every \( t = [0, \ldots, T - 1] \) and \( \hat{c}_T = c_T \). Assuming he has enough own funds simply guarantees that raising some external financing such that it induces optimal effort is always possible, while the borrower prefers to minimize the use of his own investment, due to lexicographic preferences.

Recall that under the optimal contract there is no need for own investments if the borrower operates in "success lowers prospects" environment or if potential sales are independent. When "success raises prospects", we can find from the assumed distribution and (25)\(^{17} \) that own investment under

\(^{17}\)Using (26) and (27), it holds that \( \Delta_t(c_t) = \frac{1}{\lambda \theta_0 + e_t} \) and \( \Phi_t = \bar{e} \frac{T-t}{\lambda \theta_0 + e_t} \). As \( \mathbb{E}[s_{t+1}|c_t] = \frac{\lambda \theta_0 + e_t}{\lambda + e_t} \), we obtain

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the optimal contract is

\[ A_0 = \bar{e} \left( \sum_{t=0}^{T-2} \frac{T - 1 - t}{\lambda + t} \right) \]

Consider first an equity contract. From earlier results, we already know that the simple equity contract is the optimal contract if potential sales are i.i.d. and own investment needed is \( A^{E,Ind}_0 = 0 \). Setting the incentive compatibility constraints (12) to bind, we obtain that \( \alpha^{Ind}_E = \bar{e}/\theta_0 \).

If there is learning, then all incentive compatibility constraints (12) must hold for the borrower to make an effort at all dates. Given that the equity contract is linear in sales,

\[ \alpha_E E[s_{t+1}|s_1, \ldots, s_t] \geq \bar{e} \]

must hold at all dates and under all sales revenues. This is achieved if this constraint is binding when the borrower’s expectations of the next sale are at their lowest. If "success raises prospects" the borrowers expectations are at their lowest when there have not been any sales until date \( T - 1 \).

This implies that

\[ \alpha^{SBS}_E = \frac{\bar{e}}{E[s_T|s_1 = 0, \ldots, s_{T-1} = 0]} = \frac{\bar{e} (\lambda + T - 1)}{\lambda \theta_0}. \]

If "success lowers prospects" then the borrower’s expectations are at their lowest when there has been a sale at every period until \( T - 1 \), this implies that

\[ \alpha^{SBF}_E = \frac{\bar{e}}{E[s_T|s_1 = 1, \ldots, s_{T-1} = 1]} = \frac{\bar{e} N - (T - 1)}{\theta_0 N - (T - 1)}. \]

In both cases the borrower’s own investment equals \( E[\hat{w}_E (c_T)] - T \bar{e} = \alpha_E E[c_t] - T \bar{e} \). Given the above, the own investment needed in "success raises prospects" and "success lowers prospects" environments are respectively

\[
A^{E,SBS}_0 = T \theta_0 \frac{\bar{e} (\lambda + T - 1)}{\lambda \theta_0} - T \bar{e} = T \bar{e} \frac{T - 1}{\lambda} > A_0
\]

\[
A^{E,SBF}_0 = T \theta_0 \bar{e} \frac{N - (T - 1)}{\theta_0 N - (T - 1)} - T \bar{e} = T \bar{e} \frac{(1 - \theta_0) (T - 1)}{N \theta_0 - (T - 1)} > 0.
\]

In both cases, the own investment needed is higher if there is more information that the borrower can learn after signing the contract (i.e., when parameters \( \lambda \) and \( N \) are lower).

\[
E[\Phi_t s_{t+1}] = E[E[\Phi_t s_{t+1}|c_t]] = E[\Phi_t E[s_{t+1}|c_t]] = \bar{e} T^{1-1}/(\lambda + t).
\]

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Second, consider simple debt contract. It is clear that incentive compatibility constraint (12) is hardest to satisfy at date $T - 1$, if there have been no sales until that point in time. This implies that no debt contract with face value $d > 1$ can incentivise the borrower to always make first best effort decisions. Any debt contract that can incentivise the borrower to make an effort at all dates must have $d < 1$ and satisfy (12) at $T - 1$ when $c_{T-1} = 0$. This implies that the probability of making just one sale at the last date must be sufficient to cover $d$, and the maximum face value of debt must satisfy

$$\Pr (s_{T-1} = 0 | c_{T-1} = 0) \mathbb{E} [s_T - d | c_{T-1} = 0, s_T - d \geq 0] = \bar{e}$$

As $s_T - d \geq 0$ only if $s_T = 1$, this simplifies to

$$(1 - d) \Pr (s_T = 1 | c_{T-1} = 0) = \bar{e}.$$

This highlights that debt gives poor incentives to continue after unlucky outcomes. Using the distributional assumptions, we find that $d$ is determined in the three cases as

$$d^{\text{Ind}} = 1 - \bar{e} \frac{1}{\theta_0},$$
$$d^{\text{SBS}} = 1 - \bar{e} \frac{\lambda + T - 1}{\lambda \theta_0},$$
$$d^{\text{SBF}} = 1 - \bar{e} \frac{N - (T - 1)}{\theta_0 N},$$

The borrower’s own investment is then given by the lenders break-even constraint as $I^{FB} = A_0^D = d + \Pr (c_T \leq d) \mathbb{E} [s_T - d | c_T \leq d] = d (1 - \Pr (c_T = 0))$, where $\Pr (c_T = 0) = (1 - \theta_0)^T$ if sales are independent, $\Pr (c_T = 0) = \frac{\Gamma (\lambda (1 - \theta_0) + T)}{\Gamma (\lambda (1 - \theta_0))} \frac{\Gamma (\lambda)}{\Gamma (\lambda + T)}$ if "success raises prospects" and

$$\Pr (c_T = 0) = \binom{N}{T}^{-1} \binom{\theta_0 N}{0} \binom{N (1 - \theta_0)}{T}$$

if "success lowers prospects", where $\Gamma (x)$ is the gamma function and $\binom{x}{y}$ is the binomial coefficient.

Given all this we can numerically compare these cases. Assume that $T = 12$, $I^{FB} = 5$, $\bar{e} = 1/12$, $\theta_0 = 0.5$, $N = 30$ and $\lambda = 4$ as in the examples considered in Section 3.4. The borrower’s own
investment needed to cover $I^{FB}$ is in Table 1.

<table>
<thead>
<tr>
<th></th>
<th>Optimal contract</th>
<th>Simple Equity</th>
<th>Simple Debt</th>
</tr>
</thead>
<tbody>
<tr>
<td>Independent</td>
<td>0</td>
<td>0</td>
<td>4.167</td>
</tr>
<tr>
<td>&quot;Success raises prospects&quot;</td>
<td>0.627</td>
<td>2.75</td>
<td>4.635</td>
</tr>
<tr>
<td>&quot;Success lowers prospects&quot;</td>
<td>0</td>
<td>1.375</td>
<td>4.026</td>
</tr>
</tbody>
</table>

Simple equity is the first best when sales are independent, but still requires substantial own investment from the borrower when sales reveal more information. As a ratio to investment cost, the potential gains from switching to optimal contract derived from simple equity can be substantial, as in Table 1. As shown in Section 3.4, a changing effort cost is an additional reason that would make simple equity suboptimal, even in the case of i.i.d. sales opportunities.

We can see that it is very difficult to maintain the borrower's incentives to always make an effort with a debt contract: the borrower would need to almost fully self-finance to be able to undertake a project with the highest optimal investment cost $I = I^{FB}$. Unlike equity, the own investment needed is substantial even if potential sales are i.i.d., and debt is particularly restrictive in a "success raises prospects" environment.

It was assumed here that the borrower has enough own funds and simply prefers not to use these. When $I < I^{FB}$, the above table also gives the maximum investment a borrower with no own funds can undertake while still having the highest possible utility. Such maximum investment cost is calculated by subtracting the values in the table from $I^{FB}$. For example, in the "success raises prospects" case, investment costs up to $5 - 0.627 = 4.373$ can be always covered via the optimal contract, while investment costs up to only $5 - 2.75 = 2.25$ and $5 - 4.635 = 0.365$ can be covered via equity and debt contract, respectively.

Borrowers that have higher investment costs could still get financing via debt or equity, but financing in such a case must always be costlier for them, which in turn reduces their expected utility from the project. To see this, consider the example of a debt contract. If a debt contract with face value $d \geq 1$ is feasible to sign, then the borrower and the lender both expect that the borrower will quit making an effort after observing some sufficiently bad sales history. Because the realized sales $\tilde{c}_T$ is a sum that is increasing in effort decisions, it follows that the distribution on
realized sales $\tilde{c}_T$ would be first order stochastically dominated by the distribution of $c_T$, hence the expected value of the project would be lower. It is then immediate from the lender’s break even constraint (2) that at any given wealth of the borrower and investment cost, the reward that the borrower receives is also lower, which reduces the borrower’s expected returns. It then follows that there exists a threshold investment level, lower than the one under the optimal contract, above which debt financing is impossible.

To illustrate this argument, consider a borrower with no own funds, i.e., $A = 0$ and with a project that can generate i.i.d. potential sales, each with probability 0.5, as above. Figure 4 plots the distribution of total realized sales, $\tilde{c}_T$, in the case that the borrower has low enough investment cost such that he chooses to make an effort each period (solid line) and when the borrower’s investment cost is at the threshold level where external financing is still possible (dotted line). From the above analysis, a borrower with investment cost up to $I = 0.83$ maintains his incentives to make an effort every date. By considering the possible investment costs and effort strategies, this simulation shows that the highest investment cost that can be covered via a debt contract is $I = 3.78$. In such a case the borrower stops making efforts if he does not achieve the following benchmarks: 1) at least one successful sale by date 4; 2) at least two successful sales by date 6; 3) at least three successful sales by date 8 and 4) at least four successful sales by date 10. The distribution of realized sales with $I = 0.83$ is the same as the distribution of potential sales, and it
first order stochastically dominates the realized sales distribution when investment cost is higher, in particular when it is at the threshold $I = 3.78$. First, this negatively affects the borrower’s utility as his project generates lower revenues in expectations. We can see from Table 1 that if the borrower would use optimal contract (or equity) he could cover this investment cost at no utility loss. Second, debt becomes more expensive because the borrower makes effort decisions frequently and cannot pre-commit to making an effort at each date and regardless of early sales outcomes. A discount bond that covers investment cost $I = 3.78$ must have a face value 4.51, i.e., interest rate of 19%. Notice that if the borrower could somehow pre-commit to make efforts at each date, the distribution of total sales would be given by the solid line on Figure 4, with such distribution, the face value of a discount bond that covers the same investment cost would be 3.87, i.e., interest rate of 2%.

This also explains why the results in the paper are different from Innes (1990) and other static models. In static settings, the borrowers essentially choose between the distributions at different effort costs. If a borrower would be given the choice between distributions represented by the solid and dotted line on Figure 4, he could be incentivised to choose the first order stochastically dominant line solid line with a debt contract. However, this argument does not hold when a borrower cannot realistically commit to continue making efforts after observing low early sales revenues: he knows that most of the future revenues will in such a case go to the lender. Anticipating this, a rational lender would only agree to a debt contract which offers high interest rate as dynamic moral hazard increases default risk. Such an incentive problem would be amplified in a "success raises prospects" environment where the borrower further becomes more pessimistic about future selling prospects following low early sales outcomes.

5 Increasing frequency of learning and effort decisions

In order to explore the effect of more frequent decision making, let us consider $T = 12$, i.e., one widget can be sold every "month", and we consider a "year". Consider that the total cost of effort is fixed, and there were three possibilities: 1) the borrower commits to effort in January; 2) the borrower chooses effort in "January" and "July"; 3) the borrower chooses effort every month.
Assume that the total effort cost is the same under these scenarios. It is straightforward that under the January commitment, all contracts are equivalent (as shown by of Modigliani-Miller), and own investment need is zero. If the borrower commits "twice" in the year then:

Table 2: Own investment needed for first best effort when $I = I^{FB} = 5$

<table>
<thead>
<tr>
<th></th>
<th>Optimal contract</th>
<th>Simple Equity</th>
<th>Simple Debt</th>
</tr>
</thead>
<tbody>
<tr>
<td>Independent</td>
<td>0</td>
<td>0</td>
<td>2.07</td>
</tr>
<tr>
<td>&quot;Success raises prospects&quot;</td>
<td>0.25</td>
<td>1.50</td>
<td>3.91</td>
</tr>
<tr>
<td>&quot;Success lowers prospects&quot;</td>
<td>0</td>
<td>0.33</td>
<td>1.40</td>
</tr>
</tbody>
</table>

The results in the case of the borrower's "monthly" commitment are in Table 1. Comparing Table 1 and Table 2, highlights the following patterns. The optimal contract in the environment where "success lowers prospects" leads to the first best outcome regardless of the frequency of decision making. In the "success raises prospects" environment, the borrowers would be better off if they could somehow commit to make decisions less frequently. Overall, we can see that the difference between optimal contract, simple equity and debt becomes increasingly important when the frequency of decision making increases and there is more to learn from sales history, making these traditional contracts increasingly restrictive.

The trends towards better capabilities to analyze data, and more frequent decision making, would make financial contracting more difficult in "success raises prospects" environment also under the optimal contract. It is not obvious how one could slow down learning and decision making. There is a somewhat hypothetical possibility as different blockchain transactions are confirmed at different frequencies, e.g., Bitcoin transactions are confirmed less frequently than Litecoin transactions. Suppose a borrower sells a good that can only be bought online in exchange of some cryptocurrency payment. The analysis here suggests that firms operating in a "success raises prospects" environment may benefit from restricting the means of payments to be one that confirms transactions in bulk, and perhaps slower.

Overall this analysis suggests that the possibility to write flexible and self-adjusting "smart contracts" could become increasingly valuable over time. It also shows that it is necessary to identify in the type of environment the firm operates to understand the risks lenders face.
6 Conclusion

This paper explored how the blockchain environment could change the borrowing-lending relationship. Blockchain brings efficiency gains such as costless verification and automatic enforcement, and all possible financing contracts would benefit from these gains. At the same time, the blockchain technology is emerging in an environment where agents can learn from their cash flow data (due to for example advances in computer power and big data analytics), update their beliefs about future prospects and make decisions faster. If anything, blockchain is likely to facilitate such faster learning and decision making.

I show that an ideal borrowing contract in this environment would be an automatically adjusting profit sharing agreement, and for this reason reliable timestamps of records is an important feature of blockchain records. How the optimal contract should adjust depends on the fundamental characteristics of the market where the firm is selling its products. Dynamics differ depending on whether the firm operates in a market where better than expected sales outcomes are associated with expected higher future demand or lower future demand. With such a contract, agents are able to test out their ideas with minimal need for own funds, and the economic outcomes are closer to those of a frictionless market.

An environment with frequent decisions is particularly bad for standard debt contracts because it gives poor incentives for the borrower to make effort after unlucky outcomes. If one were to be restricted to use only debt and simple equity contracts, then equity would dominate debt contracts. When compared to the optimal contract, the equity contract is almost as good when sales do not reveal a lot of new information, and noticeably worse if sales data are very informative about future prospects. Equity contracts also do not optimally adjust to the dynamics of effort cost.

It can be argued that even without blockchain, debt contracts could become worse in the future, given that there is a trend towards shorter contracts, and improving capabilities for faster data analysis and decision making. If the negative effects of frequent decision making are foreseen by lenders, it can make debt contracts more expensive for borrowers, and if not foreseen, it could increase the frequency of defaults. Perhaps the most compelling reason for using debt contracts has
been the costs associated with verifying cash flow and optimizing monitoring efforts. As verification costs are becoming smaller, it could encourage the creation of new types of financing contracts that are "equity-like" and ideally more flexible than equity; the recent emergence and success of equity crowdfunding and initial coin offerings seems to be a movement in this direction.

My setting assumes that there is no information asymmetry at the time of contracting and the potential sales distribution is known to both parties. One would obtain the same results if the borrower and lender agreed on how the borrower’s incentives are likely to change following realized sales, even if the prior joint probability distribution is subjective. It is also plausible that the borrower could have superior information about his target market at the time of contracting. While this could lead to additional difficulties, there are other contemporary developments that would enable the contracting parties to mitigate this information asymmetry before contracting, e.g., reward-based crowdfunding, which can be used to test the market and produce public information about the preferences of target consumers (see e.g., Chemla and Tinn 2019).

More broadly this paper contributes to the financial contracting literature by highlighting the effects of frequent learning and decision making on incentives, and characterizes how these effects depend on general distributional properties. I showed that in a realistic environment where borrowers effort choices are dynamic, the sequence of cash flow arrivals is important and this explains why my findings are in contrast to some influential existing papers on this topic.

My analysis suggests that the more important benefit of blockchain and "smart financing contracts" could be the possibility to design new, more flexible, types of contracts rather than the possibility to manage and seize collateral more easily, an argument often emphasized in this context. My model shows that there is less need for own investment and collateral under the optimal self-adjusting contract, which makes external financing noticeably more accessible.
A Appendix

A.1 Proof of Proposition 3

Let \( W_0 = \sim w(0; 0) \sim w(0; 1) \sim w(1; 0) \sim w(1; 1) \) be the vector of the borrower’s payoffs, \( P_0 = [ p(0, 0) \ p(0, 1) \ p(1, 0) \ p(1, 1) ] \) be the vector of associated probabilities, and \( b' = [ e_0 + e_1 \ e_0 \ e_1 \ e_1 ] \) be the right hand side of constraints. Defining also a matrix of coefficients

\[
G = \begin{bmatrix}
  p(0, 0) & -p(1, 0) & -\frac{p(0, 1)}{p_1(0)} & 0 \\
  p(0, 1) & -p(1, 1) & \frac{p(0, 1)}{p_1(0)} & 0 \\
  p(1, 0) & p(1, 0) & 0 & -\frac{p(1, 1)}{p_1(1)} \\
  p(1, 1) & p(1, 1) & 0 & \frac{p(1, 1)}{p_1(1)} 
\end{bmatrix}
\]

we can express the primal problem (15)-(17) as

\[
\min \mathbb{E}[\sim w(s_1; s_2)] = W'P, \text{ subject to } W'G \geq b, W \geq 0
\]  

(30)

Defining the vector \( X' = [ x_p \ x_0 \ x_1^0 \ x_1^1 ] \) as corresponding shadow prices of constraints (16)-(17), i.e., \( x_p \) corresponds to the participation constraint, \( x_0 \) to date 0 effort incentive compatibility constraint and \( x_1^0 \) to date 1 effort incentive compatibility constraint when date 1 sale outcome is \( s_1 \), there is a dual maximum problem

\[
\max V = b'X, \text{ subject to } GX \leq P, X \geq 0.
\]  

(31)

The primal and dual are linked via the Duality Theorem (Theorem 5.1 in Chvátal, 1983) and if one has a solution, then so does the other one, and values are the same.

Let us solve the dual maximum problem. The dual maximum problem simplifies to

\[
\max V = e_0 (x_p + x_0) + e_1 (x_1^0 + x_1^1 + x_p),
\]  

(32)

subject to

\[
x_p \leq 1;
\]  

(33)

\[
x_p + x_0 \leq 1;
\]  

\[
x_1^0 \leq p_1(0)(1 - x_p) + \frac{p(1, 1)p_1(0)}{p(0, 1)}x_0;
\]  

\[
x_1^1 \leq p_1(1)(1 - x_p - x_0);
\]  

\[
x_p, x_0, x_1^0, x_1^1 \geq 0
\]
Because the objective function (32) is increasing in $x_0^1$ and $x_1^1$, the third and the fourth inequalities in (33) must be binding. Using this in (32), the problem simplifies to

$$\max V = e_0 (x_p + x_0) + e_1 \left( x_p + (1 - x_p) + x_0 p_1 (0) \left( \frac{p(1,1)}{p(0,1)} - \frac{p_1(1)}{p_1(0)} \right) \right)$$

subject to $x_p \leq 1$, $x_p + x_0 \leq 1$, $x_p, x_0 \geq 0$. It then follows that the sign of $p(1,1)p(0,0) - p(1,0)p(0,1)$ determines the maximizing values of $x_0$ and $x_p$, and hence also the maximizing values of $x_0^1$ and $x_1^1$.

If $p(1,1)p(0,0) > p(0,1)p(1,0)$ then from (34) the solution of the dual maximum problem is $x_p = 0$, $x_0 = 1$, $x_0^1 = p_1(0) \left( 1 + \frac{x(1,1)}{p(0,1)} \right)$, $x_1^1 = 0$, and the maximized value is

$$V = e_0 + e_1 + e_1x_0 \frac{p(1,1)p(0,0) - p(1,0)p(0,1)}{p(0,1)}$$

Given this solution, complementary slackness (see e.g., Theorem 5.3 in Chvátal, 1983) then implies that $\tilde{w}(0,0) = 0$, the participation constraint need not be binding and the first two incentive compatibility constraints in (3) must be binding. This, and the observation that the objective function of primal problem (30) is minimized when $\tilde{w}(1,1)$ is at its lowest value, implies that all incentive compatibility constraints are binding under the optimal solution. Solving that system gives (18). Plugging this solution in the objective function of the primal minimization problem (30) verifies that $E[\tilde{w}(s_1,s_2)] = V$, as implied by the Duality Theorem.

If $p(1,1)p(0,0) < p(0,1)p(1,0)$ then from (34) $x_p = 1$, $x_0 = 0$, $x_0^1 = 0$, and $x_1^1 = 0$, and the value

$$V = e_0 + e_1$$

Complementary slackness now implies that $\tilde{w}(0,0) = 0$, the participation constraint must be binding, and incentive compatibility constraints do not need to be binding. The Duality Theorem in that case further implies that $E[\tilde{w}(s_1,s_2)] = V$, which can be confirmed by plugging the solution (20) in the primal minimization problem. Note that there can be other optimal solutions of the primal maximum problem, but the presented solution is the only one that holds for any values of $e_0, e_1 \geq 0$, which satisfy 14.
If \( p(1,1)p(0,0) = p(0,1)p(1,0) \), then the two contracts are equivalent.\(^{19}\) This proves the optimality of the solution in Proposition 3.

### A.2 Proof of Propositions 5 and 7

The solution of the two date problem, suggests that solution of the primal minimization problem, given by (11) and (3), takes the form described in the propositions 5 and 7. In order to prove optimality, we again benefit from the duality and complementary slackness theorems. Denote \( x_p \) the shadow price of the participation constraint, and \( x_t^{s_1...s_T} \) the shadow price of the incentive compatibility constraint following realization history \( s_1, ..., s_t \). The dual maximization problem is

\[
\begin{align*}
\text{max } V &= x_p \sum_{t=0}^{T-1} e_t + e_0 x_0 + ... + e_T \sum_{s_1=0}^{1} ... \sum_{s_{T-1}=0}^{1} x_t^{s_1...s_T} + ... + e_{T-1} \sum_{s_1=0}^{1} ... \sum_{s_{T-2}=0}^{1} x_T^{s_1...s_T} \\
\text{subject to } \quad p(s_1, ..., s_T)x_p + (-1)^{1-s_1}p(1, s_2, ..., s_T)x_0 + ... + \\
&\quad (-1)^{1-s_t} \frac{p(s_1, ..., s_{t-1}, 1, s_{t+1}, ..., s_T)}{p(s_1, ..., s_{t-1})} x_{t-1}^{s_1...s_{t-1}} + ... + \\
&\quad (-1)^{1-s_T} \frac{p(s_1, ..., s_{T-1}, 1)}{p(s_1, ..., s_{T-1})} x_{T-1}^{s_1...s_{T-1}} \leq p(s_1, ..., s_T)
\end{align*}
\]

and the non-negativity constraints \( x_t^{s_1...s_t} \geq 0 \) for every \( t = 1, ..., T - 1 \).

By Theorem 5.1-5.3 in Chvátal (1983) the proof that the solutions in Propositions 5 and 7 are optimal requires checking that these solutions are feasible, using then complementary slackness and confirming that the implied solution of the dual maximum is feasible as well.

**Proof of Proposition 5.**

*Step 1) Proof that the primal guess is feasible.* First, we can prove by induction that all incentive compatibility constraints hold with equality under the guessed solution. It is clear that this is true if \( t = T - 1 \): using (23) in (12) gives

\[
\mathbb{E} \left[ \frac{e_{T-1} e_T}{\mathbb{E} [s_T | s_1, ..., s_{T-1}]} | s_1, ..., s_{T-1} \right] = e_{T-1}.
\]
with equality at some period \( t + 1 \), i.e.,

\[
E \left[ \bar{w} (s_1, \ldots, s_{t+1}, s_{t+2}, s_{t+3}, \ldots, s_T) - \bar{w} (s_1, \ldots, s_{t+1}, 0, s_{t+3}, \ldots, s_T) \mid s_1, \ldots, s_{t+1} \right] = e_{t+1}. \quad (36)
\]

It then follows that at \( t \)

\[
E \left[ \bar{w} (s_1, \ldots, s_t, s_{t+1}, s_{t+2}, \ldots, s_T) - \bar{w} (s_1, \ldots, s_t, 0, s_{t+2}, \ldots, s_T) \mid s_1, \ldots, s_t \right]
= E \left[ E \left[ \bar{w} (s_1, \ldots, s_t, s_{t+1}, s_{t+2}, \ldots, s_T) - \bar{w} (s_1, \ldots, s_t, 0, s_{t+2}, \ldots, s_T) \mid s_1, \ldots, s_t \right] \right]
= E \left[ \bar{w} (s_1, \ldots, s_{t+1}, 0, s_{t+3}, \ldots, s_T) \mid s_1, \ldots, s_t \right] + e_{t+1} - E \left[ \bar{w} (s_1, \ldots, s_t, 0, s_{t+2}, \ldots, s_T) \mid s_1, \ldots, s_t \right]
= \alpha_{t+1} (s_1, \ldots, s_t) E [s_{t+1} | s_1, \ldots, s_t] - \alpha_{t+2} (s_1, \ldots, s_t, 0) E [s_{t+2} | s_1, \ldots, s_t] + e_{t+1} +
\begin{align*}
& E \left[ (\alpha_{t+3} (s_1, \ldots, s_t, s_{t+1}, 0) - \alpha_{t+2} (s_1, \ldots, s_t, 0, s_{t+2})) s_{t+3} \mid s_1, \ldots, s_t \right] + \ldots + \\
& E \left[ (\alpha_T (s_1, \ldots, s_{t+1}, 0, s_{t+3}, \ldots, s_{T-1}) - \alpha_T (s_1, \ldots, s_t, 0, s_{t+2}, \ldots, s_{T-1})) s_T \mid s_1, \ldots, s_t \right]
= \alpha_{t+1} (s_1, \ldots, s_t) E [s_{t+1} | s_1, \ldots, s_t] - \alpha_{t+2} (s_1, \ldots, s_t, 0) E [s_{t+2} | s_1, \ldots, s_t] + e_{t+1} = e_t
\end{align*}
\]

where the first equality follows from the law of iterated expectations, the second equality uses (36), the third equality uses (23), the fourth follows from exchangeability, which implies that \( \alpha_{t+k} (s_1, \ldots, s_t, s_{t+1}, 0, s_{t+3}, \ldots, s_{t+k-1}) = \alpha_{t+k} (s_1, \ldots, s_t, 0, s_{t+2}, s_{t+3}, \ldots, s_{t+k-1}) \) for any \( k = 3, \ldots, T - t - 1 \), and the fifth follows from (24) and exchangeability, which implies that \( E [s_{t+1} | s_1, \ldots, s_t] = E [s_{t+2} | s_1, \ldots, s_t] \). This proves that all incentive compatibility constraints are binding under the guessed solution of the primal minimization problem.

In order to confirm that the participation constraint (11) holds, note that \( \Delta_t (s_1, \ldots, s_t) \), defined in (27), is non-negative as \( \Delta_t (s_1, \ldots, s_t) \geq 0 \) if \( E [s_{t+2} | s_1, \ldots, s_t] \geq E [s_{t+2} | s_1, \ldots, s_t, 0] \Leftrightarrow E [s_{t+2} | s_1, \ldots, s_t, 0] \leq E [s_{t+2} | s_1, \ldots, s_t, 1] \). The latter holds because

\[
\frac{p (s_1, \ldots, s_t, 0, 1)}{p (s_1, \ldots, s_t, 0)} \leq \frac{p (s_1, \ldots, s_t, 1)}{p (s_1, \ldots, s_t)} \Leftrightarrow p (s_1, \ldots, s_t, 0, 0) p (s_1, \ldots, s_t, 1, 1) \geq p (s_1, \ldots, s_t, 1, 0) p (s_1, \ldots, s_t, 0, 1)
\]

and a "success raises prospects" environment assumes stochastic affiliation. Furthermore, the inequality is strict unless potential sales are i.i.d. It then follows from (23), (24) and the definitions
(26) and (27) that the participation constraint holds, and is not-binding (unless potential sales are i.i.d).

**Step 2) Proof of optimality.** Given the results in step 1, complementary slackness implies that \( x_p = 0 \) (as the participation constraint is not binding) and constraints (35) must hold with equality whenever at least one \( s_t \neq 0 \) (as \( \tilde{w}(s_1, ..., s_T) > 0 \) for all sequences of \( s_1, ..., s_T \) except the sequence of zeros only).

To prove optimality, we need to that the solution of the dual maximization problem under these constraints is non-negative. To shorten the notation, define \( z_t(s_1, ..., s_t) = \frac{x_{s_1}^{s_1}...s_t}{p(s_1, ..., s_T)} \), which has the same sign as \( x_{s_1}^{s_1}...s_t \). Hence it needs to be proven that \( z_t(s_1, ..., s_t) \geq 0 \) for any \( t = 0, ..., T - 1 \). We already know from Proposition 3 that it \( T = 2 \) then there is a feasible solution of the dual problem, and we can extend the proof to \( T > 2 \) by induction.

Suppose that there exists a feasible solution \( z_0, z_1(s_1), ..., z_{\tau - 1}(s_1, ..., s_{\tau - 1}) \geq 0 \) when \( T = \tau \), which satisfies the complementary slackness conditions, i.e., for all \( t = 0, ..., \tau - 1 \) it holds that

\[
(-1)^{1-s_1}p(1, s_2, ..., s_{\tau})z_0 + ... +
\]

\[
(-1)^{1-s_t}p(s_1, ..., s_{t-1}, 1, s_{t+1}, ..., s_{\tau})z_{t-1}(s_1, ..., s_{t-1}) + ... +
\]

\[
(-1)^{1-s_{\tau}}p(s_1, s_2, ..., s_{\tau-1}, 1)z_{\tau-1}(s_1, ..., s_{\tau-1}) \leq (=) p(s_1, s_2, ..., s_{\tau}),
\]

where the weak inequality holds for any \( s_t \) and the equality holds whenever at least one \( s_t \neq 0 \). To prove optimality, it must then follow that there also exists a feasible solution \( z_{\tau}(s_1, ..., s_{\tau}) \geq 0 \) when \( T = \tau + 1 \). As by complementary slackness, it then follows that the equivalent to (37) holds when \( T = \tau + 1 \) and as \( s_{\tau+1} = \{0, 1\} \), we must show that

\[
(-1)^{1-s_1}p(1, s_2, ..., s_{\tau}, 0)z_0 + ... +
\]

\[
(-1)^{1-s_{\tau}}p(s_1, s_2, ..., s_{\tau-1}, 1, 0)z_{\tau-1}(s_1, ..., s_{\tau-1}) +
\]

\[
(-1)p(s_1, s_2, ..., s_{\tau-1}, s_{\tau}, 1)z_{\tau}(s_1, ..., s_{\tau}) \leq (=) p(s_1, s_2, ..., s_{\tau}, 0),
\]
where equality holds as long as at least one \( s_t \neq 0 \) in set \( \{s_1, \ldots, s_\tau\} \), and

\[
(-1)^{1-s_1} p(1, s_2, \ldots, s_\tau, 1) z_0 + \ldots + \\
(-1)^{1-s_\tau} p(s_1, s_2, \ldots, s_{\tau-1}, 1, 1) z_{\tau-1} (s_1, \ldots, s_{\tau-1}) + \\
(+1) p(s_1, s_2, \ldots, s_{\tau-1}, s_\tau, 1) z_\tau (s_1, \ldots, s_\tau) = p(s_1, s_2, \ldots, s_\tau, 1)
\]

for any realizations of \( s_t \in \{s_1, \ldots, s_\tau\} \). Notice that adding up (38) and (39) gives (37). This implies that the solutions \( z_t (s_1, \ldots, s_t) \) for \( t = 0, \ldots, \tau - 1 \) do not change when we increase \( T \) from \( \tau \) to \( \tau + 1 \).

What is left to be shown is that \( z_\tau (s_1, \ldots, s_\tau) \geq 0 \) for any sequence \( s_1, \ldots, s_\tau \).

First, suppose that \( s_\tau = 0 \). If \( s_t = 0 \) also for any \( t = 1, \ldots, s_{\tau-1} \), then (39) implies that

\[
z_\tau (0, \ldots, 0) = 1 + p(1,0,\ldots,0,1) z_0 + \ldots + p(0,\ldots,0,1) z_{\tau-1} (0, \ldots, 0) > 0,
\]

As \((-1)^{1-0} = -1\), (38) is satisfied as well, and does not need to be binding.

If \( s_\tau = 0 \) and at least one \( s_t \neq 0 \) in set \( \{s_1, \ldots, s_{\tau-1}\} \) then (38) must hold with equality. By the assumption of existence of a feasible solution for \( T = \tau \), exchangeability (which implies that \((-1)^{1-0} \frac{p(s_1, s_2, \ldots, s_{\tau-1}, 1, 0)}{p(s_1, s_2, \ldots, s_{\tau-1}, 1)} = -1\)) and (38), it follows that

\[
z_\tau (s_1, \ldots, s_{\tau-1}, 0) = \\
(-1)^{1-s_1} \left( \frac{p(1, s_2, \ldots, s_{\tau-1}, 0, 0)}{p(s_1, s_2, \ldots, s_{\tau-1}, 1, 0)} - \frac{p(1, s_2, \ldots, s_{\tau-1}, 0)}{p(s_1, s_2, \ldots, s_{\tau-1}, 1)} \right) z_0 + \ldots + \\
(-1)^{1-s_{\tau-1}} \left( \frac{p(s_1, s_2, \ldots, s_{\tau-1}, 0, 0)}{p(s_1, s_2, \ldots, s_{\tau-1}, 1, 0)} - \frac{p(s_1, s_2, \ldots, s_{\tau-1}, 0)}{p(s_1, s_2, \ldots, s_{\tau-1}, 1)} \right) z_{\tau-2} (s_1, \ldots, s_{\tau-2}) + \\
\left( \frac{p(s_1, s_2, \ldots, s_{\tau-1}, 0, 0)}{p(s_1, s_2, \ldots, s_{\tau-1}, 1, 0)} - \frac{p(s_1, s_2, \ldots, s_{\tau-1}, 0)}{p(s_1, s_2, \ldots, s_{\tau-1}, 1)} \right) z_\tau (s_1, \ldots, s_\tau, 0)
\]

Notice that exchangeability implies that

\[
(-1)^{1-s_t} \left( \frac{p(s_1, s_2, \ldots, s_{t-1}, 1, s_t+1+\ldots+s_{\tau-1}, 0, 0)}{p(s_1, s_2, \ldots, s_{t-1}, s_t+1+\ldots+s_{\tau-1}, 1, 0)} - \frac{p(s_1, s_2, \ldots, s_{t-1}, 1, s_t+1+\ldots+s_{\tau-1}, 0)}{p(s_1, s_2, \ldots, s_{t-1}, s_t+1+\ldots+s_{\tau-1}, 1)} \right) = 0 \text{ whenever } s_t = 0.
\]

We can then simplify

\[
z_\tau (s_1, \ldots, s_{\tau-1}, 0) = \\
\left( \frac{p(s_1, s_2, \ldots, s_{\tau-1}, 0, 0)}{p(s_1, s_2, \ldots, s_{\tau-1}, 1, 0)} - \frac{p(s_1, s_2, \ldots, s_{\tau-1}, 0)}{p(s_1, s_2, \ldots, s_{\tau-1}, 1)} \right) (s_1 z_0 + \ldots s_{\tau-2} z_{\tau-1} (s_1, \ldots, s_{\tau-1}) - 1) \cdot
\]

As by the solution of \( T = 2 \) and (40) \( z_0 \geq 1, z_1 (0) \geq 1, z_2 (0, 0) \geq 1 \) etc., it holds that \((s_1 z_0 + \ldots s_{\tau-2} z_{\tau-1} (s_1, \ldots, s_{\tau-1}) - 1) \geq 0\). Furthermore, "success raises prospects"/stochastic af-
filiation implies that
\[ \frac{p(s_1, s_2, \ldots, s_{r-1}, 0)}{p(s_1, s_2, \ldots, s_{r-1}, 1)} \geq \frac{p(s_1, s_2, \ldots, s_{r-1}, 0)}{p(s_1, s_2, \ldots, s_{r-1}, 1)} \iff \frac{p(s_1, s_2, \ldots, s_{r-1}, 1)}{p(s_1, s_2, \ldots, s_{r-1}, 0)} \geq \frac{p(s_1, s_2, \ldots, s_{r-1}, 1)}{p(s_1, s_2, \ldots, s_{r-1}, 0)}. \]

This proves that \( z_r(s_1, \ldots, s_{r-1}, 0) \geq 0. \)

If \( s_r = 1 \), then (39) and (37), imply that
\[ z_r(s_1, \ldots, s_{r-1}, 1) = \]
\[ (-1)^{1-s_1} \left( \frac{p(s_1, s_2, \ldots, s_{r-1}, 1)}{p(s_1, s_2, \ldots, s_{r-1}, 1)} - \frac{p(s_1, s_2, \ldots, s_{r-1}, 1)}{p(s_1, s_2, \ldots, s_{r-1}, 1)} \right) z_0 + \ldots + \]
\[ (-1)^{1-s_{r-1}} \left( \frac{p(s_1, s_2, \ldots, s_{r-1}, 1)}{p(s_1, s_2, \ldots, s_{r-1}, 1)} - \frac{p(s_1, s_2, \ldots, s_{r-1}, 1)}{p(s_1, s_2, \ldots, s_{r-1}, 1)} \right) z_{r-1}(s_1, \ldots, s_{r-1}), \]

which is non-negative as \((-1)^{1-s_t} \left( \frac{p(s_1, s_2, \ldots, s_{r-1}, 1)}{p(s_1, s_2, \ldots, s_{r-1}, 1)} - \frac{p(s_1, s_2, \ldots, s_{r-1}, 1)}{p(s_1, s_2, \ldots, s_{r-1}, 1)} \right) = 0 \) if \( s_t = 1 \), and \((-1)^{1-s_t} \left( \frac{p(s_1, s_2, \ldots, s_{r-1}, 1)}{p(s_1, s_2, \ldots, s_{r-1}, 1)} - \frac{p(s_1, s_2, \ldots, s_{r-1}, 1)}{p(s_1, s_2, \ldots, s_{r-1}, 1)} \right) \geq 0 \) if \( s_t = 0 \) due to stochastic affiliation. This proves that \( z_r(s_1, \ldots, s_{r-1}, 0) \geq 0 \), which completes the proof of optimality of the primal guess.

The value of the primal and the dual, and the implied information rents and own investment needed for \( I^{FB} \) follow from (23) and (24).

**Proof of Proposition 7.** Using the law of iterated expectations, (28) and (29), it is easy to confirm that the participations constraint (11) holds and is binding. The guessed solution is feasible as
\[ \mathbb{E}[\tilde{w}(s_1, \ldots, s_t, s_{t+1}, s_{t+2}, \ldots, s_T) - \tilde{w}(s_1, \ldots, s_t, 0, s_{t+2}, \ldots, s_T) | s_1, \ldots, s_t] = \]
\[ c_t + c_{t+1} \mathbb{E}\left[ \frac{s_t+2}{\mathbb{E}[s_{t+2}|s_1, \ldots, s_{t+1}]} - \frac{s_t+2}{\mathbb{E}[s_{t+2}|s_1, \ldots, s_{t+1}, s_{t+2}, \ldots, s_T]} s_1, \ldots, s_t \right] + \]
\[ + \ldots + c_{T-1} \mathbb{E}\left[ \frac{s_T}{\mathbb{E}[s_T|s_1, \ldots, s_{t+1}, s_{t+2}, \ldots, s_T]} - \frac{s_T}{\mathbb{E}[s_T|s_1, \ldots, s_{t+1}, s_{t+2}, \ldots, s_T]} s_1, \ldots, s_T \right] \]

which satisfies (3) as by the law of total expectations it holds for any \( k = 2, \ldots, T - t \) that
\[ \mathbb{E}\left[ \frac{s_{t+k}}{\mathbb{E}[s_{t+k}|s_1, \ldots, s_{t+k-1}]} - \frac{s_{t+k}}{\mathbb{E}[s_{t+k}|s_1, \ldots, s_{t+k-1}, s_{t+1}, \ldots, s_{t+k-1}]} \right] = \]
\[ = \mathbb{E}\left[ \frac{s_{t+k}}{\mathbb{E}[s_{t+k}|s_1, \ldots, s_{t+k-1}]} - \frac{s_{t+k}}{\mathbb{E}[s_{t+k}|s_1, \ldots, s_{t+k-1}, s_{t+1}, \ldots, s_{t+k-1}]} \right] \]
\[ \mathbb{E}[s_{t+2}|s_1, s_t, s_{t+1} + \ldots + s_{t+k}] = \mathbb{E}[s_{t+2}|s_1, s_t, 0, s_{t+2} + \ldots + s_{t+k}] \] if \( s_{t+1} = 0 \), and by the definition of a "success lowers prospects" environment in (1)

\[ \mathbb{E}[s_{t+2}|s_1, s_t, 1, s_{t+2} + \ldots + s_{t+k}] \leq \mathbb{E}[s_{t+2}|s_1, s_t, 0, s_{t+2} + \ldots + s_{t+k}] \]. This proves that the guessed solution (28) and (29) is feasible. The proof of optimality is trivial as \( x_p = 1 \) and \( x_0 = \ldots = x_t^{s_1 \ldots s_t} = 0 \) for any \( t = 1, \ldots, T - 1 \) clearly satisfies (35) and the primal and the dual have the same value.

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