Abstract

Embodied technological change (or investment-specific technological change, or IST), reflected in the declining price of new investment goods, has been recognized as an important driver of long-run economic growth (Greenwood et al. [1997]), business cycle fluctuations (Greenwood et al. [2000], Fisher [2006]), and asset prices (Papanikolaou [2011], Kogan and Papanikolaou [2014]). Most studies measure the rate of embodied progress using a price index for investment goods. There is a large heterogeneity among capital types (such as the different types of equipment and of structures) in the behavior of these prices. In particular, the price declines are concentrated in very few categories (computers and communication equipment). The aggregate importance of these declining prices hence depends importantly on how one weights the different capital types. The overwhelming majority of the studies use investment-flow weighted indices, such as the Bureau of Economic Analysis (BEA) national income account deflators. These indices weight each category according their share of investment spending. We show, however, that a standard model implies a different aggregation scheme: one should weight the categories according to their share in the rental cost of capital, as this proxies their impact on the marginal product. (This is akin to the measure of capital services produced by the Bureau of Labor Statistics (BLS) productivity program, based on the work of Jorgensen.) We use this new measure of investment-specific technological change to revisit its contribution to long-run growth, to business cycles, and to asset prices. We find that the long-run contribution is smaller than suggested by Greenwood et al. [1997]. We also study the role of capital heterogeneity in accounting for the decline of investment as studied by Gutierrez and Philippon [2017].

1 Introduction

2 Theory

Basic production framework

Suppose that the consumption good, which has price normalized to 1, is produced with by a competitive representative firm with a constant-returns-to-scale technology $F$ using labor $L$ and $n$ types $K_1, \ldots, K_n$ of capital as inputs, with potentially time-varying TFP $A_t$:

$$Y_t = A_t F(L_t, K_{1t}, \ldots, K_{nt})$$

Suppose that there is a a technology to convert $p_{it}$ units of the consumption good to 1 unit of investment in capital type $i$ (this is the price of type $i$ investment). We will write the model in continuous time, and have
capital depreciate at type-specific rate $\delta_i$. We can then write:

$$Y_t = C_t + \sum_{i=1}^{n} p_{it} I_{it}$$

$$= C_t + \sum_{i=1}^{n} p_{it}(\dot{K}_{it} + \delta K_{it})$$

(2)

where $\dot{K}_{it}$ denotes the time derivative of capital $K_{it}$.

Equations (1) and (2) summarize the production side.

**Price changes, user cost of capital, and production**

Let $\dot{p}_{it}$ denote the time derivative of $p_{it}$. Also assume there is a real interest rate $r_t$ that is the cost of financing all capital types. Then type $i$ has user cost, which we will denote by $R_{it}$, of

$$R_{it} = p_{it} \left( r_t + \delta_i - \frac{\dot{p}_{it}}{p_{it}} \right)$$

(3)

and it is demanded by the firm according to the first-order condition

$$A_t F_{K,t} = R_{it}$$

(4)

We can decompose gross income $Y_t$ at time $t$ into its recipients (equalizing the wage $W_t = A_t F_L$ and marginal product of labor):

$$Y_t = W_t L_t + \sum_{i=1}^{n} \left( r_t + \delta_i - \frac{\dot{p}_{it}}{p_{it}} \right) p_{it} K_{it}$$

(5)

and, moving terms to the left, can write either income net of capital depreciation

$$Y_t - \sum_{i=1}^{n} \delta_i p_{it} K_{it} = W_t L_t + \sum_{i=1}^{n} \left( r_t - \frac{\dot{p}_{it}}{p_{it}} \right) p_{it} K_{it}$$

(6)

or income net of capital depreciation and capital gains

$$Y_t - \sum_{i=1}^{n} \delta_i p_{it} K_{it} + \sum_{i=1}^{n} \dot{p}_{it} K_{it} = W_t L_t + \sum_{i=1}^{n} r_t p_{it} K_{it}$$

(7)

**Various (Divisia) aggregate price indices**

We can now define several aggregate price indices. For convenience, write $I_{it}^{\text{car}} \equiv \sum_{i=1}^{n} p_{it} I_{it}$ and $K_{i}^{\text{car}} \equiv \sum_{i=1}^{n} p_{it} K_{it}$ to be investment and capital aggregated in current consumption cost units.

- The most common aggregate price index for capital, the *gross investment-weighted* price index $p^I$, is

$$\frac{p^I_t}{p^I_t} = \sum_{i=1}^{n} \left( \frac{p_{it} I_{it}}{I_{it}^{\text{car}}} \right) \frac{\dot{p}_{it}}{p_{it}}$$

(8)

This can be interpreted as the rate of change in cost (in consumption units) of the current real investment bundle.
Another aggregate price index is the stock-weighted price index \( p^K \),
\[
\frac{\dot{p}^K_t}{p^K_t} = \sum_{i=1}^{n} \left( \frac{p_i K_{it}}{K_{it}} \right) \frac{\dot{p}_{it}}{p_{it}} \tag{9}
\]
This can be interpreted as the rate of change in current investment cost (in consumption units) of the current capital stock.

Yet another aggregate price index is the user cost-weighted price index \( p^R \),
\[
\frac{\dot{p}^R_t}{p^R_t} = \sum_{i=1}^{n} \left( \frac{R_{it} K_{it}}{Y_t - W_t L_t} \right) \frac{\dot{p}_{it}}{p_{it}} \tag{10}
\]
This can be interpreted as the rate of change in the cost of purchasing capital that delivers the same bundle of capital services. It’s dual to a quantity index aggregated using user costs, which correctly measures the change in aggregate input to the production function. (Note, however, that it’s distinct from an index measuring the aggregate rental cost itself, which requires knowing how \( r_t \) and \( \dot{p}_{it} \) change.)

Which of these indices are most relevant for us? To decide, we’ll have to make more parametric assumptions.

**Cobb-Douglas and balanced growth path**

Suppose that \( F \) is Cobb-Douglas with weights \( \alpha_L, \alpha_{K_1}, \ldots, \alpha_{K_n} \) summing to 1:
\[
Y_t = A_t L^\alpha L_t K_1^{\alpha_{K_1}} \cdots K_n^{\alpha_{K_n}} \tag{11}
\]
Also suppose that there are constant rates of growth \( g_A \equiv \dot{A} / A \) and \( g_{p_i} \equiv \dot{p}_i / p_i \), where the latter corresponds to the inverse of investment-specific technological change. Finally, suppose that we have preferences that guarantee a BGP with constant labor, e.g.
\[
U = \int_0^\infty e^{-\rho t} \left( \log C_t - v(L_t) \right) dt \tag{12}
\]
Since we assume efficiency, (11), (12), and (2) can be treated as an optimization problem.

We have constant \( N_t, C_t / Y_t, \) and \( p_{it} L_{it} / Y_t \) for all \( i \) along the BGP. Let us denote the latter two, shares of aggregate spending on consumption and each investment type, as \( s_C \) and \( s_{I_i} \).

**Result 1: utility effects of unexpected change.** Suppose that we start on a BGP, and that at time 0 there is an unexpected one-time proportional change \( \hat{p}_i \) in prices \( \{ p_{i0} \} \) of investment goods, after which they resume their previous BGP growth rates \( g_{p_i} \) (so that effectively the entire path is shocked by the proportion \( \hat{p}_i \)). How can we measure the effect on utility (12)?

Since we start from an optimum, we can make an envelope argument: suppose that the household makes exactly the same investment decisions, leading to exactly the same paths for \( K_{it} \), and the shocks to the paths of \( p_{it} \) are absorbed by consumption. This should give us the first-order effect on utility.
From (2), the proportional consumption benefit from the shocks \( \{\hat{p}_i\} \) at each date is given by

\[
\frac{dC_t}{C_t} = -\sum_{i=1}^{n} \frac{p_{it}I_{it}}{Y_t} \hat{p}_i
\]

\[= -\sum_{i=1}^{n} s_i \hat{p}_i \tag{13}\]

which is not time-varying! This directly translates, from (12), to a utility effect of

\[-\frac{1}{\rho} \sum_{i=1}^{n} s_i \hat{p}_i \tag{14}\]

but, of course, we might want to stick with (13) as the consumption equivalent.

Letting \( s_I \equiv \sum s_i = I^{cur}/Y = 1 - s_C \) be the share of aggregate spending on investment, we can rewrite (13) as

\[-s_I \sum_{i=1}^{n} \left( \frac{p_{it}I_{it}}{I_{it}^{cur}} \right) \hat{p}_i = -s_I \hat{p}_i \tag{15}\]

where \( \hat{p}_i \) is the proportional shock to the gross investment-weighted price index a la (8).

Hence we conclude that for the date-0 welfare effect of an unexpected shock to investment prices, the proper aggregate measure is given by the usual gross investment-weighted price index, multiplied by the gross investment share.

**Result 2: formula for consumption growth rate, or output growth rate in consumption prices, along BGP.**

Given some constant real interest rate \( r \) on the BGP, we can obtain the user costs

\[R_{it} = (r + \delta_i - g_{pi}) p_{it}\]

and then from Cobb-Douglas (11) we have

\[
\frac{R_{it}K_{it}}{Y_t} = \alpha_{K_i}\]

for each \( i \). It follows that

\[
\frac{K_{it}}{Y_t} = \left( \frac{\alpha_{K_i}}{r + \delta_i - g_{pi}} \right) p_{it}^{-1} \propto p_{it}^{-1} \tag{17}\]

where we get proportionality since the term in parentheses is constant. Then we write

\[
Y_t = A_t L_t^{a_L} K_{1t}^{a_{K_1}} \ldots K_{nt}^{a_{K_n}}
\]

\[
\left( \frac{Y_t}{L_t} \right)^{a_L} = A_t \left( \frac{K_{1t}}{Y_t} \right)^{a_{K_1}} \ldots \left( \frac{K_{nt}}{Y_t} \right)^{a_{K_n}}
\]

\[
Y_t \propto A_t^{a_L - a_L^{a_{K_1}} a_{K_1}} \ldots p_{it}^{-a_L^{a_{K_n}} a_{K_n}}
\]

\[\frac{4}{4}\]
where we make use of $L$ being constant. Remember that $Y_t$ is output in consumption prices, and that $C_t/Y_t$ is constant, so that we have

$$g_C = g_Y = a_L^{-1} (g_A - \alpha_K g_p - \ldots - \alpha_K g_p)$$

$$= a_L^{-1} (g_A - \alpha_K g_p)$$  \hspace{1cm} (18)$$

where we write $\alpha_K = \alpha_K_1 + \ldots + \alpha_K_n$ as the total capital share, and then use the user cost-weighted price index $p^R$ from (10), whose growth rate is the proper measure for the contribution of investment-specific technological change to either consumption growth or output growth in consumption units.\(^{1}\)

Result 3: user cost index as the average of stock and flow-weighted indices. From (17) we have

$$\frac{p_{it} K_{it}}{Y_t} = \frac{\alpha_{K_i}}{r + \delta_i - g_{p_i}}$$

Note that the right is constant. Log-differentiating with respect to time, we have

$$\frac{\dot{p}_t}{p_t} + \frac{\dot{K}_{it}}{K_{it}} \frac{Y_t}{Y_t} = 0$$

$$\frac{\dot{K}_{it}}{K_{it}} = g_Y - g_{p_i}$$

Gross investment expenditure, in consumption goods, on capital type $i$ is then

$$p_{it} I_{it} = p_{it} \dot{K}_{it} + p_{it} \delta_i K_{it} = (g_Y + \delta_i - g_{p_i}) p_{it} K_{it}$$  \hspace{1cm} (19)$$

Note that the weights on $\frac{p_{it}}{p_t}$ in the user cost-weighted (10) price index are proportional to, in our BGP,

$$(r + \delta_i - g_{p_i}) p_{it} K_{it} = (r - g_Y) p_{it} K_{it} + (g_Y + \delta_i - g_{p_i}) p_{it} K_{it}$$  \hspace{1cm} (20)$$

where the first term on the right is proportional to the weight in the capital stock-weighted index (9) and the second term on the right is proportional to investment expenditure (19) and therefore to the weight in the gross investment-weighted index (8). And adding up the left across all $i$ gives $\alpha_K Y_t$, the total capital share of income, while adding up the right term across all $i$ gives $s_I Y_t$, the total investment share of output.

Putting all this together, we can write

$$g_{p^R} = \left(1 - \frac{s_I}{\alpha_K}\right) g_{p^L} + \frac{s_I}{\alpha_K} g_{p^I}$$  \hspace{1cm} (21)$$

i.e. the rate of change in the user cost-weighted price index $p^R$ equals the rate of change in the investment-weighted price index $p^I$, weighted by investment spending over capital income, plus the rate of change in the stock-weighted price index $p^K$, weighted by the remainder.

This offers a simple way to get at $g_{p^R}$ in terms of aggregates that are more often computed.

\(^{1}\)Output in consumption prices is the measure used by Greenwood, Hercowitz, and Krusell (1997). A more conventional measure of real output growth, like than in national accounts, would use the gross output deflator instead of the consumption deflator, delivering an additional term $-s_I g_{p_i}$ in (18) equal to the share of gross investment in gross output times the growth rate of the gross investment-weighted deflator.
Result 4: characterizing net investment. Note that net investment is, from (19),

\[ p_{it} I_{it} - p_{it} \delta_{it} K_{it} = (g_Y - g_{p_i}) p_{it} K_{it} \]

Summing up across all \( i \) and looking as a share of \( Y \), we get

\[ g_Y \sum_{i=1}^{n} \frac{p_{it} K_{it}}{Y_t} - \sum_{i=1}^{n} \frac{g_{p_i} p_{it} K_{it}}{K_{i}^{cur} / Y_t} = \left( g_Y - \sum_{i=1}^{n} \frac{g_{p_i} p_{it} K_{it}}{K_{i}^{cur} / Y_t} \right) \frac{K_{i}^{cur}}{Y_t} \]

so that net investment to GDP equals aggregate growth minus the stock-weighted rate of change in investment prices, times the capital-output ratio. (Of course, \( K_{i}^{cur} / Y_t \) is endogenous to investment prices.)

Summary. We pointed out the existence of three interesting aggregate price indices for capital: gross investment-weighted (8), stock-weighted (9), and user cost-weighted (10).

Given balanced-growth assumptions, we have shown:

- The investment-weighted index is relevant for measuring the date-0 welfare effect of a surprise one-time permanent change in investment prices (result 1).
- The user cost-weighted index is relevant for measuring the effect of investment price growth rates in a BGP on the rate of consumption or output (in consumption prices) growth (result 2).
- This user cost-weighted index can be viewed as an average of the investment and stock-weighted indices (result 3).
- The stock-weighted index is relevant for measuring the effect of investment price growth rates in a BGP on the rate of net investment, given the nominal capital-output ratio \( K_{i}^{cur} / Y_t \) (result 4).
3 Measurement

3.1 Data
3.2 Prices
3.3 Quantities

4 Long Run Growth

4.1 Accounting
4.2 Equilibrium analysis

5 Business Cycles: VAR analysis

5.1 Data and replication of Fisher [2006]
5.2 Results

6 The decline of investment

7 Conclusion

References


